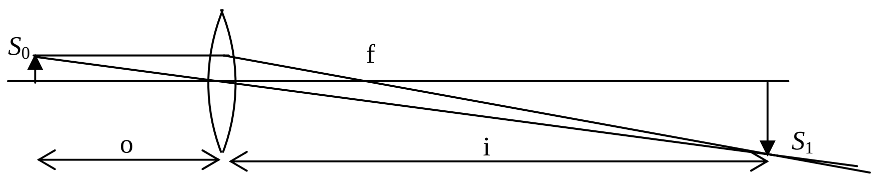
PHYS 210
Spring 2006
Optics and Microscopy

In this lecture we will briefly discuss optics and various types of microscopes.

Geometric optics



Simple equations for the effects of thin lenses can be obtained following the rules of geometrical optics:

1) Rays going parallel to the axis on one side of the lens pass through the focal point on the other side
2) Rays going through the center of the lens are not deflected.

From these rules it follows from simple geometry that

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$$

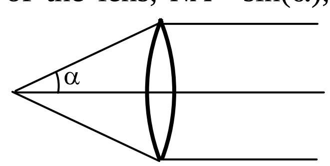
where *o* is the distance from the object to a convergent lens, *i* is the distance from the lens to the image and *f* is the focal length of the lens. Similarly, one can show that the magnification is $S_1/S_0 = i/o$. For example, to obtain large magnification one needs *iπo* and therefore *o*~ *f*.

Wave optics

On length scales comparable to the wavelength of light its propagation is no longer governed by geometric optics. Light diffraction begins to play a large role, for example, allowing the light to pass around small obstruction. Diffraction also limits the minimum size of an object that can be seen with light to approximately the wavelength of light.
More precise calculations show that the spatial resolution of a microscope is given by

$$d_{\min} = 1.22\lambda / NA,$$

where the numerical aperture *NA* defines the light collection ability of the lens, *NA*= sin(α), where α is the opening angle, see picture to the right. Numerical aperture is larger for shorter focal length lenses, but is generally smaller than 1. Thus the maximum spatial resolution that can be obtained with visible light is on the order of 0.5 μm.



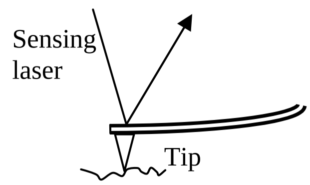
Wave Microscopy

As shown above, the maximum resolution one can obtain with optical microscope is on the order of 0.5 μm. It can be improved somewhat using shorter wavelength and sub-wavelength interference tricks. For example, optical lithography used in modern chip manufacturing reaches resolution of about 50 nm with laser wavelength of 193 nm. Another way to increase the resolution is to use other particles, such as electrons, with short de Broglie wavelength. An electron accelerated to 200 keV, for example, has a de Broglie wavelength $\lambda = h/p = 5\,\text{pm}$.

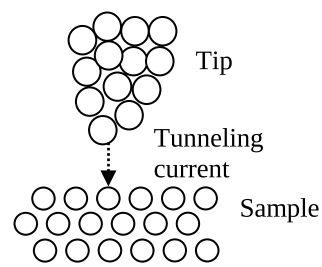
Instead of lenses, electric and magnetic fields are used to shape and image electron beams. However, since possible configurations of electromagnetic fields are limited by Maxwell equations, it is much harder to achieve diffraction-limited performance. Only recently development of effective correction coils for spherical aberration enabled resolution below 0.1 nm.

Force microscopy

Another way to image the top layer of a sample is to literally drag a mechanical tip along the surface. This technique is known as atomic force microscopy. The height of the tip can be measured using laser interferometry with better than 0.1 nm resolution. A surface image is obtained by scanning the tip across the sample. The horizontal resolution is about 1 nm, limited by how sharp the tip is, typically a few atoms across.



A higher resolution image for conductive samples can be obtained using electron quantum tunneling effects. The main features of quantum tunneling can be obtained from a simple solution to the Schrödinger equation assuming a constant potential V>E in the gap between the conducting tip and the sample:



$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)+V\psi(x)=E\psi(x)$$

$$\psi(x)=\psi(0)e^{-kx}$$

$$k=\frac{\sqrt{2m(V-E)}}{\hbar}$$

$$P(x)=|\psi(x)|^2=|\psi(0)|^2e^{-2kx}$$

Hence, the tunneling current depends exponentially on the distance between the tip and the sample. This allows one to obtain sub-atomic level horizontal resolution because electrons will tunnel only from the very tip of the sample. Scanning tunneling microscopes (STM) are fairly simple and with good vibration isolation can achieve a resolution of 0.1 nm or better.