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New Experimental Test of Coulomb's Law: A Laboratory Upper Limit on the Photon Rest Mass

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A high-frequency test of Coulomb's law is described. The sensitivity of the experiment is given in terms of a finite photon rest mass using the Proca equations. The null result of our measurement expressed in the form of the photon rest mass squared is $\mu^2 = (1.04 \pm 1.2)\times 10^{-19}$ cm^{-2}. Expressed as a deviation from Coulomb's law of the form $1/r^{2+q}$, our experiment gives $q = (2.7 \pm 3.1)\times 10^{-16}$. This result extends the validity of Coulomb's law by two orders of magnitude.

The testing of Coulomb's law (Gauss's law) by means of a null experiment dates back to Cavendish (1773).[1] The now classical test of Plimpton and Lawton[2] was performed in 1936, and showed that any difference in the exponent from 2 was smaller than 1×10^{-9}. Recently two other groups have extended the accuracy of that result by two[3,4] and four[5] orders of magnitude. The result reported here represents an extension in accuracy over that obtained by Plimpton and Lawton by six orders of magnitude.

The experiment described here is a "high-frequency" null test of Coulomb's law. We make use of the fact that a $1/r^2$ force law does not give rise to any electric field on the inside of a closed conductor. A conducting shell that is about $1\frac{1}{2}$ m in diameter is charged to 10 kV peak to peak with a 4-MHz sinusoidal voltage. Centered inside of this charged conducting shell is a smaller conducting shell. Any deviation from the $1/r^2$ force law is detected by measuring the line integral of the electric field between these two shells with a detection sensitivity of about 10^{-12} V peak to peak.

The results of the experiment can be expressed in terms of the Proca equations,[6,7] a relativistically invariant linear generalization of Maxwell's equations, which are appropriate to describe the experimental system when a finite rest mass is assumed. Proca's equations for a particle of spin 1 and mass m_0 are[8]

$$(\Box + \mu^2)A_\nu = (4\pi/c)J_\nu, \tag{1}$$

where $\mu = m_0 c/\hbar$. In three-dimensional notation, Gauss's law becomes

$$\nabla\cdot\vec{E} = 4\pi\rho - \mu^2\varphi. \tag{2}$$

To calculate the sensitivity of the system, consider an idealized geometry consisting of two concentric, conducting, spherical shells of radii R_1 and R_2 ($R_2 > R_1$) with an inductor across (in parallel with) this spherical capacitor. To the outer shell is applied a potential $V_0 e^{i\omega t}$. An iterative solution for the field between the spheres may easily be found. Forming a spherical Gaussian surface at radius r between the two shells and then using the approximation $\varphi(r) = V_0 e^{i\omega t}$ for this interior region, the integral of Eq. (2) over the volume interior to the Gaussian surface becomes

$$\int[\nabla\cdot\vec{E} - 4\pi\rho + \mu^2 V_0 e^{i\omega t}]d^3x = 0. \tag{3}$$

Therefore $\vec{E}(r)$ is given by

$$\vec{E}(r) = (qr^{-2} - \tfrac{1}{3}\mu^2 V_0 e^{i\omega t} r)\hat{r}, \tag{4}$$

where q is the total charge on the inner shell.

A complete solution of the fields inside a symmetrically charged single sphere of radius R_2 gives[9,10]

$$\vec{E}(r) = \frac{\mu^2}{k^2r^2}\frac{R_2V_0e^{i\omega t}}{e^{-ikR_2}-e^{ikR_2}} \times [ikr(e^{-ikr}+e^{ikr})-(e^{-ikr}-e^{ikr})]\hat{r} \tag{5}$$

and also $\vec{H}=0$. Here $k^2=\omega^2/c^2-\mu^2$. A power series expansion of $\vec{E}(r)$ where $kr<1$ [near zone] and $\omega/c>\mu$ gives

$$\vec{E}(r) = -\tfrac{1}{3}\mu^2V_0e^{i\omega t}r(1-\tfrac{1}{10}k^2r^2+\tfrac{1}{6}k^2R_2^2\cdots)\hat{r}. \tag{6}$$

On neglecting the second-order term, which for this experiment produces an error of less than 1%, this equation reduces to a nonzero rest mass term which is the same as was derived rather simply above.

Since $\partial\vec{H}/\partial t$ is zero inside, $\oint\vec{E}\cdot dl=0$. The voltage appearing across the inductor is then simply given by

$$\int_{R_1}^{R_2}\vec{E}\cdot d\vec{l} = \frac{q}{C} - \mu^2\frac{V_0e^{i\omega t}}{6}(R_2^2-R_1^2). \tag{7}$$

The differential equation which describes this LC circuit in the case of a nonzero rest mass is then

$$L\frac{d^2q}{dt^2}+r\frac{dq}{dt}+\frac{q}{C}=\mu^2\frac{[V_0e^{i\omega t}]}{6}(R_2^2-R_1^2). \tag{8}$$

The signal-to-noise ratio of a system described by this differential equation can be analyzed using conventional circuit theory. On doing this it turns out that the use of a high frequency, high-Q circuits, a large apparatus, and as high as possible applied voltage V_0 serve to maximize the experimental sensitivity.

The experimental apparatus (Fig. 1) consists of five concentric icosahedrons. A 4-MHz rf voltage between the outer two shells (4 and 5) is obtained by pumping energy into the resonant circuit formed by the two shells and a high-Q water cooled coil. A peak-to-peak voltage of 10 kV was achieved. A battery-powered lock-in amplifier located inside the innermost shell is used to detect the voltage appearing across the inductor. The reference signal for this phase detector derived directly from the voltage on the outer charged shell is phase shifted continuously at a linear rate of 720°/h ($\varphi'=\omega't$) and sent in-

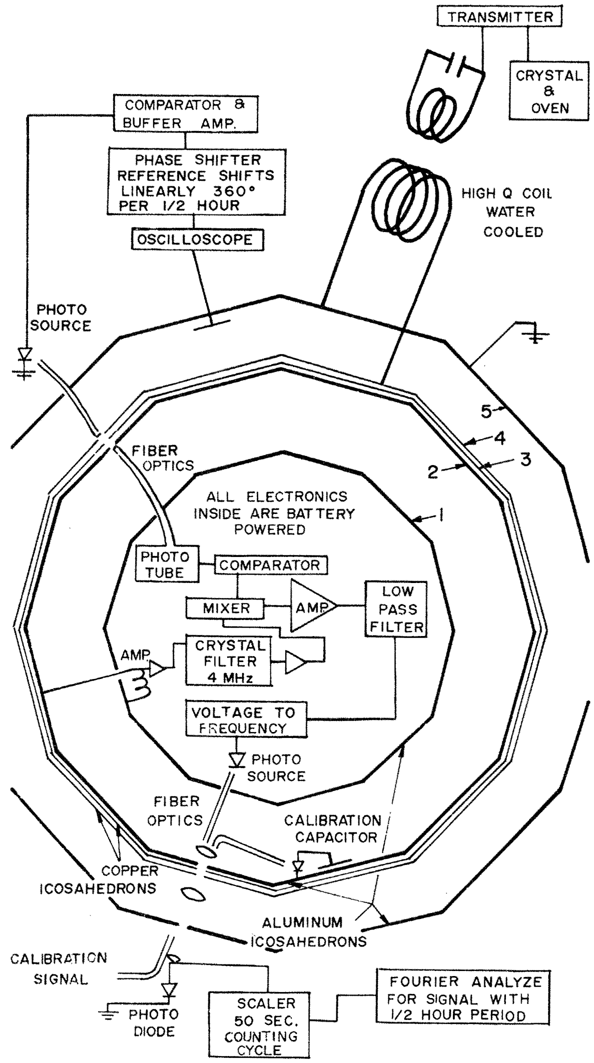

FIG. 1. Schematic drawing of the apparatus. A 4-MHz voltage is applied between shells 4 and 5. A signal of less than 10^{-12} V could have been detected between shells 1 and 2.

side on a light beam. The optical photons are passed through a small pin hole which does not pass the rf for which the hole constitutes a waveguide below cutoff frequency. The output of the lock-in is amplified, sent through a voltage-to-frequency converter, and then returned outside the shells on another light beam. This output frequency is detected and counted for 50-sec intervals. Finally this digital output is Fourier analyzed for the phase-shifting frequency ω'.

Stray electric and magnetic fields are shielded against by the skin effect in the conducting shells which attenuates them exponentially with the distance of perpendicular penetration into the conductor. In practice this shielding is limited not

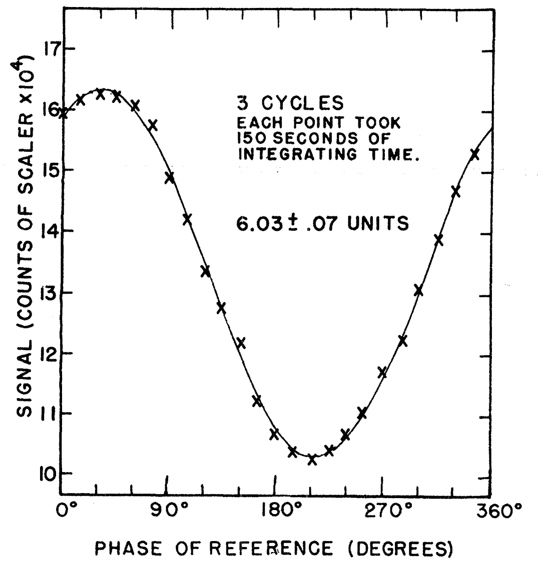

FIG. 2. Signal output of the system versus phase of the reference signal when a calibrating voltage of 3.8×10^{-10} V is introduced between the two icosahedrons.

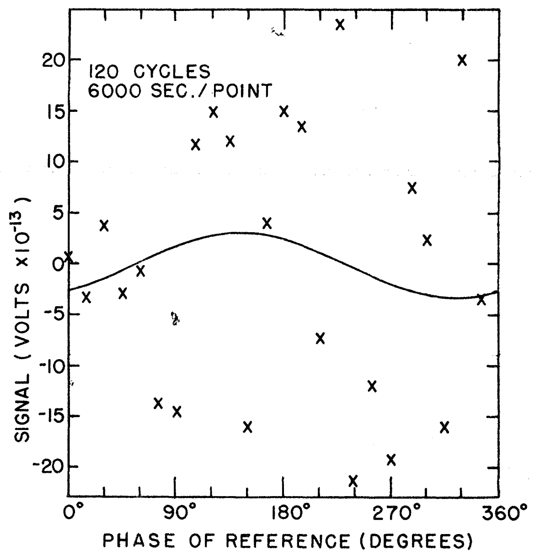

FIG. 3. Data from which our experimental result is derived. The solid curve is a least squares fit of the data by a sinusoid of angular frequency ω', the phase-shifting frequency. The amplitude of this fitted curve is $(6.4 \pm 7.3) \times 10^{-13}$ V peak to peak.

by the number of skin depths but by the soldered seams which are not perfect. By using three shells for shielding, this limitation was overcome. The fields which exist inside any one shell due to these leaks are shielded out to a great extent by the next shell.

All fields that are consistent with Maxwell's equations can be shielded out; the radially symmetric field predicted by Proca's equations, however, cannot be shielded. From Eq. (2) radially symmetric fields can be produced by a net charge on the innermost shell or by a violation such as the form suggested by Proca.

To ensure that adequate shielding has been obtained for a data run the following procedure is followed. Before the last triangular member of the outermost copper shell (No. 4) is soldered into place, the high voltage is applied. On doing this, a leakage signal of about 2×10^{-9} V is observed. Since a similar shell (No. 3) gave about seven orders of magnitude of shielding on completely closing it shut, on soldering the last member of this outer shell (No. 4) into place, any signal that might result because of leaks should be well below the observed Johnson noise level of approximately 10^{-12} V.

During a data run to ensure that the gain and phase of the amplifier have remained constant, a calibration voltage is periodically introduced into the system on a third light beam. This beam on striking a light-sensitive diode induces a voltage on a capacitance divider which is located just inside shell No. 2.

Figure 2 shows the output averaged over three cycles of ω' when a calibration signal of 3.8×10^{-10} V peak to peak is inserted.

The results of a three-day run are given in Fig. 3. The curve is a least squares fit of a sinusoidal wave of frequency ω' to the data. The result for this run, expressed in the form of a rest mass squared, is given by $\mu^2 = (1.04 \pm 1.2) \times 10^{-19}$ cm^{-2}. This result is consistent with an earlier though somewhat less sensitive 3-day run and includes all known uncertainties. Our experimental result is statistically consistent with the assumption that the photon rest mass is identically zero.

Expressed as a deviation from Coulomb's law in the form $1/r^{2+q}$ as was suggested by Maxwell,[11] our result gives

$$q = (2.7 \pm 3.1) \times 10^{-16}.$$

As a test of Coulomb's law, this measurement is the most stringent to date (see Table I).

The value we obtained for μ^2 complements a geomagnetic limit for a photon rest mass which was first derived by Schrödinger.[13] As a test of Gauss's law rather than Ampere's law, the work reported here represents an independent test.

Table I. Results of various tests of Coulomb's law and tests for a nonzero photon rest mass.

	Coulomb's Law violation of form r^{2+q} q	$\mu^2 = (\frac{m_o c}{\hbar})^2$	Photon rest mass m_o
Cavendish (1773)	2×10^{-2}		
Coulomb (1785)	4×10^{-2}		
Maxwell (1873)	4.9×10^{-5}		
Plimpton and Lawton (1936)	2.0×10^{-9}	1.0×10^{-12} cm^{-2}	$\leq 3.4 \times 10^{-44}$ g
Cochran and Franken (1967)	9.2×10^{-12}	7.3×10^{-15} cm^{-2}	$\leq 3 \times 10^{-45}$ g
Bartlett, Goldhagen, Phillips (1970)	1.3×10^{-13}	1×10^{-16} cm^{-2}	$\leq 3 \times 10^{-46}$ g
Williams, Faller, Hill	$(2.7 \pm 3.1) \times 10^{-16}$	$(1.04 \pm 1.2) \times 10^{-19}$ cm^{-2}	$\leq 1.6 \times 10^{-47}$ g
Schroedinger (1943)	Test of Ampere's Law from Geomagnetic Data	3×10^{-19} cm^{-2}	$\sim 2 \times 10^{-47}$ g
Gintsburg (1963)	Test of Ampere's Law from Geomagnetic Data	5×10^{-20} cm^{-2}	$\leq 8 \times 10^{-48}$ g
Nieto and Goldhaber (1968)	Test of Ampere's Law from Geomagnetic Data	1.3×10^{-20} cm^{-2}	$\leq 4 \times 10^{-48}$ g
Feinberg (1969) [a]	Dispersion of light	8×10^{-14} cm^{-2}	10^{-44} g

[a]Feinberg, Ref. 12.

And though in terms of a rest mass it does not quite equal the sensitivity which has been obtained from the most recent analysis[8,14] of Earth's geomagnetic data, as a laboratory test it has the advantage that all of the experimental parameters are controlled and can be individually tested and varied. A very sensitive test using an *LCR* circuit has just recently been reported[15]; however, the authors themselves express reservations about the precise interpretation of their results.

The theoretical implications of a nonzero photon rest mass if one were found to exist would be considerable. In addition, the existence of a finite rest mass could have some practical importance when describing the magnetic field of large bodies such as Jupiter and the sun.[7] A further extension of the limit on the rest mass via laboratory tests of Coulomb's law though increasingly difficult is experimentally feasible. In view of a number of experimental unknowns which preclude much if any extension of this limit using Schrödinger's method, the greatest promise for further improving on the rest mass limit may well rest with precision tests of Coulomb's law. To this end, continuing effort on Cavendish-type experiments would appear to be worthwhile.

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