Phys 551 Homework 4

1. Density matrix

Consider a spin=1 system with a Hamiltonian given by $H = \mu_B \mathbf{B} \cdot \mathbf{J}$. The density matrix ρ(t) is given at t = 0 by

$$\langle m|\rho(0)|m'\rangle = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

in the basis with quantization axis along the *z* direction.

a) Use Liouville equation to find the evolution of the density matrix as a function of time for $\mathbf{B} = B_0 \hat{x}$. Evaluate the form of the density matrix at $t = \pi \hbar / 2\mu B$.

b) Decompose the initial density matrix ρ(0) into irreducible spherical tensors and apply a rotation by π/2 around the *x* axis. How does the form of the density matrix after rotation compare with part a)?

2. Spin squeezed state

Consider a state of spin J described by a wavefunction $|\psi\rangle = \cos\theta |J, J\rangle + \sin\theta |J, -J\rangle$ in the $|J, m_J\rangle$ basis. Evaluate $\langle J_z \rangle$, ΔJ_x, ΔJ_y and find values of θ for which the state has reduced uncertainty $\Delta J_x < \left(\langle J_z \rangle / 2\right)^{1/2}$. Verify that the Heisenberg uncertainty relationship $\Delta J_x \Delta J_y \geq |\langle J_z \rangle| / 2$ is always satisfied.