HOLOGRAPHY

Introduction and Background

The aesthetic appeal and commercial usefulness of holography are both related to the ability of a hologram to store a three-dimensional image. Unlike ordinary photographs, holograms record both phase and amplitude information. Because phase is a relative property, construction of a hologram requires a "reference beam" in addition to the light reflected from an object's surface. Additional requirements include a powerful, coherent, monochromatic light source (*e.g.* a laser, whose workings you should understand); a vibration-free flat surface; some common optical devices; and photographic plates, chemicals, and a darkroom.

A hologram can be made by exposing a photographic plate to the interference pattern made by the reference and object beams. The plate is then developed and dried. When illuminated by a reference beam similar to the original one, it recreates a three dimensional picture of the object. There is nothing mystical about this recreation, and you can understand how it works with the help of some Fourier transform mathematics.

Mathematical Overview

Consider the light waves reflected from an object (see Figure 1). The details of this "object beam" obviously depend upon the shape and nature of the particular object. We will take a general approach and say that the object beam is a superposition of plane waves,

$$\psi_0(\mathrm{r},t) = A_0 \iint F_0(k_x,k_y) e^{i(\omega t - k_x x - k_y y - k_z z)} dk_x dk_y \,,$$

where $(k_x^2 + k_y^2 + k_z^2)^{1/2} = k = 2\pi/\lambda$. Let's assume that we have permitted ψ_0 to interfere with a reference beam given by

$$\psi_1(\mathrm{r},t) = A_1 e^{i(\omega t - kz)} \,,$$

and recorded the resulting intensity pattern on a photographic plate in the plane $z = 0$. The recorded intensity will be

$$I(x,y) = |A_1 + A_0 \iint F_0(k_x,k_y) e^{-i(k_x x - k_y y)} dk_x dk_y|^2 \,.$$

When the right-hand side of this equation is expanded, it will have terms proportional to $|\psi_0|^2$, $|\psi_1|^2$, and $|\psi_0 \psi_1^*|$.

We will assume that the reference beam is much more intense than the object beam, so that $|\psi_1| >> |\psi_0|$. Then the term in $|\psi_0|^2$ can be ignored, and the intensity can be written

$$I(x,y) \approx |a_1|^2 + A_0 A_1^* \iint F_0(k_x,k_y) e^{-i(k_x x + k_y y)} dk_x dk_y + A_1 A_0^* \iint F_0(k_x,k_y) e^{+i(k_x x + k_y y)} dk_x dk_y \,.$$

The developed plate will have a transmission function $T(x,y) = 1 - \gamma I(x,y)$, where γ is a function of the exposure and developing processes. The hologram will therefore have a transmission function

$$T(x,y) = C_0 - \gamma A_0 A_1^* \iint F_0(k_x,k_y) e^{-i(k_x x + k_y y)} dk_x dk_y - \gamma A_1 A_0^* \iint F_0(k_x,k_y) e^{+i(k_x x + k_y y)} dk_x dk_y \,,$$

where C_0 is a constant. Put in words, the hologram contains scaled versions of the two-dimensional Fourier transform of the object beam, and the inverse two-dimensional Fourier transform of the

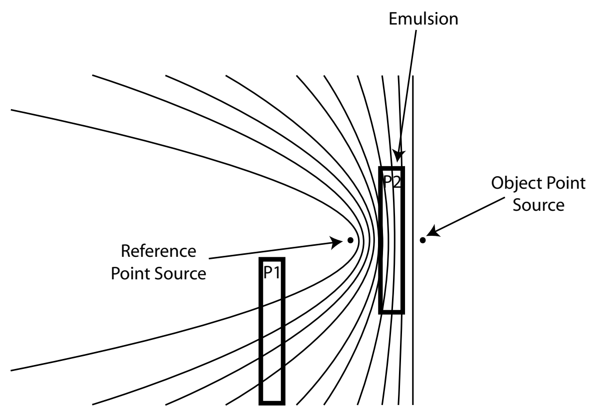

Figure 1: A two dimensional projection of the interference patterns produced. A photographic plate at P1 would result in a transmission hologram, a plate at P2 in a reflection hologram.

object beam, superimposed on each other. If you illuminate this hologram with a plane wave like the original reference beam, another Fourier transformation takes place, and what emerges is

$$\psi(\mathrm{r}, t) = \iint F(k'_x, k'_y) e^{i(\omega t - k'_x x - k'_y y - k'_z z)} dk'_x dk'_y$$

where, as is usual in Fourier optics,

$$F(k'_x, k'_y) = \left(\frac{1}{2\pi}\right)^2 \iint T(x, y) e^{i(k'_x x + k'_y y)} dx dy \,.$$

The three terms constituting $T(x, y)$ obligingly integrate as delta functions (the term we threw out above would not have been so obliging), and we find

$$F(k'_x, k'_y) = -\gamma A_1^* A_0 F_0(k'_x, k'_y) - \gamma A_1 A_0^* F_0^*(-k'_x, -k'_y) + C_0 \delta(k'_x) \delta(k'_y) \,.$$

Therefore the wave that emerges from the suitably illuminated hologram (see Figure 2) consists of three parts: (1) a term proportional to the original object beam, reconstructing the object in all details of amplitude and phase; (2) a so-called "twin reconstruction" that can be shown to be the original object beam reflected in the plane of the photographic plate and run backwards in time; and (3) an undeflected piece of the reference beam traveling along the z axis.

Transmission and Reflection Holograms

The first holograms were examples of transmission holography. Transmission holograms require a monochromatic reconstructive beam. In other words, the image is only visible when the hologram is illuminated by either a laser or other single frequency source. This limitation arises from the way a hologram stores information. The interference patterns produced from an object and reference beam, whose formalism has been derived above, are actually three dimensional hyperbolae of revolution[3]. The position of the photographic plate relative to the object determines how this interference pattern is recorded onto the photographic emulsion.

Figure 1 shows two possible placements of the photographic emulsion relative to the object. Position one (P1) shows the development of a transmission hologram. Note how the interference pattern is recorded perpendicular to the plane of the plate. Consequently, any frequency of light

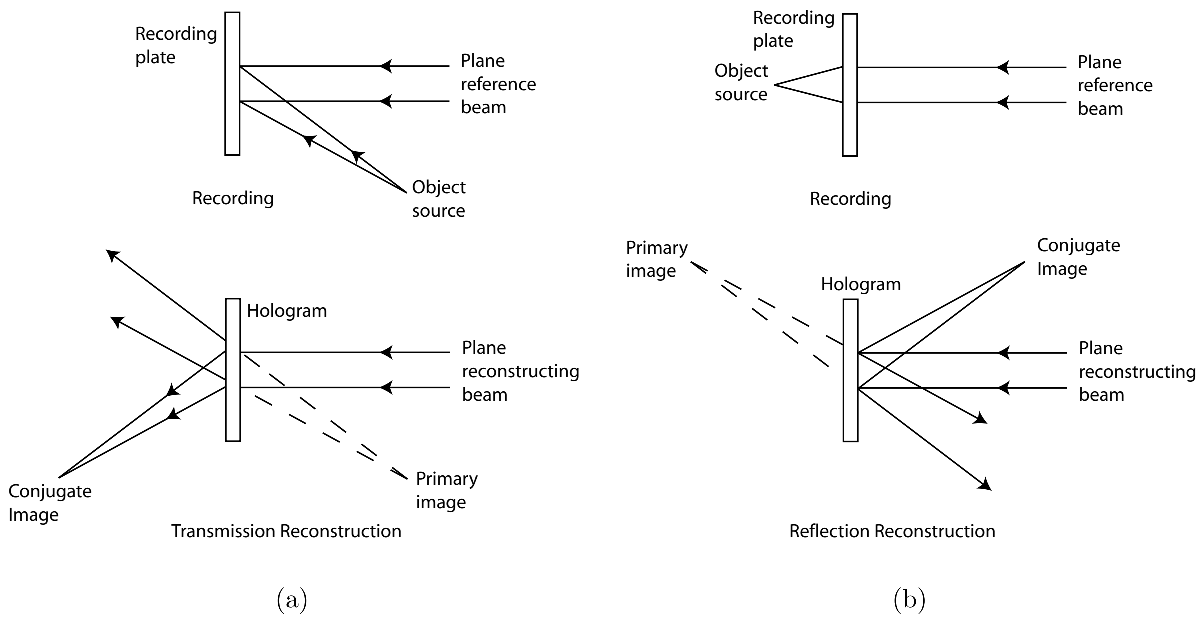

Figure 2: The recording and reconstruction setups for (a) transmission and (b) reflection holograms. The transmission figure was taken from [3].

can pass through the plate. If this hologram were viewed in white light, *every* frequency would create an image, the combined effect simply being a wash-out. This is why transmission holograms are not visible in white light.

However if we place the photographic plate at P2, the interference pattern is recorded *parallel* to the plate and along the thickness of the emulsion. This thickness, irrelevant to transmission holograms is essential for reflection holograms. Reflection holograms get their name from the fact that instead of creating an image by transforming the reconstructive beam into a replica of the object beam as it passes through the plate, they reflect a reconstruction of the object beam. Figure 2 illustrates the difference between the two types of holograms.

The unique characteristic of reflection holograms is that they can be viewed in white light. Therefore reflection holograms are commonly known as white light holograms. Exactly how does this happen? As we mentioned earlier, the thickness of the emulsion is crucial. In reflection holography, the interference pattern is recorded along the thickness of the emulsion, creating what are known as Bragg diffraction planes. These planes act similarly to half-silvered mirrors: some of the light used to illuminate the object reflects off each embedded interference fringe. As shown in figure 3, there is a path difference between light rays reflecting off different Bragg diffraction planes. These reflected rays then interfere with each other, and if they are not in phase, the interference will be destructive. Consequently, the only light rays reflected off a Bragg diffraction grating have path length differences equal to an integral number of wavelengths.

The geometry shown in figure 3 allows us to derive a quantitative description of Bragg diffraction [5]. The beam passing through the top grating and reflecting off the lower layer travels a distance $AB + BC$ longer than the beam reflected off the top layer. The angle of incidence θ is equal to the angle of reflection, so $AB = BC = d \sin\theta$. The path length difference must be equal to an integral multiple of wavelengths for constructive interference to occur. Therefore, the condition for

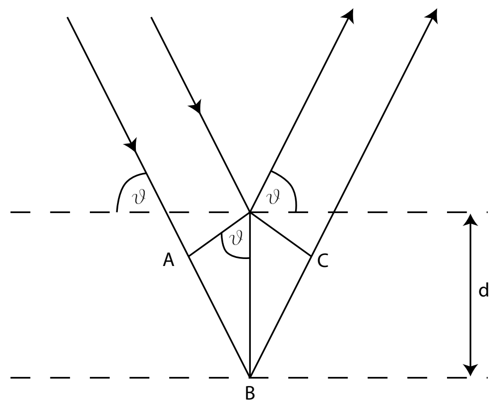

Figure 3: Diagram representing Bragg diffraction. The path length difference is $AB - BC$ and the angle of reflection is θ. Figure taken from [5].

constructive interference is

$$n\lambda = 2d\sin\theta, \tag{1}$$

which is known as Bragg's Law.

Bragg's law allows only a particular wavelength to be reflected at a specific angle. Therefore, when viewed in white light, a reflection hologram will essentially select one particular frequency to reconstruct the image, preventing the wash-out effect seen in transmission holograms.

Additional Details

Following certain procedures will help you to ensure success in making your holograms.

1. Recall that in the derivation above we ignored the contribution of $|\psi_0|^2$ in comparison with $|\psi_1|^2$; that is, we assumed that the reference beam was more intense than the object beam. However, the derivation did not tell us *how much* more intense the reference beam should be. It has been found empirically that a reference/object intensity ratio between 3 and 10 yields good results. A photocell is provided to measure the intensities of the reference and object beams.

2. Higher intensities (consistent with a suitable reference/object ratio), will need shorter exposure times, and, will, in general, produce better holograms.

3. A clean, uniform reference beam is needed for a high quality hologram. A pinhole apparatus can help you achieve this goal. Set it up carefully in order to minimize the diffraction and interference patterns from imperfect optical surfaces (mirrors, beam splitters, and lenses) in the path of the reference beam. In addition, you should make sure that the optical surfaces are clean and free of dust.

4. The path lengths of the reference and object beams should differ by less that the "coherence length" of the laser. You should understand what this means about the purity of the laser light.

Experimental Preparations

1. CAUTION: be careful with laser light! Never look directly into the undiverged beam.

2. During initial checkout of the laser and optical devices, set up a Michelson interferometer so that you can gain some familiarity with the various pieces of equipment. Demonstrate interference phenomena to yourselves, and investigate the coherence length of this particular helium-neon laser. Some questions for thought: why does the interference pattern show up as concentric rings of light and dark zones? Why do the zone spacings depend as they do on the path-length difference in the two arms of the interferometer? What happens if you increase the path-length difference well beyond the coherence length? What does this behavior suggest about the spectral distribution of the laser light? Think carefully about what you are doing here, and record your measurements and the details of your interferometer setup to the necessary degree of accuracy for at least a semi-quantitative result.

3. Spend some time playing with the lens-pinhole apparatus used for generating a clean reference beam. The pinhole acts as a "spatial filter" to assure that you have clean wavefronts, and makes the system relatively forgiving of dirty or imperfect optics upstream of the pinhole.

4. Now you can make some holograms! Instructions for the photography aspects of the experiment can be found in the darkroom. After you have made a successful hologram following this prescription, vary the parameters. For example, you can see if, indeed, the pinhole makes a difference. You can adjust the reference/object intensity ratio, to see how magical are the limits 3 to 10. And you can play with the exposure times or try unequal path lengths for the object and reference beams. Use your imagination to discover which factors are physically important, and discuss your findings in your write-up.

5. Here are some questions to consider when making white light holograms. What emulsion thickness is required to make white light holograms? What does this specifically mean for our setup? Check to make sure that the photographic plates fulfill these requirements. Can white light holograms still be viewed with laser light, and what differences are there between reflection and transmission holograms when illuminated by laser light? When viewed in white light, your hologram will appear in different colors when viewed at different angles. Why is this? When doing white light holography, is it more or less important to have a strong reference beam? How should exposure time be different for white light versus transmission holograms?

6. There are several variations of the traditional white light holography setup. Things you might want to try are using two object beams or recording different objects at different angles. What factors are important when using two object beams? How can you avoid contaminating your beam with a more complicated optical setup? See what other creative setups you can think of.

References

[1] Gabor, D. 1972, "Holography, 1948-1971," *Science,* **177,** 299-313 .

[2] Jeong, T. H. 1970, "Holograms," in *Physics Demonstration Experiments,* ed. H. F. Meiners (New York: Ronald Press), vol. 2, chapter 37.

[3] Beesley, M. J., 1971. *Lasers and their Applications,* (London: Taylor and Francis), chapter 12.

[4] *Lasers and Light,* (San Francisco: W.H. Freeman and Company, 1969), especially Sections 6 and 7.

[5] Schields, P. "Bragg's Law and Diffraction". *http://www.journey.sunysb.edu /ProjectJava/Bragg/home.html*. 4/10/02.

The paper by Gabor is the author's Nobel Prize address. Jeong gives a very helpful discussion of experimental approaches, and the last reference is a collection of articles from *Scientific American.*

Revised January, 1990
J. H. Taylor
Revised April, 2002
A. Erickcek, Z. Kermish