Introduction to Econometrics (3rd Updated Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 13

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13.1. For students in kindergarten, the estimated small class treatment effect relative to being in a regular class is an increase of 13.90 points on the test with a standard error 2.45. The 95% confidence interval is $13.90 \pm 1.96 \times 2.45 = [9.098, 18.702]$.

For students in grade 1, the estimated small class treatment effect relative to being in a regular class is an increase of 29.78 points on the test with a standard error 2.83. The 95% confidence interval is $29.78 \pm 1.96 \times 2.83 = [24.233, 35.327]$.

For students in grade 2, the estimated small class treatment effect relative to being in a regular class is an increase of 19.39 points on the test with a standard error 2.71. The 95% confidence interval is $19.39 \pm 1.96 \times 2.71 = [14.078, 24.702]$.

For students in grade 3, the estimated small class treatment effect relative to being in a regular class is an increase of 15.59 points on the test with a standard error 2.40. The 95% confidence interval is $15.59 \pm 1.96 \times 2.40 = [10.886, 20.294]$.

13.3. (a) The estimated average treatment effect is $\bar{X}_{TreatmentGroup} - \bar{X}_{Control}$ = 1241 - 1201 = 40 points.

(b) There would be nonrandom assignment if men (or women) had different probabilities of being assigned to the treatment and control groups. Let p_{Men} denote the probability that a male is assigned to the treatment group. Random assignment means p_{Men} = 0.5. Testing this null hypothesis results in a t-statistic of $t_{Men} = \frac{\hat{p}_{Men} - 0.5}{\sqrt{\frac{1}{n_{men}}\hat{p}_{Men}(1-\hat{p}_{Men})}} = \frac{0.55-0.5}{\sqrt{\frac{1}{100}0.55(1-45)}} = 1.00$, so that the null of random assignment cannot be rejected at the 10% level. A similar result is found for women.

13.5. (a) This is an example of attrition, which poses a threat to internal validity. After the male athletes leave the experiment, the remaining subjects are representative of a population that excludes male athletes. If the average causal effect for this population is the same as the average causal effect for the population that includes the male athletes, then the attrition does not affect the internal validity of the experiment. On the other hand, if the average causal effect for male athletes differs from the rest of population, internal validity has been compromised.

(b) This is an example of partial compliance which is a threat to internal validity. The local area network is a failure to follow treatment protocol, and this leads to bias in the OLS estimator of the average causal effect.

(c) This poses no threat to internal validity. As stated, the study is focused on the effect of dorm room Internet connections. The treatment is making the connections available in the room; the treatment is not the use of the Internet. Thus, the art majors received the treatment (although they chose not to use the Internet).

(d) As in part (b) this is an example of partial compliance. Failure to follow treatment protocol leads to bias in the OLS estimator.

13.7. From the population regression

$$Y_{it} = \alpha_i + \beta_1 X_{it} + \beta_2 (D_t \times W_i) + \beta_0 D_t + v_{it},$$

we have

$$Y_{i2} - Y_{i1} = \beta_1 (X_{i2} - X_{i1}) + \beta_2 [(D_2 - D_1) \times W_i] + \beta_0 (D_2 - D_1) + (v_{i2} - v_{i1}).$$

By defining DY_i = Y_{i2} - Y_{i1}, DX_i = X_{i2} - X_{i1} (a binary treatment variable) and u_i = v_{i2} - v_{i1}, and using D_1 = 0 and D_2 = 1, we can rewrite this equation as

$$\Delta Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i,$$

which is Equation (13.5) in the case of a single W regressor.

13.9. The covariance between $\beta_{1i}X_i$ and X_i is

$$\begin{aligned}\text{cov}(\beta_{1i}X_i,\, X_i) &= E\{[\beta_{1i}X_i - E(\beta_{1i}X_i)][X_i - E(X_i)]\}\\ &= E\{\beta_{1i}X_i^2 - E(\beta_{1i}X_i)X_i - \beta_{1i}X_iE(X_i) + E(\beta_{1i}X_i)E(X_i)\}\\ &= E(\beta_{1i}X_i^2) - E(\beta_{1i}X_i)E(X_i)\end{aligned}$$

Because X_i is randomly assigned, X_i is distributed independently of b_{1i}. The independence means

$$E(\beta_{1i}X_i) = E(\beta_{1i})E(X_i) \qquad \text{and} \qquad E(\beta_{1i}X_i^2) = E(\beta_{1i})E(X_i^2).$$

Thus $\text{cov}(\beta_{1i}X_i, X_i)$ can be further simplified:

$$\begin{aligned}\text{cov}(\beta_{1i}X_i, X_i) &= E(\beta_{1i})[E(X_i^2) - E^2(X_i)]\\ &= E(\beta_{1i})\sigma_X^2.\end{aligned}$$

So

$$\frac{\text{cov}(\beta_{1i}X_i, X_i)}{\sigma_X^2} = \frac{E(\beta_{1i})\sigma_X^2}{\sigma_X^2} = E(\beta_{1i}).$$

13.11. Following the notation used in Chapter 13, let π_{1i} denote the coefficient on state sales tax in the "first stage" IV regression, and let -β_{1i} denote cigarette demand elasticity. (In both cases, suppose that income has been controlled for in the analysis.) From (13.11)

$$\hat{\beta}^{TSLS} \xrightarrow{p} \frac{E(\beta_{1i}\pi_{1i})}{E(\pi_{1i})} = E(\beta_{1i}) + \frac{Cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})}$$
$$= \text{Average Treatment Effect} + \frac{Cov(\beta_{1i}, \pi_{1i})}{E(\pi_{1i})},$$

where the first equality uses the uses properties of covariances (equation (2.34)), and the second equality uses the definition of the average treatment effect. Evidently, the local average treatment effect will deviate from the average treatment effect when $Cov(\beta_{1i}, \pi_{1i}) \neq 0$. As discussed in Section 13.6, this covariance is zero when β_{1i} or π_{1i} are constant. This seems likely. But, for the sake of argument, suppose that they are not constant; that is, suppose the demand elasticity differs from state to state (β_{1i} is not constant) as does the effect of sales taxes on cigarette prices (π_{1i} is not constant). Are β_{1i} and π_{1i} related? Microeconomics suggests that they might be. Recall from your microeconomics class that the lower is the demand elasticity, the larger fraction of a sales tax is passed along to consumers in terms of higher prices. This suggests that β_{1i} and π_{1i} are positively related, so that $Cov(\beta_{1i}, \pi_{1i}) > 0$. Because $E(\pi_{1i}) > 0$, this suggests that the local average treatment effect is greater than the average treatment effect when β_{1i} varies from state to state.