

# Buy on Rumors - Sell on News: A Manipulative Trading Strategy

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## Abstract

This paper shows that a trader who receives a signal about a future public announcement can exploit this private information twice. First, when he receives his signal, and second, at the time of the public announcement. The second round advantage occurs because the early-informed trader can best infer the extent to which his information is already reflected in the current price. We also show that the early-informed trader trades very aggressively when he receives his signal. He tries to manipulate the price in order to enhance his informational advantage at the time of the public announcement. In addition, he speculates by building up a position in period one, which he partially unwinds ‘on average’ in period two. The analysis also shows that information leakage makes prices prior to public announcements more informative but reduces informational efficiency in the long run.

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# 1 Introduction

People trade assets for various reasons. Some trade to diversify or to hedge their risky endowment. Others trade to speculate on the stock market. This latter class of traders is willing to take on a less diversified position in order to exploit their superior private information. Large hedge funds might even try to trade to manipulate prices and mislead the market. A good understanding of the various incentives for trade and of the trading patterns that result is a prerequisite for the design and evaluation of trading regulations. This knowledge can also facilitate the monitoring of trading activities and the enforcement of existing regulations by the Securities and Exchange Commission (SEC).

This paper analyzes the optimal trading strategy of a large trader who receives some early private information - perhaps in the form of a rumor - about a forthcoming public announcement. The analysis provides several novel insights on insider trading by enriching the information structure typically employed in the prior literature. In our model prices reflect both information related to the forthcoming public announcement as well as other private information dispersed in the economy. Given this generalized information structure, the early informed insider's trading strategy exhibits three features: (i) He can exploit his private information twice; once before the public announcement and a second time after it. (ii) He trades for speculative reasons; that is, he intends to unwind the acquired position after the public announcement. (iii) He engages in a special form of market manipulation.

The intuition behind these results is as follows. Prior to the public announcement, the early informed trader makes use of his informational advantage by trading based on his imprecise signal. At the same time, other informed trading occurs due to information which is unrelated to the forthcoming public announcement. Both trading activities move the price. All traders employ technical analysis after the public announcement to tease out the remaining information from the past price. The early informed trader's technical analysis is more informative than the other traders' analyses since he knows the exact extent to which he has moved the past price. This provides him an additional informational advantage even after the public announcement. This is in spite of the fact that the public announcement is more precise than his original private signal. Paradoxically, it is the imprecision of the early-informed trader's signal that induces the uninformed market participants to make an error in their technical analyses, thereby giving him an informational advantage even after the public announcement.

In addition to showing that an early-informed trader can exploit his information twice, we demonstrate that he also trades for speculative reasons. We define 'speculative trading' as trading that is undertaken with the intention to unwind the acquired position after the public announcement. In this setting, the early-informed trader can exploit his knowledge about the error others make in conducting technical analysis. The formal analysis shows that the early informed trader 'on average' partly reverses the position that he built up

in the previous trading round. After receiving a positive (negative) imprecise signal the trader buys (sells) stocks that he expects to sell (buy) at the time of the public announcement. In other words, he follows the well known trading strategy: “Buy on Rumors - Sell on News.” This trade reversal has conceptually distinct roots than those typically discussed in the prior literature.

Our analysis also introduces a novel form of trade-based *price manipulation*. Manipulative trading is defined as active trading with the intention of moving the price such that the informational advantage is enhanced at a later time. The model shows that the early-informed insider trades in order to manipulate the price in his favor. His future capital gains result from correcting the other market participants’ error in technical analysis. If an early-informed agent trades very aggressively prior to the public announcement, his private signal’s imprecision has a larger impact on the current price. This imprecision makes it harder for the other market participants to infer other relevant information from past prices after the public announcement. Hence, by trading more aggressively in the first trading round, he increases his expected future capital gains in later trading rounds. Put more bluntly, he generates a larger informational advantage by ‘throwing sand in the eyes of the other traders’. This manipulative trading behavior is in sharp contrast to Kyle (1985) where the insider trades less aggressively today in order to save his informational advantage for future trading rounds. In our setting, the insider trades more aggressively now in order to enhance his future informational advantage. Therefore, the optimal trading strategy could more appropriately be called “Trade ‘Aggressively’ on Rumors - Sell on News”.

This paper also highlights the importance of other traders’ information in the interpretation of prices and runs counter to the notion of informational efficiency of markets. It illustrates that in some situations, knowledge about what other market participants know can be more valuable than direct knowledge about the fundamental value of a stock. This is in the spirit of Keynes’ well known beauty contest argument (Keynes 1936). If it is important to know other traders’ information in order to interpret the price, then price alone cannot be a sufficient statistic for all individual signals. This sheds new light on the strong-form informational efficiency of markets. For the Grossman-Stiglitz Paradox to arise, it is, therefore, not only necessary that all traders are price takers, as illustrated in Jackson (1991), but also that each market participant knows how his information is related to the information of other agents. Rumors are especially detrimental for achieving informationally efficient markets. Even after the truth is announced, rumors still distort the price and should therefore be avoided.

The model also has policy implications. It is well known that insider trading can have detrimental effects on risk-sharing. Using a more realistic information structure, our model illustrates that insider trading can also make prices less informationally efficient. In addition, we find that the insider’s optimal trading strategy involves buying and selling shares before and after public announcements. Thus, our analysis provides new support

for the short swing rule (Rule 16b of the Securities Exchange Act (SEA)), which prohibits corporate insiders from buying and selling the same shares within a period of six months.

This paper builds on the prior literature on technical analysis, speculation and manipulation in several important ways. The prior literature on *technical analysis*, such as Brown and Jennings (1989) and Grundy and McNichols (1989), analyzes the inference of information from past prices in a competitive rational expectations model setup. Since public announcements affect all traders symmetrically in these models, no individual trader can gain an informational advantage over the other traders. In contrast, in our model the early informed trader enjoys a larger informational advantage even after the public announcement due to his superior ability to interpret the past price.

Treynor and Ferguson (1985) demonstrate the usefulness of technical analysis in a setting where a trader does not know whether his information is already known to all the other market participants or not. In our model, traders who are not early-informed know that the early-informed trader has received an imprecise signal prior to the public announcement, but they do not know the extent to which this information is already incorporated in the past stock price. This makes it hard for them to infer information unrelated to the public announcement from the past stock price. Our model provides a micro foundation for Treynor and Ferguson's reasoning and demonstrates that it only works if the price before the public announcement also reflects information other than the information related to the public announcement.

Hirshleifer, Subrahmanyam, and Titman (1994) also generate *speculative trading* wherein risk averse insiders unwind part of their risky position as soon as their private information is revealed to a larger group of traders. However, in their model speculation would not occur without risk aversion. In contrast, in our model, the insider speculates even though he is risk neutral. Therefore, our model provides a conceptually distinct explanation for speculative behavior: the insider partially unloads his position due to informational reasons and not due to risk aversion.

The prior literature on *manipulation* distinguishes between trade-based, information-based and action-based stock price manipulation (Allen and Gale 1992). Our model falls in the class of trade-based manipulation models.<sup>1</sup> One form of trade-based manipulation is due to differences in market liquidity. Kumar and Seppi (1992) illustrate price manipulation if futures are settled by cash rather than by physical delivery. The intuition is that 'cash settlement' acts as an infinitely liquid market in which pre-existing futures positions are closed out relative to the less liquid spot market. In Allen and Gorton (1992)

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<sup>1</sup>Information-based and action-based manipulation are more distant from the current analysis because they are not directly caused by trading activity. Information-based manipulation involves the spreading of false rumors to manipulate the price (Vila (1989), Benabou and Laroque (1992)), while action-based manipulation occurs when corporate insiders entangle corporate decisions with their private stock market activities.

trade-based manipulation is possible since buy orders are more likely to be from informed traders than sell orders. Therefore, the market is less liquid for upswings than for downturns. Unlike these papers, manipulation in our model is not driven by differences in liquidity but by the desire to generate a future informational advantage.

Allen and Gale (1992) illustrate manipulation due to informational considerations in a setting with higher order uncertainty. In their model, all traders are price takers except for one large trader, who is either an informed trader or an uninformed manipulator. His information set includes two dimensions: the actual information and knowledge that he is informed. The authors show that if the large trader is uninformed, he still acts as if he has received good news. This pretense helps him drive up the price. This is optimal in Allen and Gale (1992) because there is asymmetry in the timing of good and bad news announcements. Chakraborty (1997) also illustrate manipulation by a potentially informed insider in a generalized Easley and O'Hara (1987,1992) setting that includes less informed followers who receive an imprecise signal of whether the insider is informed.

A related branch of literature looks at manipulative trading induced by the introduction of a mandatory disclosure rule for insider trading activities. Fishman and Hagerty (1995) initiated this line of research by showing that the mandatory disclosure of individual trading activities under Rule 16a of the SEA can lead to manipulative trading by uninformed insiders. John and Narayanan (1997) extend their analysis by showing that even an informed trader can manipulate the market if good and bad news do not occur with equal probability. More recently, Huddart, Hughes, and Levine (2000) analyze the introduction of the mandatory disclosure rule within a Kyle (1985) framework. They illustrate that an insider will apply a mixed strategy in order to preserve his private information for future trading rounds.

In all these models, the potential manipulator is endowed with superior information, even if it is only information about whether he is informed or not. He manipulates the price in order to hide his own information or lack of information. The manipulation that arises in our model is conceptually different. In our setting, it is common knowledge that an early-informed insider has received a noisy signal about the forthcoming announcement. The novelty of our form of manipulation is that the insider can still manipulate the price in order to make it harder for others to conduct technical analysis, while maintaining his own ability to infer information from past price. Thus, he jams the signal of others as in the signalling jamming industrial organization literature (Fudenberg and Tirole 1986). Furthermore, in contrast to most of the prior models in the finance literature, manipulative trading in our model is derived without the imposition of any restriction on the traders' order size. That is, the insider's strategy space is richer than in other models rooted in the framework of Glosten and Milgrom (1985).

The remainder of the paper is organized as follows. Section 2 outlines the model. It shows that an early-informed trader still has an informational advantage at the time of the

public announcement and that he trades for speculative as well as manipulative reasons. The impact of information leakage on informational efficiency is illustrated in Section 3. Section 4 extends the analysis to address mixed strategies, optimal signal precision, and multiple informed traders. Conclusions and topics for future research are presented in Section 5.

## 2 Analysis

### 2.1 Model Setup

There are two assets in the economy: a risky stock and a risk-free bond. For simplicity we normalize the interest rate of the bond to zero. Market participants include risk-neutral informed traders, liquidity traders and a market maker. The informed traders' sole motive for trading is to exploit their superior information about the fundamental value of the stock. Liquidity traders buy or sell shares for reasons exogenous to the model. Their demand typically stems from information which is not of common interest, such as from their need to hedge against endowment shocks or private investment opportunities in an incomplete market setting.<sup>2</sup> A single competitive risk-neutral market maker observes the aggregate order flow and sets the price. Traders submit their market orders to the market maker in two consecutive trading rounds taking into account the price impact of their orders. The market maker sets the price in each round after observing the aggregate order flow and trades the market clearing quantities. As in Kyle (1985) the market maker is assumed to set informationally efficient prices; thus his expected profit is zero. The underlying Bertrand competition with potential rival market makers is not explicitly modelled in this analysis.<sup>3</sup> Informed traders receive their signal before trading begins in  $t = 1$ . The public announcement occurs prior to trading in  $t = 2$ . The timeline in Figure 1 illustrates the sequence of moves.

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<sup>2</sup>See Brunnermeier (2000) or O'Hara (1995) for a detailed discussion of the different reasons why liquidity traders trade, and for a discussion on the distinction between information of common versus private interest.

<sup>3</sup>Alternatively, one could also employ a setting where many competitive risk-neutral traders like scalpers, floor brokers etc. submit limit order schedules. The analysis would be formally identical and the price would be determined by market clearing. Therefore, when we speak of the information set of 'market participants' we are referring to the information set of the single market maker in our formal analysis.

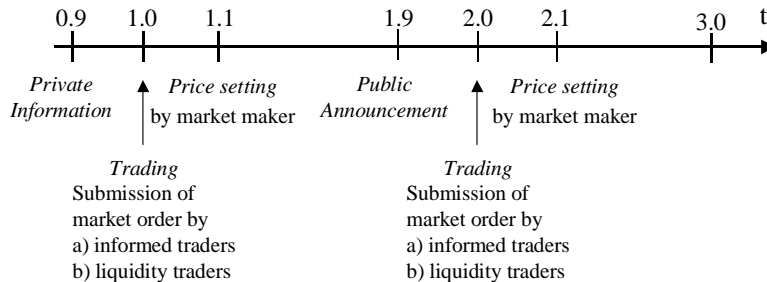


Figure 1: Timeline

Traders face a price risk submitting market orders since they do not know the price at which their trade will be executed. In contrast, limit orders allow the trader to specify a price at which the order will be executed. Traders can create demand schedules which allow them to trade conditionally on the current price by combining many limit and stop orders. Unfortunately, limit order models make the analysis less tractable without adding any significant insight. Therefore, we opt for a market order setting similar to Kyle (1985), Admati and Pfleiderer (1988) and Foster and Viswanathan (1996).

Many different events can provide information about the equity value of a company. Events like earnings announcements, a major contract with a new client, legal allegations, a new CEO, macroeconomic news etc. can have a significant impact on the market value of a stock. Let us restrict our attention to only two events,  $A$  and  $B$ . Their impact on the value of the stock is modelled by the two random variables  $\delta^A$  and  $\delta^B$ , which are independently normally distributed with mean zero. The liquidation value of the stock  $v = \delta^A + \delta^B$  is paid out in  $t = 3$ . Event  $A$  is publicly announced before the second trading round and the price impact  $\delta^A$  of event  $A$  becomes common knowledge to all market participants. Prior to the announcement some imprecise information about the event  $A$  leaks already to some trader(s). This information,  $\delta^A + \varepsilon$ , leaks possibly in the form of a rumor in period one. The error term  $\varepsilon$  reflects the imprecision of the rumor. Other trader(s) receive information about event  $B$  in period one. Their information is long-run, since  $\delta^B$  is only made public at the end of the trading game. Due to the private information about event  $A$  and  $B$ , the price of the asset in  $t = 1$ ,  $p_1$ , reflects both information events  $A$  and  $B$ . The past price  $p_1$  still carries information about event  $B$  in  $t = 2$  after  $\delta^A$  is made public. In period three  $\delta^B$  is publicly announced, that is, the true value of the stock  $v = \delta^A + \delta^B$  is known to everybody in  $t = 3$ . Liquidity traders do not receive any information and their aggregate trading activity is summarized by the random variables  $u_1$  in period one and  $u_2$  in period two.

Ideally, one would like to analyze a setting where many early informed  $A$ -traders receive the signal  $\delta^A + \varepsilon$  prior to the forthcoming public announcement of  $\delta^A$  in  $t = 2$  and many  $B$ -traders receive a piece of news about the event  $B$ . Since event  $B$  should capture all the other relevant information in the market place, a model setup where information



about event  $B$  is dispersed among many  $B$ -traders would be most realistic. That is each individual  $B$ -trader receives a noisy signal  $\delta^B + \varepsilon^{B_i}$  about  $\delta^B$ . However, such analysis would be notationally very cumbersome and would lead to the same qualitative results. Therefore, as a first step we opt to analyze a setting with a single  $A$ -trader and a single  $B$ -trader who observes  $\delta^B$ . This setup, simplifies the notation while still retaining all the main economic insights. The implications of a multi-insider setup are discussed in Section 4.3.

The information structure is summarized in the following table.

Player $i$	Period $t = 1$	Period $t = 2$	Period $t = 3$
Market maker	$X_1$	$\delta^A, p_1, X_2$	$\delta^B, p_2$
Trader $A$	$\delta^A + \varepsilon$	$\delta^A, p_1$	$\delta^B, p_2$
Trader $B$	$\delta^B$	$\delta^A, p_1$	$\delta^B, p_2$

Table 1: Information Structure

where  $X_1 = x_1^A + x_1^B + u_1$  is the aggregate order flow in  $t = 1$  and  $X_2 = x_2^A + x_2^B + u_2$  is the order flow in  $t = 2$ . For notational simplicity, we denote the signal of trader  $i \in \{A, B\}$  at time  $t$  by  $S_t^i$ . The random variables  $\delta^A, \delta^B, \varepsilon, u_1$  and  $u_2$  are independently normally distributed with mean zero. For symmetry, let  $Var[\delta^A] = Var[\delta^B]$ .

This information structure is common knowledge among all market participants, i.e. we assume that everybody knows that trader  $A$  has received some noisy information about a forthcoming public announcement. However, they do not know the content of trader  $A$ 's information.<sup>4</sup> The information structure implies that trader  $A$ 's and trader  $B$ 's information sets do not stochastically dominate each other. In other words, the information sets are non-hierarchical or non-nested even in this simplified setting. It is also easy to see that perturbing trader  $B$ 's signal in  $t = 1$  with an idiosyncratic noise,  $\varepsilon^B$  does not alter the analysis. Since trader  $B$  is risk neutral,  $\delta^B$  can simply be replaced by  $E[\delta^B | \delta^B + \varepsilon^B]$ .

An analysis of price manipulation is ruled out in a Rational Expectations Equilibrium setting because all traders are assumed to be price-takers. In a Bayesian Nash Equilibrium setting, however, all traders take the strategies of all other players as given. That is, they are aware that their trade affects the price. All informed traders submit their market orders,  $x_t^i$ , to the market maker in each trading round. Note that in period two, each trader  $i$  knows not only his signal, the price  $p_1$  and the public information  $\delta^A$  but also his demand in  $t = 1$ ,  $x_1^i$ . The risk-neutral market maker sets the execution price  $p_t$  after

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<sup>4</sup>This problem can also be captured in a model with higher order uncertainty, i.e. information leakage occurs only with a certain probability. In that case, trader  $A$  receives two pieces of information. In addition to the actual signal, he knows whether some information has leaked or not. Trader  $A$ 's informational advantage at the time of the public announcement in  $t = 2$  stems from his knowledge of whether he had received an early signal or not. Such models are not pursued in this paper because they are very intractable without restricting the trading size.

observing the aggregate net order flow. The price is semi-strong informationally efficient, i.e. the price is the best estimate given the market maker's information. A different price would lead to an expected loss or an expected profit for the market maker. The latter is ruled out because the market maker faces Bertrand competition from potential rival market makers. For ease of exposition, the strategy for the market maker is exogenously specified. He has to set informationally efficient prices in equilibrium, i.e.  $p_1 = E[v|X_1]$  and  $p_2 = E[v|X_1, \delta^A, X_2]$  due to potential Bertrand competition.

A sequentially rational Bayesian Nash Equilibrium of this trading game is given by strategy profile  $\{\{x_1^{i,*}(\cdot), x_2^{i,*}(\cdot)\}_{i=\{A,B\}}, p_1^*(\cdot), p_2^*(\cdot)\}$  such that

- (1)  $x_2^{i,*} \in \arg \max_{x_2^i} E[x_2^i(v - p_2)|S_1^i, x_1^i, p_1, \delta^A] \forall i \in \{A, B\}$ ,
- (2)  $x_1^{i,*} \in \arg \max_{x_1^i} E[x_1^i(v - p_1) + x_2^{i,*}(v - p_2)|S_1^i] \forall i \in \{A, B\}$ , and prices  $p_1^* = E[v|X_1^*]$  and  $p_2^* = E[v|X_1^*, \delta^A, X_2^*]$ ,

where the conditional expectations are derived using Bayes' Rule to ensure that the beliefs are consistent with the equilibrium strategy.

## 2.2 Characterization of Linear Equilibrium

Proposition 1 characterizes a sequentially rational Bayesian Equilibrium in linear pure strategies. It has the elegant feature that each trader's demand is the product of his trading intensity (or aggressiveness) and the difference in the trader's and market maker's expectations about the value of the stock. Linear strategies have the advantage that all random variables remain normally distributed. In addition, the pricing rules are linear as a consequence of the Projection Theorem.<sup>5</sup> In period one the market maker's pricing rule is  $p_1 = \lambda_1 X_1$  and in period two it is  $p_2 = \delta^A + E[\delta^B|X_1, \delta^A] + \lambda_2 X_2$  in equilibrium. As in Kyle (1985)  $\lambda_t$  reflects the price impact of an increase in market order by one unit. This price impact restricts the trader's optimal order size. Kyle interpreted the reciprocal of  $\lambda_t$  as market depth. If the market is very liquid, i.e.  $\lambda_t$  is very low, then an increase in the trader's demand only has a small impact on the stock price. For expositional clarity, we denote the regression coefficient of  $y$  on  $x$  by  $\phi_x^y := \frac{Cov[x,y]}{Var[x]}$ .

**Proposition 1** *A sequentially rational Bayesian Nash Equilibrium in which all pure trading strategies are of the **linear** form*

$$\begin{aligned} x_1^i &= \beta_1^i(S_1^i), \\ x_2^i &= \beta_2^i(E[v|S_1^i, p_1, \delta^A] - E[v|p_1, \delta^A]), \end{aligned}$$

and the market maker's pricing rule

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<sup>5</sup>Since all variables are normally distributed, the orthogonal projection of  $v$  on the space of linear-affine functions of  $S$  is equal to the projection of  $v$  (in the sense of  $\mathcal{L}^2$ ) on the space  $\mathcal{L}^2(S)$  of quadratic integrable functions of  $S$ . Consequently,  $E[v|\mathbf{S}] = E[v] + (\mathbf{S} - E[\mathbf{S}])^\tau \mathbf{Var}^{-1}[\mathbf{S}]Cov[v, \mathbf{S}]$ , which allows us to calculate the conditional expectations.

$p_1 = E[v|X_1] = \lambda_1 X_1$ ,  
 $p_2 = E[v|X_1, \delta^A, X_2] = \delta^A + \phi_{S_2^{p_1}}^{\delta^B} S_2^{p_1} + \lambda_2 X_2$ , with  $S_2^{p_1} = \frac{X_1 - \beta_1^A \delta^A}{\beta_1^B}$ ,  
 is determined by the fixed points of the following system of equations

$$\begin{aligned}
 \beta_1^{A,*} &= \frac{1}{2 \left( \lambda_1 - \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon \right)} \phi_{S_1^A}^{\delta^A} \\
 \beta_1^{B,*} &= \frac{1}{2\lambda_1} \left( 1 - 2\lambda_2 \beta_2^B \gamma_2^B \left( 1 - \phi_{S_2^{p_1}}^{\delta^B} \right) \right)
 \end{aligned}$$

where

$$\lambda_1 = \frac{\beta_1^{A,*} \text{Var}[\delta^A] + \beta_1^{B,*} \text{Var}[\delta^B]}{\text{Var}[\beta_1^{A,*}(\delta^A + \varepsilon) + \beta_1^{B,*}(\delta^B) + u_1]} \quad \lambda_2 = \frac{\text{Cov}[\delta^B, X_2 | S_2^{p_1}]}{\text{Var}[X_2 | S_2^{p_1}]}$$

with

$$\begin{aligned}
 \beta_2^A &= \frac{1}{2\lambda_2} \frac{1}{2} \frac{1}{1 - \frac{1}{4} \phi_{S_2^{A,w}}^w \phi_{S_2^{A,w}}^w} \phi_{S_2^{A,w}}^w & \gamma_2^A &:= \frac{\beta_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w} \\
 \beta_2^B &= \frac{1}{2\lambda_2} \frac{1 - \frac{1}{2} \phi_{S_2^{A,w}}^w \phi_{S_2^{A,w}}^w}{1 - \frac{1}{4} \phi_{S_2^{A,w}}^w \phi_{S_2^{A,w}}^w} & \gamma_2^B &:= \frac{1}{2\lambda_2} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} - \frac{1}{2} \frac{\beta_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w}
 \end{aligned}$$

if the second order conditions  $\lambda_2 > \lambda_1 \max \left\{ \left[ \frac{b_2^A \phi_{S_2^{p_1}}^{\delta^B}}{b_1^B \phi_{S_2^{A,w}}^w} \right]^2, \left[ \frac{1}{b_1^B} \phi_{S_2^{p_1}}^{\delta^B} - \frac{1}{2} \frac{b_2^A \phi_{S_2^{p_1}}^{\delta^B}}{b_1^B \phi_{S_2^{A,w}}^w} \right]^2 \right\}$ ,  $\lambda_2 > 0$   
 (with  $b_t^i := 2\lambda_t \beta_t^i$ ) are satisfied.

The interested reader is referred to the Appendix for a complete proof of the proposition. The proof makes use of backward induction. In order to solve the continuation game in  $t = 2$ , the information structure prior to trading in  $t = 2$  has to be derived. For this purpose, let us propose an arbitrary action rule profile,  $\{\{\beta_1^i\}_{i \in \{A,B\}}, p_1(X_1)\}$  for  $t = 1$ , which is mutual knowledge and is considered to be an equilibrium profile by all agents. In  $t = 2$  all market participants can derive the aggregate order flow  $X_1 = \beta_1^A(\delta^A + \varepsilon) + \beta_1^B \delta^B + u_1$  from price  $p_1$ . After knowing  $\delta^A$ , the price signal is  $S_2^{p_1} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{1}{\beta_1^B} u_1$ . Since all market participants know  $S_2^{p_1}$ , it is useful to state each traders' information relative to the publicly known symmetric information, i.e. to orthogonalize the signals with respect to  $S_2^{p_1}$ . The stock is split into an expected part  $E[v|S_2^{p_1}, \delta^A] = \delta^A + E[\delta^B | S_2^{p_1}]$  and an unexpected part  $w := \delta^B - E[\delta^B | S_2^{p_1}]$ . This 'virtual' split of the stock  $v$  into a risk-less bond and a risky asset  $w$  is possible, without loss of generality. Even if a trader deviates in  $t = 1$ , other market participants still assume that he has played his equilibrium strategy. This is because the liquidity traders order size  $u_1$  is normally distributed and thus any aggregate order flow from  $(-\infty, +\infty)$  can arise in equilibrium. This makes it unnecessary to specify off-equilibrium beliefs as the market maker and the other traders do not see an order flow that could not be observed in equilibrium. In  $t = 2$  traders face a generalized static Kyle-trading-game with the usual trade-off. On the one hand, a risk-neutral trader

wants to trade very aggressively in order to exploit the gap between his estimate of the fundamental value of the stock and the price of the stock. On the other hand, very aggressive trading moves the price at which his order will be executed towards his estimate of the asset's value since it allows the market maker to infer more of the trader's information from the aggregate order flow. This latter price impact reduces the value-price gap from which the trader can profit and restrains the traders from trading very aggressively.

Using backward induction one has to check whether a single player wants to deviate in  $t = 1$  from the proposed action rule profile,  $\{\{\beta_1^i\}_{i \in \{A,B\}}, p_1(X_1)\}$ . Trading in  $t = 1$  affects not only the capital gains in  $t = 1$  but also the future prospects for trading in  $t = 2$ . Any deviation in  $t = 1$  alters the price  $p_1$ . Since other market participants infer wrong information from  $p_1$ , their trading and price setting in  $t = 2$  is also affected. An equilibrium is reached if no trader wants to deviate from the proposed action rule profile in  $t = 1$ . In other words, the sequentially rational Bayesian Nash Equilibrium is given by the fixed point described in Proposition 1.

Proposition 1 also presents two inequality conditions. They result from the second order conditions in the traders' maximization problems. They guarantee that the quadratic objective functions for each period have a maximum rather than a minimum. In economic terms, they require that the market is sufficiently liquid/deep in trading round one relative to trading round two. These inequality restrictions rule out the case where it is optimal to trade an unbounded amount in  $t = 1$ , move the price, and make an infinitely large capital gain in  $t = 2$ .

### 2.3 Exploiting Information Twice

Information about the fundamental value of the stock as well as information about other traders' demand affects the traders' optimal order size. In period two, traders can infer some information from the past price,  $p_1$ . Brown and Jennings (1989) call this inference 'technical analysis'. If a trader's prediction of the stock's liquidation value is more precise than the market maker's prediction, then the trader has an informational advantage. Proposition 2 shows that trader  $A$  still has an informational advantage in period two over the market maker. Trader  $A$  can, therefore, *exploit his private information twice*. First, when he receives his signal, and second, at the time of the public announcement. This is surprising since one might think that the public announcement is a sufficient statistic for trader  $A$ 's private information.

**Proposition 2** *Trader  $A$  retains an informational advantage in period two in spite of the public announcement in period two. Technical analysis is more informative about the value of the stock for trader  $A$  than for the market maker. Trader  $A$ 's informational advantage in period two is increasing in his trading intensity and decreasing in the trading intensity of trader  $B$  in  $t = 1$ .*

Since all traders trade conditional on their signal in period one, the price  $p_1$  reflects not only the signal about  $\delta^B$  but also the signal about  $\delta^A + \varepsilon$ . In period two all market participants try to infer information in  $t = 2$  from the past price  $p_1$ . However, only trader  $A$  knows the exact extent to which the past price,  $p_1$ , already reflects the new public information,  $\delta^A$ . That is, while the other market participants can only separate the impact of  $\delta^A$  on  $p_1$ , trader  $A$  can also deduce the impact of the  $\varepsilon$  error term on  $p_1$ .

In general, technical analysis serves two purposes. First, traders try to infer more about the fundamental value of the stock from the past price. Second, they use the past price to forecast the forecasts of others. Knowing others' estimates is useful for predicting their market orders in  $t = 2$ . This in turn allows traders to estimate the execution price  $p_2$  more precisely. Trader  $B$  trades conditional on  $p_1$  in  $t = 2$  in order to improve his forecasts of trader  $A$ 's market order in  $t = 2$ . Since trader  $B$  already knows  $\delta^B$ , he knows the fundamental value  $v = \delta^A + \delta^B$  when  $\delta^A$  is publicly announced in  $t = 2$ . Thus, he does not need to conduct technical analysis to get a better estimate of the fundamental value.<sup>6</sup>

When conducting technical analysis, the market maker and trader  $B$  are aware that price  $p_1$  is affected by the error term  $\varepsilon$ . The price,  $p_1 = \lambda_1 X_1$  depends on the individual demand of trader  $A$ ,  $x_1^A$ , and thus on the signal  $\delta^A + \varepsilon$ . Trader  $A$ 's informational advantage in  $t = 2$  is his knowledge of the error  $\varepsilon$ . He can infer  $\varepsilon$  from the difference between his signal in  $t = 1$  and the public announcement in  $t = 2$ . If trader  $A$  would have abstained from trading in  $t = 1$ , the public announcement  $\delta^A$  would be a sufficient statistic for trader  $A$ 's private information,  $\delta^A + \varepsilon$ . However, since trader  $A$  traded in  $t = 1$ , trader  $B$  and the market maker would like to know the extent to which his trading activities changed price,  $p_1$ . Knowledge not only of  $\delta^A$  but also of  $\varepsilon$  would allow them to infer even more information from the price,  $p_1$ . Hence, the public announcement in  $t = 2$  is not a sufficient statistic of  $\delta^A + \varepsilon$  for interpreting the past price,  $p_1$ .

Trader  $A$  applies technical analysis in order to infer more information about the fundamental value of the stock. This information is also valuable for predicting trader  $B$ 's demand in  $t = 2$ . The additional information about the value of the stock provided by technical analysis is higher for trader  $A$  than for the market maker. For trader  $B$ , technical analysis only provides information about trader  $A$ 's forecast since trader  $B$  already knows the liquidation value  $v$  in  $t = 2$ . Since trader  $A$  knows his own demand, he can infer  $\frac{1}{\beta_1^B}(\frac{p_1}{\lambda_1} - x_1^A) = \delta^B + \frac{1}{\beta_1^B}u_1$ , which is trader  $B$ 's signal perturbed by the demand of the noise traders. The market maker can infer  $(\delta^B + \frac{1}{\beta_1^B}u_1) + \frac{\beta_1^A}{\beta_1^B}\varepsilon$ , which is trader  $A$ 's price signal perturbed by the additional error term,  $\varepsilon$ . Therefore, trader  $A$ 's informational advantage

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<sup>6</sup>However, as discussed in more detail in Section 5.2, in a more general setting with multiple  $B$ -traders where each  $B$ -trader receives only a noisy idiosyncratic signal  $\delta^B + \varepsilon^{Bi}$ ,  $B$ -traders conduct technical analysis to learn more about the fundamental value. In this generalized setup each  $B$ -trader tries to infer the other  $B$ -traders signals from the past price  $p_1$ .

is given by  $\frac{\beta_1^A}{\beta_1^B}\varepsilon$ , which increases with his trading intensity,  $\beta_1^A$ , and decreases with the trading intensity of trader  $B$ ,  $\beta_1^B$ . Intuitively, if trader  $A$  trades more aggressively in  $t = 1$  his signal's imprecision has a higher impact on the price,  $p_1$ .

## 2.4 Speculative and Manipulative Trading

In general, trading occurs for risk sharing purposes or for informational reasons. Since all traders are risk-neutral in this setting, their only motive to trade is to exploit their informational advantage. As illustrated in Proposition 2, current trading affects future informational advantages. In Kyle (1985), the single insider reduces his trading intensity in order to save information for future trading rounds. The single insider faces a trade-off. Taking on a larger position in period one can result in higher profits today but also leads to worse prices for current and future trading rounds. Thus in a Kyle (1985), setting the insider restrains his trading activity with the objective of not trading his informational advantage away.

In contrast to the literature based on Kyle (1985), trader  $A$  in our model trades more aggressively in period one. He incurs myopically non-optimal excessive trades in period one and then recuperates the losses and makes additional profit in period two. Trading more aggressively in period one changes the price in such a way that his informational advantage in the next trading round is enhanced. Trading with the sole intention of increasing one's informational advantage in the next period is defined as *manipulative trading*. *Speculative trading* is defined as trading with the expectation to unwind one's position in the next period. The following definitions restate the two trading objectives:

**Definition 1** *Speculative trading is carried out with the expectation of unwinding the acquired speculative position in the next period.*

Speculative trading can also be manipulative.

**Definition 2** *Manipulative trading is intended to move the price in order to enhance the informational advantage in the next period.*

Manipulative trading is excessive in the sense that it is the component of trading intensity that exceeds the optimal myopic trading intensity, holding the other market participants' strategies fixed. The myopic trading intensity does not take into account the fact that trader  $A$  could enhance his informational advantage in period two by trading more aggressively in period one.

Proposition 3 shows that trader  $A$  trades for speculative reasons since he expects to unwind part of his accomplished position in period two. Furthermore, he trades excessively with the objective of manipulating the price.

**Proposition 3** *In period one, trader A trades conditional on his current information in order to build up a long-term position, and also for speculative and manipulative reasons.*

*Speculative trading is given by  $\gamma_2^A \phi_{S_1^A}^\varepsilon \beta_1^A S_1^A$ .*

*Manipulative trading is given by  $\lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon \beta_1^A S_1^A$ ,*

*where the coefficients in front of  $S_1^A$  are strictly positive.*

The proof in the appendix shows that if trader  $A$  receives a positive signal, all trading objectives induce the trader to take a long position in the stock. Similarly, if trader  $A$  receives a negative signal he sells the stock. He does not apply a contrarian trading strategy.

Proposition 3 introduces a novel form of stock price manipulation. The underlying purpose of trader  $A$ 's manipulative trading is to extend the informational gap in the second trading round.<sup>7</sup> In contrast, to the previous literature, trader  $A$  does not trade in order to hide his own information or lack of information. The novelty of this form of manipulation is that trader  $A$ 's aggressive trading worsens the other market participants' ability to infer trader  $B$ 's information from the past price in period two, while he retains his full ability to conduct technical analysis. More specifically, by trading excessively in  $t = 1$ , trader  $A$  confounds the other market participants' price signal  $S_2^{p1}$  in  $t = 2$ . The reason is that the imprecision of trader  $A$ 's signal  $\varepsilon$  has a larger impact on  $p_1$  if he trades more aggressively. Consequently, the larger is  $\beta_1^A$ , the less the price signal  $S_2^{p1} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{1}{\beta_1^B} u_1$  reveals about the fundamental value  $\delta^B$ . This increases trader  $A$ 's informational advantage in  $t = 2$  with respect to the market maker. It also makes trader  $B$ 's forecast about trader  $A$ 's  $\delta^B$  forecast worse. Since trader  $B$  already knows the fundamental value  $\delta^B$ , his only motive for conducting technical analysis is to achieve a better prediction of trader  $A$ 's market order and thus the execution price in  $t = 2$ ,  $p_2$ . Trader  $A$ 's market order in  $t = 2$  is based on his information,  $\delta^B + \frac{1}{\beta_1^B} u_1$  and  $S_2^{p1}$ . The only term trader  $B$  does not know is  $u_1$ , trader  $A$ 's error in predicting the fundamental value  $\delta^B$ . The price signal  $S_2^{p1}$  allows him to derive the signal  $(S_2^{p1} - \delta^B) \beta_1^B = \beta_1^A \varepsilon + u_1$ , which helps him to forecast trader  $A$ 's order size. However, he can not forecast it perfectly since his signal is perturbed by  $\beta_1^A \varepsilon$ , the imprecision of the rumor times trader  $A$ 's trading intensity in period one. In short, if trader  $A$  trades more aggressively in period one, he builds up a larger informational advantage with respect to the market maker and also reveals less of his informational advantage to his competitor, trader  $B$ . Overall, more aggressive trading in period one increases trader  $A$ 's expected future capital gains. The proof in the appendix shows that in equilibrium the trading intensity of trader  $A$  is higher if he takes the impact on future expected capital gains into account, given the strategies of all other players. It is the expected knowledge of the  $\varepsilon$ -term in  $t = 2$  which induces manipulative trading.

Speculation in this model is driven purely by trading for informational reasons. A positive (negative) signal for trader  $A$  has two implications. First, he buys (sells) shares

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<sup>7</sup>This relates to the literature on signal-jamming in industrial organization. Fudenberg and Tirole (1986) were the first to analyze signal-jamming in the context of predatory pricing.

in the first trading round and second, he expects that  $\varepsilon$  is positive (negative), that is  $E[\varepsilon|S_1^A] = \text{Var}[\varepsilon](\text{Var}[\delta^A] + \text{Var}[\varepsilon])^{-1}S_1^A > (<)0$ . Other market participants' technical analysis in  $t = 2$  is based on  $S_2^{p1} = \delta^B + \frac{\beta_1^A}{\beta_1^B}\varepsilon + \frac{1}{\beta_1^B}u_1$ . That is, if  $\varepsilon$  is positive (negative), the market maker overestimates (underestimates)  $\delta^B$  in period two. Since trader  $A$  can infer  $\varepsilon$  in period two, he expects to make money by correcting the market maker's overoptimism (pessimism). In short, trader  $A$  expects to sell (buy) shares in period two. Therefore, trader  $A$  expects to trade in the opposite direction in period two. 'On average', he partially unwinds his position in period two. This is solely due to informational reasons since trader  $A$  expects the price to overshoot in  $t = 2$ . Given however, the information of the market maker or of any other outsider who only observes the past prices and the public announcement, the price follows a Martingale process, i.e. it neither overshoots nor undershoots.

Speculative trading is also caused by the imprecision of trader  $A$ ' signal,  $\varepsilon$ . Consequently, an increase in trading intensity in period one due to manipulative behavior also leads to more speculation. Trader  $A$  expects to unwind a larger position in  $t = 2$ . The imprecision  $\varepsilon$  of trader  $A$ 's early signal plays a crucial role in this analysis.<sup>8</sup>

Hirshleifer, Subrahmanyam, and Titman (1994) appeal to traders' risk-aversion and thus provide a very distinct explanation for speculative behavior. In their setting early-informed risk averse traders are willing to take on a riskier position in order to profit from their superior private information. After a larger group of traders receives the same information one period later, they partially unwind their position to reduce their risk exposure. In their model, no speculation would occur without risk aversion, while in our setting trader  $A$  speculates even though he is risk neutral. His speculation is driven by informational reasons. It is easy to visualize a generalized setting with risk averse traders where these traders speculate due to risk aversion and informational reasons.

### 3 Impact of Information Leakage on Informational Efficiency

The information structure analyzed above also provides new insights on how information leakage affects market efficiency. Information leakage leads to insider trading which in general reduces liquidity trading and the amount of risk sharing. It might even lead to market breakdowns. Therefore, information leakage typically reduces allocative efficiency. The argument follows a similar line of reasoning as in Akerlof's (1970) "market for lemons." On the other hand, if there is some information leakage prices might adjust faster to be in line with the true asset value. This section focuses solely on the implication

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<sup>8</sup>Note that if  $\delta_A$  and  $\delta_B$  could be traded separately neither speculative nor manipulative trading would arise. Questions such as whether trader  $A$  has an incentive to generate some additional noise of his own or how his expected profit varies as  $\text{Var}[\varepsilon]$  varies are relegated to Section 4.1 and 4.2.



of information leakage on informational efficiency and, therefore, the amount of liquidity trading is assumed to be exogenously fixed. More specifically, this section illustrates how the noisy information leakage of  $\delta^A + \varepsilon$  to trader  $A$  prior to the official public announcement of  $\delta^A$  in  $t = 2$  affects the informational content revealed by prices. The benchmark is the setting where trader  $A$  receives no signal prior to the public announcement. A dynamic trade-off is illustrated: while information leakage can make prices more informative in the very short-run, it reduces informational efficiency in the long-run.

Before diving into the analysis, let us first define two different measures of the degree of information revelation by prices. A market is (strong-form) *informationally efficient* if the price is a sufficient statistic for all the information dispersed among all market participants. In this case, the market mechanism perfectly aggregates all information available in the economy, and the price reveals it to everybody. In general, if traders trade for informational as well as non-informational reasons, the price is not informationally efficient. This is also the case in our setting where some traders try to exploit their superior information and others trade for liquidity reasons. Nevertheless, one can distinguish between more and less informationally efficient markets. A measure of informational efficiency should reflect the degree to which information dispersed among many traders can be inferred from the price (process) together with other public information. Consider the forecast of the fundamental value of the stock  $v$ , given the pool of all available information in the economy at a certain point in time. If the price (process) is informationally efficient then the price(s) and other public information up to this time yields the same forecast. Consequently, the variance of this forecast conditional on prices and other public information is zero. This conditional variance increases as the market becomes less informationally efficient. Therefore, we choose the reciprocal of this conditional variance, i.e. the precision, as a measure of the degree of informational efficiency. Note that the degree of informational efficiency depends crucially on the pool of information in the economy. To illustrate this, consider a world without asymmetric information. In that setting, any price process is informationally efficient even though it is uninformative. While informational efficiency is relative to the information dispersed in the market, *informativeness* of a price process is absolute. The conditional variance of the stock value itself captures how informative the price (process) and the other public information are.<sup>9</sup> This variance term, therefore, also measures the risk a liquidity trader faces when trading this stock. This conditional variance is zero if all public information, including the price process, allows one to perfectly predict the liquidation value of the stock. In this case everybody knows the true stock value. The following definitions define both measures more formally.

**Definition 3** *The reciprocal of the variance  $\text{Var}[E[v|\{p_t, S_t^{\text{public}}, \{S_t^i\}_{i \in I}\}_{t \leq \tau} | \{p_t, S_t^{\text{public}}\}_{t \leq \tau}]$  conditional on the public information,  $S_t^{\text{public}}$ , and the pool of private information up to time  $\tau$  measures the degree of informational efficiency at time  $\tau$ . The reciprocal of the conditional variance  $\text{Var}[v|\{p_t, S_t^{\text{public}}\}_{t \leq \tau}]$  measures how informative*

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<sup>9</sup>Note that all public information at the beginning of the trading game is incorporated in the common priors.

the price (process) and the public information are.

Equipped with these measures, we can now analyze how the information leakage of  $\delta^A + \varepsilon$  to trader  $A$  affects informational efficiency and informativeness of the price (process). In addition, these measures also allow us to address the role of the imprecision of the rumor.

Since these definitions are time dependent, let us analyze informational efficiency and informativeness at the time after the first trading round, after the public announcement of  $\delta^A$ , and after the second trading round. Let us assume for the following proposition that there is a sufficient amount of liquidity trading in  $t = 1$ . More precisely,  $Var[u_1] > \frac{6}{25} \sqrt{\frac{2}{5} Var[u_2] Var[\delta^B]}$ .

**Proposition 4** *In  $t = 1$ , information leakage makes the price  $p_1$  more informative but less informationally efficient if the information leakage is sufficiently precise. However, after the public announcement in  $t = 2$ , the price is less informative and less informationally efficient.*

Leakage of information makes the price  $p_1$  in  $t = 1$  more informative, if  $Var[\varepsilon]$  is not too high. Trader  $A$  trades on his information  $\delta^A + \varepsilon$  and thus price  $p_1$  reveals information about not only  $\delta^B$  but also about  $\delta^A$ . Trader  $A$ 's market activity increases informed trading relative to liquidity trading. This allows the market maker as well as the public to infer more information from the aggregate order flow  $X_1$ . Note that this might not be the case for very high  $Var[\varepsilon]$  since aggressive manipulative trading activity could increase the non-informative component of the aggregate order flow.

On the other hand, information leakage makes the market less informationally efficient in  $t = 1$ . In this case, the information dispersed in the economy is not only  $\delta^B$  but also  $\delta^A + \varepsilon$ . If there is no leakage,  $p_1$  reveals more about  $\delta^B$  than  $p_1$  reveals about  $E[v|\delta^B, \delta^A + \varepsilon] = \delta^B + \phi_{S_1^A}^{\delta^A}(\delta^A + \varepsilon)$  in the case of a leakage. The reason is that sufficiently precise information leakage leads to a higher  $\lambda_1$  which reduces the trading intensity of trader  $B$ ,  $\beta_1^B$ . Therefore less information can be inferred about  $\delta^B$ . In addition,  $\delta^A + \varepsilon$  can only be partly inferred from the price  $p_1$ . Both effects together result in a lower informational efficiency for  $p_1$  in the case of a precise leakage.

After the public announcement in  $t = 2$ ,  $\delta^A$  as well as  $\delta^B$  are known to some traders in the economy, (i.e. the best forecast of  $v$  given the pooled information is  $v$ ). Consequently, the measures of informational efficiency and informativeness coincide from that moment onwards. Since  $\delta^A$  is common knowledge, the conditional variance stems solely from the uncertainty about  $\delta^B$ . The proof in the appendix shows that sufficiently precise information leakage leads to a less liquid market, i.e. to a higher  $\lambda_1$ . This reduces  $\beta_1^B$  and thus makes the price signal about  $\delta^B$  less precise. In addition, the price signal  $S_2^{p_1} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{1}{\beta_1^B} u_1$  is perturbed by the  $\varepsilon$ -error term. Therefore, information leakage

makes the price  $p_1$  after the public announcement less informative and less informationally efficient. The same is true after the second trading round for the price process  $\{p_1, p_2\}$ .

In summary, information leakage reduces informational efficiency at each point in time. It makes the price process more informative prior to the public announcement and less informative afterwards.

## 4 Extensions

The propositions in Section 3 demonstrated that trader  $A$ 's informational advantage as well as his speculative and manipulative trading result from the imprecision of the rumor. The noise term  $\varepsilon$  is crucial for these results. Three interesting extensions come to mind: (i) is it possible for trader  $A$  to generate some (additional) imprecision himself in equilibrium by trading above or below his optimal level in period one; (ii) what is the optimal level of imprecision for trader  $A$ ; and (iii) how does the analysis change if we have multiple informed traders? These questions are addressed in the following subsections.

### 4.1 Mixed Strategy Equilibria

Before addressing the comparative static question (i), let us analyze the case where trader  $A$  adds some noisy component  $\gamma_1^A \zeta$  to his optimal order size. That is, his demand is of the form  $x_1^A = \beta_1^A (\delta^A + \varepsilon) + \gamma_1^A \zeta$  and he follows a mixed (or behavioral) strategy.<sup>10</sup> In order to preserve normality for all random variables, assume  $\zeta \sim \mathcal{N}(0, 1)$ . The addition of a random demand  $\gamma_1^A \zeta$  in trading round one makes the market more liquid in  $t = 1$ , but less liquid in  $t = 2$ . This occurs because trader  $A$  trades in  $t = 2$  on information generated by  $\gamma_1^A \zeta$ . The changes in the liquidity measure,  $\lambda_t$ , also alters the trading intensities,  $\beta_t^i$ . All this affects the new price signal  $S_2^{p_1} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{\gamma_1^A}{\beta_1^B} \zeta + \frac{1}{\beta_1^B} u_1$ , which has the additional error term  $\frac{\gamma_1^A}{\beta_1^B} \zeta$ . This additional term is known to trader  $A$ , but not to the other market participants. Therefore, trader  $A$ 's informational advantage in  $t = 2$  consists of his knowledge of  $\frac{\beta_1^A}{\beta_1^B} \varepsilon$  as well as of  $\frac{\gamma_1^A}{\beta_1^B} \zeta$ . These error terms differ in two respects. First, whereas trader  $A$  knows  $\zeta$  already in  $t = 1$ , he learns the precise value of  $\varepsilon$  only at the time of the public announcement. Second, if trader  $A$  wants to increase the importance of the error term  $\frac{\beta_1^A}{\beta_1^B} \varepsilon$  by varying  $\beta_1^A$ , he must also trade more aggressively on his information in  $t = 1$ . In contrast, trader  $A$  can control the impact of the error term  $\frac{\gamma_1^A}{\beta_1^B} \zeta$  on the price signal  $S_2^{p_1}$  separately by adjusting  $\gamma_1^A$ . The trade-off is that while he acts like a noise trader in

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<sup>10</sup>Pagano and Röell (1993) conjecture a mixed strategy equilibrium in a model which analyzes front-running by brokers. Investors submit their orders to the broker who forwards it to the market maker. Prior to trading the broker observes the aggregate order flows for the next two trading rounds. Hence he has more information than the market maker in the first trading period. In the first trading round he front-runs by adding his own (possibly random) orders. In Huddart, Hughes, and Levine (2000) the single insider employs a mixed strategy since the market maker can observe his order ex-post.

$t = 1$  incurring trading costs on the one hand, on the other hand he also increases his informational advantage in  $t = 2$ .

The analysis of the continuation game in  $t = 2$  is analogous to the one in Proposition 1. The only difference stems from the less informative price signal  $S_2^{p1}$ . This alters the stock split and trader  $B$ 's forecast of traders 1's forecast. Formally,  $\phi_{S_2^{p1}}^{\delta^B} = Var[\delta^B](Var[\delta^B] + (\frac{\beta_1^A}{\beta_1^B})^2 Var[\varepsilon] + (\frac{\gamma_1^A}{\beta_1^B}) + (\frac{1}{\beta_1^B})Var[u_1])^{-1}$  and  $\phi_w^{S_2^{A,w}} = \frac{(\beta_1^A)^2 Var[\varepsilon] + (\gamma_1^A)^2}{(\beta_1^A)^2 Var[\varepsilon] + (\gamma_1^A)^2 + Var[u_1]}$  change due to the additional  $\gamma_1^A$ -terms. This affects  $\beta_2^i$ ,  $\gamma_2^i$  and  $\lambda_2$ . In  $t = 1$  trader  $A$  expects a larger informational advantage for the second trading round due to randomization,  $E[S_2^{A,w}|S_1^A] = -(\phi_{S_2^{p1}}^{\delta^B} + \frac{1}{\beta_1^B}\phi_{S_2^{p1}}^{u_1})\frac{1}{\beta_1^B}[\beta_1^A\phi_{S_1^A}^\varepsilon S_1^A + \gamma_1^A\zeta]$ . Trader  $A$ 's trading rule only exhibits the proposed form  $x_1^A = \beta_1^A S_1^A + \gamma_1^A \zeta$ , if  $\lambda_1 = \psi^A = \lambda_2(\gamma_2^A)^2$ . This implies  $\beta_1^A = \frac{1}{2\lambda_1}$ .

For a mixed strategy to sustain in equilibrium, trader  $A$  has to be indifferent between any realized pure strategy, i.e. between any realization of  $\zeta$ . Since the random variable  $\zeta$  can lead to any demand with positive probability, he has to be indifferent between any  $x_1^A$  in equilibrium. This requires that the marginal trading costs in  $t = 1$  exactly offset the expected marginal gains in  $t = 2$ . Trader  $A$ 's objective function consists of two parts: the expected capital gains in  $t = 1$ ,  $(E[v|S_1^A] - \lambda_1 x_1^{A,dA})x_1^{A,dA}$  and the expected value function for capital gains in  $t = 2$ . The expected value function is also quadratic and can be written as  $(x_1^{A,dA})^2[\psi^A] + x_1^{A,dA}[-\tau^A E[S_2^{A,w}|S_1^A] - 2\psi^A x_1^A] + C_1$ . The constant  $C_1$  captures the terms which do not depend on the choice of  $x_1^{A,dA}$ .  $\psi^A := \lambda_2(\gamma_2^A)^2$  and all other variables are defined in the appendix. Figure 2 illustrates both components of the value function.

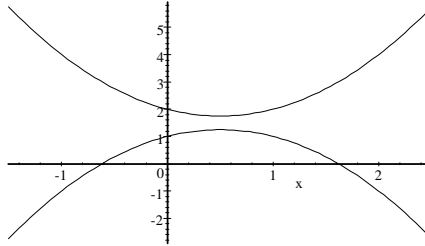


Figure 2: Components of Trader  $A$ 's Objective Function

Trader  $A$  is only indifferent between all realizations of  $\zeta$  if his objective function  $(x_1^{A,dA})^2[-\lambda_1 + \psi^A] + x_1^{A,dA}[\phi_{S_1^A}^{\delta^A} S_1^A - \tau^A E[S_2^{A,w}|S_1^A] - 2\psi^A x_1^A] + C_1$  reduces to a constant,  $C_1$ . In summary, the necessary conditions for a mixed strategy equilibrium are that  $\lambda_1 = \psi^A = \lambda_2(\gamma_2^A)^2$  and  $\phi_{S_1^A}^{\delta^A} S_1^A - \tau^A E[S_2^{A,w}|S_1^A] - 2\psi^A x_1^A = 0$ . The second necessary condition simplifies to  $1 - 2\lambda_2(\gamma_2^A)\beta_1^A = 0$ . Both conditions also imply that  $\beta_1^A = \frac{1}{2\lambda_1}$ .

Proposition 5 exploits the fact that the second order condition of trader  $A$  is binding ( $\lambda_1 = \psi^A$ ) in any mixed strategy equilibrium and that the second order condition for trader  $B$  ( $\lambda_1 > \psi^B$ ) also has to be satisfied. This allows us to rule out mixed strategy equilibria as long as the market is not very liquid in  $t = 2$ .

**Proposition 5** *There does not exist a mixed strategy equilibrium for sufficiently small  $Var[u_2]$ .*

See Appendix A.5 for the proof of this proposition. Two additional remarks are appropriate at this point. Note that the second order conditions also require that the trading round one is sufficiently liquid relative to trading round two, as stated in Proposition 1. Note also that the indifference condition requires that the expected overall profits from randomization are strictly positive. Since trader  $A$  has to be indifferent between all possible realizations of  $\zeta$ , one can restrain the attention to the realization of  $\zeta = 0$ . For  $\zeta = 0$ , he faces no randomization costs in  $t = 1$ , but still has an informational advantage in  $t = 2$ . Even if trader  $A$  receives no signal  $S^A$  in  $t = 1$  his informational advantage in  $t = 2$  in the case of  $\zeta = 0$  is  $S_2^{A,w} = E[\delta^B | \delta^B + \frac{1}{\beta_1^B} u_1] - E[\delta^B | S_2^{p1}] = (\phi_{\delta^B + \frac{1}{\beta_1^B} u_1}^{\delta^B} - \phi_{S_2^{p1}}^{\delta^B})(\delta^B + \frac{1}{\beta_1^B} u_1)$ .

## 4.2 Optimal $Var[\varepsilon]$ - Sale of Information

Trader  $A$  is only believed to follow a mixed strategy  $x_1^A = \beta_1^A(\delta^A + \varepsilon) + \gamma_1^A \zeta$  if he is indifferent between any realization of  $\zeta$ . This dramatically restricts the degrees of freedom to vary the variance of  $\gamma_1^A \zeta$ . However, we can vary  $Var[\varepsilon]$  since trader  $A$  does not observe  $\varepsilon$  in  $t = 1$ . It would be interesting to see how the expected capital gains for trader  $A$  vary as  $Var[\varepsilon]$  changes. As pointed out earlier if  $Var[\varepsilon] = 0$ , trader  $A$  will know  $\delta^A$  perfectly already in  $t = 1$  and will have no informational advantage in the second trading round. Consequently, no manipulative trading or speculative trading will occur in this case. In the other limiting case of  $Var[\varepsilon] = \infty$ , trader  $A$ 's signal is totally uninformative. This case served as benchmark case in Section 3. If  $Var[\varepsilon] = \infty$ , trader  $A$  will not engage in trading given that he did not build up an informational advantage by trading in the first trading round. In order to conduct the comparative static exercise for any possible  $Var[\varepsilon] \in (0, \infty)$ , it is necessary to explicitly derive the expected profit function of trader  $A$ . Unfortunately, a closed form expression of the expected profit function could not be obtained due to the complexity of the information structure. Any further analysis is therefore limited to numerical simulations.<sup>11</sup>

Someone who knows  $\delta^A$  in  $t = 1$  and considers selling his news to a single trader would also be interested in knowing the level of  $Var[\varepsilon]$  that would lead to the highest level of

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<sup>11</sup>Numerical analysis shows that the insider's ex-ante expected capital gains are maximized for  $Var[\varepsilon] = 0$  in a simplified setting with only one trader who also trades for manipulative reasons. Our intuition suggests that this result will also generalize to our setting. The numerical calculations are available from the author upon request.

expected profits for trader  $A$ . Given our conjecture based on numerical analysis, trader  $A$ 's profit is highest for  $Var[\varepsilon] = 0$ . Hence, the seller of information would not prefer to sell a noisy version of his signal instead of his precise information  $\delta^A$ . In Admati and Pfleiderer (1986), the information monopolist prefers to sell personalized noisy signals. Using a static rational expectations setting, they show that more traders acquire a personalized signals if the information monopolist adds an idiosyncratic personalized noise term to his information. This increases the seller's revenue. In contrast to their model setup, we employ a strategic setting with a single  $A$  trader, but two trading rounds. The noise  $\varepsilon$  of trader  $A$ 's acquired information provides him an additional informational advantage in the second trading round. In a generalized setting with endogenous information acquisition and potentially many  $A$ -traders, it might also be the case that the number of traders interested in acquiring a signal about  $\delta^A$  increases if the information monopolist sells personalized signals. The role of multiple traders is discussed in the next subsection.

### 4.3 Multiple Traders

In reality there are many informed traders active in the market. One question which might arise is whether the results derived in Section 2 also hold in a setting with many informed traders. Before increasing the number of traders let us investigate what distinguishes trader  $A$  who received a signal about  $\delta^A$  from trader  $B$  who received a signal about  $\delta^B$ . In the setting described earlier, trader  $A$ 's prior knowledge of event  $A$  causes him to trade for speculative as well as manipulative reasons. However, trader  $B$  does not act speculatively or manipulatively in  $t = 2$  despite his prior knowledge of the forthcoming public announcement about event  $B$  in  $t = 3$ . Neither the timing per se nor the fact that trader  $B$  got a precise signal about  $\delta^B$  - which is publicly announced in  $t = 3$  - can explain the difference. Trader  $B$  still would not speculate or try to manipulate the price even if his signal is imprecise, i.e.  $\delta^B + \varepsilon^B$ . This is in spite of the fact that the imprecision of trader  $A$ 's signal is necessary for trader  $A$ 's behavior. The distinctive feature is that when  $\delta^A$  is publicly announced in  $t = 2$ ,  $p_1$  still carries some information for market participants, which induces them to conduct technical analysis. This, in turn, makes it worthwhile for trader  $A$  to manipulate  $p_1$ . On the other hand, when  $\delta^B$  is announced, neither  $p_1$  nor  $p_2$  carry any additional information. Since everybody knows the true value of the stock,  $v = \delta^A + \delta^B$ , nobody trades conditional on  $p_2$ . Thus trader  $B$  has no incentive to manipulate the price in  $t = 2$ .

Having understood this crucial distinction, let us first analyze the impact of increasing the number of traders who receive some information about  $\delta^B$  in  $t = 1$ , and then increase the number of traders who can potentially act manipulatively in equilibrium. If there are many informed  $B$ -traders who receive different signals  $\delta^B + \varepsilon^{Bi}$ ,  $i \in \{1, \dots, I\}$  they have an additional incentive to conduct technical analysis. In  $t = 2$  they not only draw inferences from price  $p_1$  in order to improve their forecast of trader  $A$ 's forecast, they also try to learn more about the fundamental value  $\delta^B$ . They try to infer each others'  $\delta^B$ -signal from  $p_1$  although they know that the past price  $p_1$  is perturbed by the  $\varepsilon$ -error term. This makes

manipulation of  $p_1$  even more effective and, consequently, trader  $A$  speculates and trades to manipulate the price. On the other hand, trader  $A$  competes with a larger number of  $B$  traders in the second trading round, which reduces the expected capital gains in  $t = 2$  and thus the incentive to manipulate.<sup>12</sup>

In the context of rumors, it may be hard to envision an information structure where only a single trader receives some vague information about a forthcoming public announcement. Instead, there could be many traders who receive some signal. In a setting in which all early-informed traders receive the same signal with a common noise component,  $\delta^A + \varepsilon$ , manipulative trading and speculation still occur, but to a lesser degree. It is easy to see that as the number of  $A$ -traders converges to infinity, speculative trading still occurs whereas manipulative trading vanishes. The reason is that all  $A$ -traders try to free ride on the manipulative activity of the other manipulators. Manipulation is costly but benefits all other  $A$ -traders in the second trading round. Furthermore, a larger number of  $A$ -traders also enhances the competition in the second trading round. This lowers the expected capital gains in  $t = 2$  and hence the incentive to manipulate in  $t = 1$ .

A rumor is probably best described by an information structure where many  $A$ -traders receive individual signals  $\delta^A + \varepsilon + \xi^{Ai}$  with a common and a private noise term. For example, this is the case if every recipient of a rumor interprets it slightly differently. Even if all recipients agree on the informational content of the rumor, they can still disagree on how it impacts the fundamental value of the stock. The private noise term  $\xi^{Ai}$  alleviates the free rider problem. On the other hand as the number of traders who hear about the rumor increases, the price impact of the  $\xi^{Ai}$ -terms diminishes. In addition,  $\xi^{Ai}$  distorts trader  $Ai$ 's estimate of  $\varepsilon$ .

In summary, this discussion suggests that a rumor leads to more speculation as well as to more manipulative trading. The latter, however, only occurs as long as the rumor is not widely spread among many traders.

## 5 Conclusion

An understanding of trading patterns is essential for detecting insider trading and effectively enforcing regulatory measures. This analysis uncovered three novel features of insider trading by applying a more realistic information structure. We demonstrate that (i) insiders have an informational advantage even after the public announcement; (ii) they trade very aggressively prior to the public announcement in order to manipulate the others' price signal; and (iii) they partially unwind their position after the public announcement.

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<sup>12</sup>An earlier version of this paper included multiple  $B$ -traders in the information structure. We found that the analysis is most tractable if one assumes that  $\frac{1}{T} \sum_i \varepsilon^{Bi} = 0$ , i.e. the average of  $B$ -signals is  $\delta^B$ . A sketch of the proof for the multiple trader case is available from the author on request.

It is well understood that insider trading typically reduces risk sharing and allocative efficiency. This analysis shows that insider trading also reduces informational efficiency of prices in the long run. If the early-informed trader happens to be a corporate insider, our analysis offers additional support for the Short Swing Rule (Rule 16 b of the SEA). This rule prohibits corporate insiders to profit from buying and selling the same security within a period of six months and thus deprives corporate insiders their theoretically optimal trading strategy.

Some further extensions of this analysis come to mind. A higher order uncertainty model could be used to address the same questions. However, these models tend to be either very simplistic or very intractable. The same analysis could also be conducted in a different framework where the market maker sets bid and ask prices before the order of a trader arrives, e.g. a setting á la Glosten (1989). Preliminary analysis suggests that such a setting would yield similar outcomes. Additional insights could be obtained by endogenizing the information acquisition process. For example, traders might like to commit themselves to purchase less precise signals. It would also be interesting to determine when it is more profitable to buy imprecise information about a forthcoming announcement and when it is more lucrative to acquire long-lived information. The paper illustrates that information leakage reduces informational efficiency, but it does not make any normative welfare statements. In order to conduct a welfare analysis, one has to endogenize the trading activities of the liquidity traders. For example, one could consider risk-averse uninformed investors who are engaged in a private investment project. If the returns of these private investment projects are correlated with the value of stock, they trade for hedging reasons even though they face trading costs. A thorough welfare analysis would allow us to evaluate insider trading laws more explicitly. But these are all tasks for the future.



# A Appendix

## A.1 Proof of Proposition 1

**Propose an arbitrary action rule profile for  $t = 1$ ,**  $\{\{x_1^i(S_1^i)\}_{i \in \{A,B\}}, p_1(X_1)\}$ . This profile can be written as  $\{\{\beta_1^i\}_{i \in \{A,B\}}, p_1(X_1)\}$  since we focus on linear pure strategy sequentially rational Bayesian Nash Equilibria. Suppose that this profile is mutual knowledge among the agents and they all think it is an equilibrium profile.

**Equilibrium in continuation game in  $t = 2$ .**

**Information structure in  $t = 2$ .**

After  $\delta^A$  is publicly announced,  $\delta^B$  is the only uncertain component of the stock's value.

The *market maker* knows the aggregate order flow in  $t = 1$ ,  $X_1 = \beta_1^A(\delta^A + \varepsilon) + \beta_1^B(\delta^B) + u_1$  in addition to  $\delta^A$ . His price signal  $S_2^{p_1}$  (aggregate order flow signal,  $X_1$ ) can be written as  $S_2^{p_1} = \frac{X_1 - \beta_1^A \delta^A}{\beta_1^B} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{1}{\beta_1^B} u_1$ .

Since all market participants can invert the pricing function  $p_1 = \lambda_1 X_1$  in  $t = 2$ , they all know  $S_2^{p_1}$ . For expositional clarity let us 'virtually' *split the stock*  $v$  into a risk-free bond with payoff  $\delta^A + E[\delta^B | S_2^{p_1}]$  and a risky asset  $w$ . In equilibrium  $E[w | S_2^{p_1}] = 0$  and  $Var[w | S_2^{p_1}] = \left(1 - \phi_{S_2^{p_1}}^{\delta^B}\right) Var[\delta^B]$ . The 'virtual' split of the stock  $v$  into a risk-free bond and a risky asset  $w$  is possible, without loss of generality, since all traders are risk neutral. Such a split would also be possible in a more general setting as long as all traders have CARA utility functions.

*Trader A* can infer  $\varepsilon$  in  $t = 2$  and thus his price signal is  $\delta^B + \frac{1}{\beta_1^B} u_1$ . After orthogonalizing it to  $S_2^{p_1}$ , his signal can be written as  $S_2^{A,w} := w + \frac{1}{\beta_1^B} \vartheta_1$ , where  $w = \delta^B - E[\delta^B | S_2^{p_1}]$  and  $\vartheta_1 = u_1 - E[u_1 | S_2^{p_1}]$ . *Trader A's* forecasts of the fundamental value of  $w$  is  $E[w | S_2^{A,w}] = \phi_{S_2^{A,w}}^w S_2^{A,w}$ ,  $\phi_{S_2^{A,w}}^w = \frac{Cov[\delta^B, \delta^B + \frac{1}{\beta_1^B} u_1 | S_2^{p_1}]}{Var[\delta^B + \frac{1}{\beta_1^B} u_1 | S_2^{p_1}]} = \frac{Var[\delta^B]}{Var[\delta^B] + \frac{1}{(\beta_1^B)^2} Var[u_1]} = \phi_{\delta^B + \frac{1}{\beta_1^B} u_1}^{\delta^B}$ . *Trader A's* forecast of *trader B's* forecast is also  $E[w | S_2^{A,w}]$ .

*Trader B* knows the fundamental value  $w$ . His forecast of *trader A's* forecast is  $E[w + \frac{1}{\beta_1^B} \vartheta_1 | w, S_2^{p_1}] = E[w + \frac{1}{\beta_1^B} \vartheta_1 | w] = \phi_w^{S_2^{A,w}} w$ , where  $\phi_w^{S_2^{A,w}} = \frac{(\beta_1^A)^2 Var[\varepsilon]}{(\beta_1^A)^2 Var[\varepsilon] + Var[u_1]}$ .

**Action (trading) rules in  $t = 2$ .**

Due to potential Bertrand competition the risk-neutral *market maker* sets the price  $p_2 = E[v | X_1, X_2] = \delta^A + E[\delta^B | S_2^{p_1}] + \lambda_2 X_2$ . The first two terms reflect the value of the bond from the stock split and the last term  $\lambda_2 X_2 =: p_2^w$  is the price for  $w$ . Note that  $\lambda_2 = \frac{Cov[w, X_2 | S_2^{p_1}]}{Var[X_2 | S_2^{p_1}]}$ .

*Trader A's* optimization problem in  $t = 2$  is  $\max_{x_2^A} x_2^A E[w - p_2^w | S_2^{A,w}]$ . The first order condition of  $\max_{x_2^A} x_2^A E[w - \lambda_2 (x_2^A + \beta_2^B S_2^{B,w} + u_2) | S_2^{A,w}]$  leads to  $x_2^{A,*} = \beta_2^A S_2^{A,w}$ , where  $\beta_2^A = \frac{1}{2\lambda_2} (1 - \lambda_2 \beta_2^B) \phi_{S_2^{A,w}}^w$ .

Trader  $B$ 's optimization problem is  $\max_{x_2^B} x_2^B E[w - p_2^w | S_2^{B,w}]$ . The first order condition translates into  $x_2^{B,*} = \beta_2^B S_2^{B,w}$ , where  $\beta_2^B = \frac{1}{2\lambda_2} \left(1 - \lambda_2 \beta_2^B \phi_w^{S_2^{B,w}}\right)$ . The second order condition for both traders' maximization problem is  $\lambda_2 > 0$ .

The *equilibrium* for a given action (trading) rule profile in  $t = 1$  is given by

$$\beta_2^A = \frac{1}{2\lambda_2} \frac{1}{2} \frac{1}{1 - \frac{1}{4} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w} \phi_{S_2^{A,w}}^w, \quad \beta_2^B = \frac{1}{2\lambda_2} \frac{1 - \frac{1}{2} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w}{1 - \frac{1}{4} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w},$$

$$\lambda_2 = \left\{ \frac{1}{2} \frac{\text{Var}[\delta^B | S_2^{p_1}]}{\text{Var}[u_2]} \left( (b_2^A + b_2^B) - \frac{1}{2} (b_2^A + b_2^B)^2 \right) + \frac{1}{2} \frac{\text{Cov}[\delta^B, u_1 | S_2^{p_1}]}{\text{Var}[u_2]} \left( \frac{b_2^A}{\beta_1^B} (1 - (b_2^A + b_2^B)) \right) - \frac{1}{4} \frac{\text{Var}[u_1 | S_2^{p_1}]}{\text{Var}[u_2]} \left( \frac{b_2^A}{\beta_1^B} \right) \right\}^{\frac{1}{2}},$$

where  $b_2^i := 2\lambda_2 \beta_2^i$ . Note that  $b_2^i$  depends only on the regression coefficients  $\phi_w^{S_2^{A,w}}$  and  $\phi_{S_2^{A,w}}^w$  which are determined by the proposed action rule profile in  $t = 1$ .

### Equilibrium in $t = 1$ .

The proposed arbitrary action rule profile is an equilibrium if no player wants to deviate given the strategies of the others.

The *market maker's* pricing rule in  $t = 1$  is always given by  $p_1 = E[v | X_1] = \lambda_1 X_1$  with  $\lambda_1 = \frac{\text{Cov}[v, X_1]}{\text{Var}[X_1]}$ . He has to set an informationally efficient price due to (potential) Bertrand competition.

### Trader $A$ 's best response.

Deviation of trader  $A$  from  $x_1^A(S_1^A) = \beta_1^A S_1^A$  to  $x_1^{A,dA}(S_1^A)$  will not alter the subsequent trading intensities of the other market participants, i.e.  $\lambda_1, \beta_2^B, \lambda_2$ . They still believe that trader  $A$  plays his equilibrium strategy since they cannot detect his deviation. Nor does his deviation change his own price signals since he knows the distortion his deviation causes. The definition of  $w$  is also not affected by this deviation.

### Other market participants' misperception in $t = 2$ .

Trader  $A$ 's deviation, however, distorts the other players price signal,  $S_2^{p_1}$  to  $S_2^{p_1,dA}$ . This occurs because the other market participants attribute the difference in the aggregate order flow in  $t = 1$  not to trader  $A$ 's deviation, but to a different signal realization or different noise trading. Deviation to  $x_1^{A,dA}(\cdot)$  distorts the price signal by  $S_2^{p_1,dA} - S_2^{p_1} = \frac{1}{\beta_1^B} (x_1^{A,dA} - x_1^A)$ . Trader  $B$ 's signal prior to trading in  $t = 2$  is not  $w$  but  $w - \phi_{S_2^{p_1}}^{\delta^B} (S_2^{p_1,dA} - S_2^{p_1})$ . His market order in  $t = 2$  is, therefore,  $\beta_2^B w - \beta_2^B \phi_{S_2^{p_1}}^{\delta^B} \frac{1}{\beta_1^B} (x_1^{A,dA} - x_1^A)$ . Price  $p_2$  is also distorted. The market maker's best estimate of  $w$  prior to trading in  $t = 2$  is  $\phi_{S_2^{p_1}}^{\delta^B} (S_2^{p_1,dA} - S_2^{p_1})$  and after observing  $X_2^{dA}$ ,  $p_2^{w,dA} = \phi_{S_2^{p_1}}^{\delta^B} \frac{1}{\beta_1^B} (x_1^{A,dA} - x_1^A) + \lambda_2 (x_2^{A,dA} + \beta_2^B w - \beta_2^B \phi_{S_2^{p_1}}^{\delta^B} \frac{1}{\beta_1^B} (x_1^{A,dA} - x_1^A) + u_2)$ . Since  $\beta_2^A = \frac{1}{2\lambda_2} (1 - \lambda_2 \beta_2^B) \phi_{S_2^{A,w}}^w$ ,  $p_2^{w,dA} = \lambda_2 (x_2^{A,dA} + \beta_2^B w + u_2) + 2\lambda_2 \frac{\beta_2^A}{\beta_1^B} \frac{\phi_{S_2^{p_1}}^{\delta^B}}{\phi_{S_2^{A,w}}^w} (x_1^{A,dA} - x_1^A)$ .

Trader  $A$ 's optimal trading rule in  $t = 2$  after deviation in  $t = 1$  results from the adjusted maximization problem  $\max_{x_2^{A,dA}} E[x_2^{A,dA} (w - p_2^{w,dA}) | S_2^{A,w}]$ . It is given by

$x_2^{A,dA,*} = \beta_2^A S_2^{A,w} - \gamma_2^A (x_1^{A,dA} - x_1^A)$ , where  $\gamma_2^A := \frac{\beta_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w}$ , if the second order condition  $\lambda_2 > 0$  is satisfied.

Trader A's value function  $V_2^A(x_1^{A,dA}) = x_2^{A,dA,*} E[w - p_2^w | S_2^{A,w}]$ . After replacing  $x_2^{A,dA,*}$  with  $\beta_2^A S_2^{A,w} - \gamma_2^A (x_1^{A,dA} - x_1^A)$  and noting that  $(1 - \lambda_2 \beta_2^B) = 2\lambda_2 \beta_2^A$  it simplifies to  $V_2^A(x_1^{A,dA}) = \psi^A (x_1^{A,dA} - x_1^A)^2 - \tau^A S_2^{A,w} (x_1^{A,dA} - x_1^A) + \kappa^A (S_2^{A,w})^2$ , with  $\psi^A = \lambda_2 (\gamma_2^A)^2$ ,  $\tau^A = 2\lambda_2 \beta_2^A \gamma_2^A$ ,  $\kappa^A = \lambda_2 (\beta_2^A)^2$ . In  $t = 1$ , trader A forms expectations  $E[V_2^A(x_2^{A,dA}) | S_1^A]$  of the value function in  $t = 2$ .  $S_2^{A,w}$  is random in  $t = 1$ .  $E[S_2^{A,w} | S_1^A] = -\left(\phi_{S_2^{p_1}}^{\delta^B} + \frac{1}{\beta_1^B} \phi_{S_1^{u_1}}^u\right) \frac{\beta_1^A}{\beta_1^B} E[\varepsilon | S_1^A] = -\frac{\beta_1^A}{\beta_2^A} \gamma_2^A E[\varepsilon | S_1^A] = -\frac{1}{\beta_2^A} \gamma_2^A \left(1 - \phi_{S_1^A}^{\delta^A}\right) x_1^A$  since  $\phi_{S_1^{p_1}}^{\delta^B} + \frac{1}{\beta_1^B} \phi_{S_1^{u_1}}^u = \frac{\phi_{S_2^{p_1}}^{\delta^B}}{\phi_{S_2^{A,w}}^w}$ .

Trader A's optimization problem in  $t = 1$  is thus  $\max_{x_1^{A,dA}} E[x_1^{A,dA} (v - p_1^{dA}) + V_2^A(x_1^{A,dA}) | S_1^A]$ , where  $p_1^{dA} = \lambda_1 X_1^{dA} = \lambda_1 (x_1^{A,dA} + \beta_1^B S_1^B + u_1)$ . Since  $S_1^A$  is orthogonal to  $S_1^B$  the first order condition is  $E[\delta^A | S_1^A] - 2\lambda_1 x_1^{A,dA} + 2\psi^A (x_1^{A,dA} - x_1^A) - \tau^A E[S_2^{A,w} | S_1^A] = 0$ . Therefore,  $x_1^{A,dA,*} = \frac{1}{2(\lambda_1 - \lambda_2 (\gamma_2^A)^2)} (1 + 2\lambda_2 (\gamma_2^A)^2 \beta_1^A) \phi_{S_1^A}^{\delta^A} S_1^A$ . The second order condition is  $\lambda_1 > \lambda_2 (\gamma_2^A)^2$ . In Equilibrium  $\beta_1^A = \frac{1}{2(\lambda_1 - \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^{\delta^A})} \phi_{S_1^A}^{\delta^A}$ .

### Trader B's best response.

Other market participants' misperception in  $t = 2$ .

Deviation from  $x_1^B(S_1^B) = \beta_1^B S_1^B$  to  $x_1^{B,dB}(S_1^B)$  distorts the price signal by  $S_2^{p_1,dB} - S_2^{p_1} = \frac{1}{\beta_1^B} (x_1^{B,dB} - x_1^B)$ . Trader A's signal prior to trading in  $t = 2$  is not  $w + \frac{1}{\beta_1^B} \vartheta_1$  but  $w - \phi_{S_1^{p_1}}^{\delta^B} (S_2^{p_1,dB} - S_2^{p_1}) + \frac{1}{\beta_1^B} (\vartheta_1 - \phi_{S_1^{u_1}}^u (S_2^{p_1,dB} - S_2^{p_1}))$ . His market order is, therefore,  $x_2^{A,dB} = \beta_2^A \left(w + \frac{1}{\beta_1^B} \vartheta_1\right) - \beta_2^A \left(\phi_{S_1^{p_1}}^{\delta^B} + \frac{1}{\beta_1^B} \phi_{S_1^{u_1}}^u\right) \frac{1}{\beta_1^B} (x_1^{B,dB} - x_1^B)$ . Price  $p_2$  is also distorted. The market maker's best estimate of  $w$  prior to trading in  $t = 2$  is  $\phi_{S_1^{p_1}}^{\delta^B} (S_2^{p_1,dB} - S_2^{p_1})$  and after observing  $X_2^{dB}$ ,  $p_2^{w,dB} = \phi_{S_1^{p_1}}^{\delta^B} (S_2^{p_1,dB} - S_2^{p_1}) + \lambda_2 (x_2^{A,dB} + x_2^{B,dB} + u_2)$ . Let  $\gamma_2^B := \frac{1}{2\lambda_2} [\phi_{S_1^{p_1}}^{\delta^B} - \lambda_2 \beta_2^A (\phi_{S_1^{p_1}}^{\delta^B} + \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^u)] \frac{1}{\beta_1^B}$  then  $p_2^{w,dB} = \lambda_2 \left(\beta_2^A \left(w + \frac{1}{\beta_1^B} \vartheta_1\right) + x_2^{B,dB} + u_2\right) + 2\lambda_2 \gamma_2^B (x_1^{B,dB} - x_1^B)$ .

Trader B's optimal trading rule in  $t = 2$  after deviation in  $t = 1$  is the result of  $\max_{x_2^{B,dB}} E[x_2^{B,dB} (w - p_2^{w,dB}) | S_2^{B,w}]$ . The optimal order size in  $t = 2$  is  $x_2^{B,dB,*} = \beta_2^B S_2^{B,w} - \gamma_2^B (x_1^{B,dB} - x_1^B)$ , if the second order condition  $\lambda_2 > 0$  is satisfied. Note that if

we replace  $\beta_2^A$  with  $\frac{1}{2\lambda_2} (1 - \lambda_2 \beta_2^B) \phi_{S_2^{A,w}}^w$ ,  $\gamma_2^B$  simplifies to  $\frac{1}{2} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \frac{3 - \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w}{4 - \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w}$ .

Trader  $B$ 's value function  $V_2^B(x_1^{B,dB}) = x_2^{B,dB,*} E[w - p_2^w | S_2^{B,w}]$ . After replacing  $x_2^{B,dB,*}$  with  $\beta_2^B S_2^{B,w} - \gamma_2^B (x_1^{B,dB} - x_1^B)$  and noting that  $(1 - \lambda_2 \beta_2^A \phi_w^{S_2^{A,w}}) = 2\lambda_2 \beta_2^B$  it simplifies to  $V_2^B(x_1^{B,dB}) = \psi^B (x_1^{B,dB} - x_1^B)^2 - \tau^B S_2^{B,w} (x_1^{B,dB} - x_1^B) + \kappa^B (S_2^{B,w})^2$ , with  $\psi^B = \lambda_2 (\gamma_2^B)^2$ ,  $\tau^B = 2\lambda_2 \beta_2^B \gamma_2^B$ ,  $\kappa^B = \lambda_2 (\beta_2^B)^2$ . In  $t = 1$ , trader  $B$  forms expectations  $E[V_2^B(x_2^{B,dB}) | \delta^B]$  of the value function in  $t = 2$ .  $S_2^{B,w} = w$  is random in  $t = 1$ . The expectation of  $S_2^{B,w}$  is given by  $E[S_2^{B,w} | S_1^B] = E[w | \delta^B] = (1 - \phi_{S_2^{p_1}}^{\delta^B}) \delta^B$ .

Trader  $B$ 's optimization problem in  $t = 1$  is thus  $\max_{x_1^{B,dB}} E[x_1^{B,dB} (v - p_1^{dB}) + V_2^B(x_1^{B,dB}) | S_1^B]$ , where  $p_1^{dB} = \lambda_1 X_1^{dB} = \lambda_1 (\beta_1^A S_1^A + x_1^{B,dB} + u_1)$ . Since  $S_1^B$  is orthogonal to  $S_1^A$  the first order condition reduces to  $x_1^{B,dB,*} = \frac{1}{2(\lambda_1 - \lambda_2 (\gamma_2^B)^2)} \left( \left( 1 - 2\lambda_2 \beta_2^B \gamma_2^B (1 - \phi_{S_2^{p_1}}^{\delta^B}) \right) - 2\lambda_2 (\gamma_2^B)^2 \beta_1^B \right) S_1^B$ . The second order condition is  $\lambda_1 > \lambda_2 (\gamma_2^B)^2$ .

The sequentially rational **Bayesian Nash Equilibrium** is given by a fixed point in  $(\beta_1^{A,*}, \beta_1^{B,*})$ .

$$\beta_1^{A,*} = \frac{1}{2(\lambda_1 - \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^{\delta^A})} \phi_{S_1^A}^{\delta^A}$$

$$\beta_1^{B,*} = \frac{1}{2\lambda_1} \left( 1 - 2\lambda_2 \beta_2^B \gamma_2^B (1 - \phi_{S_2^{p_1}}^{\delta^B}) \right),$$

where

$$\lambda_1 = \frac{\beta_1^{A,*} \text{Var}[\delta^A] + \beta_1^{B,*} \text{Var}[\delta^B]}{\text{Var}[\beta_1^{A,*} (\delta^A + \varepsilon) + \beta_1^{B,*} (\delta^B) + u_1]}$$

with

$$\beta_2^A = \frac{1}{2\lambda_2} \frac{1}{2} \frac{1}{1 - \frac{1}{4} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w} \phi_{S_2^{A,w}}^w \quad \gamma_2^A := \frac{\beta_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w}$$

$$\beta_2^B = \frac{1}{2\lambda_2} \frac{1 - \frac{1}{2} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w}{1 - \frac{1}{4} \phi_w^{S_2^{A,w}} \phi_{S_2^{A,w}}^w} \quad \gamma_2^B := \frac{1}{2\lambda_2} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} - \frac{1}{2} \frac{\beta_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w}$$

$$\lambda_2 = \left\{ \frac{1}{2} \frac{\text{Var}[\delta^B | S_2^{p_1}]}{\text{Var}[u_2]} \left( (b_2^A + b_2^B) - \frac{1}{2} (b_2^A + b_2^B)^2 \right) + \frac{1}{2} \frac{\text{Cov}[\delta^B, u_1 | S_2^{p_1}]}{\text{Var}[u_2]} \left( \frac{b_2^A}{\beta_1^B} (1 - (b_2^A + b_2^B)) \right) - \frac{1}{4} \frac{\text{Var}[u_1 | S_2^{p_1}]}{\text{Var}[u_2]} \left( \frac{b_2^A}{\beta_1^B} \right) \right\}^{\frac{1}{2}},$$

where  $b_2^i := 2\lambda_2 \beta_2^i$   $i \in \{A, B\}$  if the second order conditions

$$\lambda_2 > \lambda_1 \max \left\{ \left[ \frac{b_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w} \right]^2, \left[ \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} - \frac{1}{2} \frac{b_2^A \phi_{S_2^{p_1}}^{\delta^B}}{\beta_1^B \phi_{S_2^{A,w}}^w} \right]^2 \right\}, \lambda_2 > 0 \text{ are satisfied. } \blacksquare$$

## A.2 Proof of Proposition 2

The  $p_1$ -price signal for the market maker as well as for trader  $B$  is given by  $S_2^{p_1} = \delta^B + \frac{\beta_1^A}{\beta_1^B} \varepsilon + \frac{1}{\beta_1^B} u_1$ . Trader  $A$  can infer  $\varepsilon$  and thus his price signal is more precise. Trader  $A$ 's informational advantage  $\frac{\beta_1^A}{\beta_1^B} \varepsilon$  increases in  $\beta_1^A$  and decreases in  $\beta_1^B$ .  $\blacksquare$

### A.3 Proof of Proposition 3

#### Speculative Trading

Trader  $A$  expects to trade  $\beta_2^A E[S_2^{A,w} | S_1^A]$  in  $t = 2$ .

Since  $E[S_2^{A,w} | S_1^A] = -\left(\phi_{S_2^{p_1}}^{\delta^B} + \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{u_1}\right) \frac{\beta_1^A}{\beta_1^B} \phi_{S_1^A}^\varepsilon S_1^A = -\frac{\beta_1^A}{\beta_2^A} \gamma_2^A \phi_{S_1^A}^\varepsilon S_1^A$  and  $\beta_1^A, \beta_2^A > 0$ , trader  $A$  expects to sell (buy) stocks in  $t = 2$  if he buys (sells) stocks in  $t = 1$ .

#### Manipulative Trading

Trader  $A$  trades excessively for manipulative reasons if  $\beta_1^A > \beta_1^{A, \text{myopic}}$  (given the strategies of the other market participants).

$\beta_1^A = \frac{1}{2\left(\lambda_1 - \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon\right)} \phi_{S_1^A}^{\delta^A}$  whereas  $\beta_1^{A, \text{myopic}} = \frac{1}{2\lambda_1} \phi_{S_1^A}^{\delta^A}$ . Thus manipulative trading is given by  $\frac{\lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon}{2\lambda_1 \left(\lambda_1 - \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon\right)} \phi_{S_1^A}^{\delta^A} S_1^A$ . The second order condition requires that  $\lambda_1 > \lambda_2 (\gamma_2^A)^2 \phi_{S_1^A}^\varepsilon$ .

Note that for  $Var[\varepsilon] = 0$ ,  $\phi_{S_1^A}^\varepsilon = 0$  neither speculative nor manipulative trading will occur. ■

### A.4 Proof of Proposition 4

This proposition compares two different equilibria: one with information leakage and one without. Let us denote all variables of the former equilibrium with upper bars and all variables of the latter equilibrium with tilde.

If Proposition 4 holds for  $Var[\varepsilon] = 0$  with strict inequalities it also holds for positive  $Var[\varepsilon]$  in the environment around  $Var[\varepsilon] = 0$ .

As  $Var[\varepsilon] \rightarrow 0$  the equilibrium strategies converge continuously to  $\bar{\beta}_2^A \rightarrow 0$ ,  $\bar{\beta}_2^B \rightarrow \frac{1}{2\lambda_2}$ ,  $\bar{\phi}_w^{S_2^{A,w}} \rightarrow 0$ ,  $\bar{\phi}_{S_1^A w}^\varepsilon \rightarrow 0$ ,  $\bar{\gamma}_2 \rightarrow \frac{1}{2\lambda_2} \frac{1}{2\bar{\beta}_1^B} \bar{\phi}_{S_2^{p_1}}^{\delta^B}$ , and thus  $\bar{\beta}_1^A \rightarrow \frac{1}{2\lambda_1}$  ( $\bar{b}_1^A \rightarrow 1$ ). For any

given  $\lambda_1, \lambda_2$  simplifies to  $\frac{1}{2} \sqrt{\frac{(1 - \phi_{S_2^{p_1}}^{\delta^B}) Var[\delta^B]}{Var[u_2]}}$  and trader  $B$ 's trading intensity is  $\beta_1^B = \frac{1}{2\lambda_1} - \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \frac{3}{4} \sqrt{(1 - \phi_{S_2^{p_1}}^{\delta^B}) \frac{Var[u_2]}{Var[\delta^B]}}$ . That is, the corresponding  $\lambda_1$  for a given  $\beta_1^B$  is given by  $\lambda_1 = \frac{1/2}{\beta_1^B + \frac{3}{4} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \sqrt{(1 - \phi_{S_2^{p_1}}^{\delta^B}) \frac{Var[u_2]}{Var[\delta^B]}}}$ .

#### Prior to public announcement

##### *Informativeness*

$\bar{p}_1$  is more informative than  $\tilde{p}_1$ , i.e.

$$Var[\delta^A + \delta^B | \bar{\beta}_1^A (\delta^A + \varepsilon) + \bar{\beta}_1^B \delta^B + u_1] < Var[\delta^B | \tilde{\beta}_1^B \delta^B + u_1] + Var[\delta^A]$$

$$Var[\delta^A + \delta^B] - \bar{\lambda}_1 \bar{\beta}_1^A Var[\delta^A] - \lambda_1 \bar{\beta}_1^B Var[\delta^B] < Var[\delta^A + \delta^B] - \tilde{\lambda}_1 \tilde{\beta}_1^B Var[\delta^B]$$

$$-\frac{1}{2} \bar{b}_1^A Var[\delta^A] - \frac{1}{2} \bar{b}_1^B Var[\delta^B] < -\frac{1}{2} \tilde{b}_1^B Var[\delta^B]$$

For  $Var[\delta^A] = Var[\delta^B]$  and  $Var[\varepsilon] \rightarrow 0$ ,  $\bar{b}_1^A \rightarrow 1$  the inequality above simplifies to

$1 > \tilde{b}_1^B - \bar{b}_1^B$ . Since  $\bar{b}_1^B, \tilde{b}_1^B \in ]0, 1[$  this is always satisfied.

### Informational Efficiency

$\bar{p}_1$  is less informationally efficient than  $\tilde{p}_1$ , i.e.

$$Var[\phi_{S_1^A}^{\delta^A}(\delta^A + \varepsilon) + \delta^B | \bar{\beta}_1^A(\delta^A + \varepsilon) + \bar{\beta}_1^B \delta^B + u_1] > Var[\delta^B | \tilde{\beta}_1^B \delta^B + u_1]$$

Since  $Cov[\phi_{S_1^A}^{\delta^A}(\delta^A + \varepsilon) + \delta^B, X_1] = Cov[v, X_1] = \lambda_1 Var[X_1]$

$$\phi_{S_1^A}^{\delta^A}(Var[\delta^A + \varepsilon]) - \bar{\lambda}_1 \bar{\beta}_1^A Var[\delta^A + \varepsilon] - \bar{\lambda}_1 \bar{\beta}_1^B Var[\delta^B] > -\tilde{\lambda}_1 \tilde{\beta}_1^B Var[\delta^B]$$

$Var[\delta^A] - \frac{1}{2} \bar{b}_1^A Var[\delta^A + \varepsilon] > \frac{1}{2} (\bar{b}_1^B - \tilde{b}_1^B) Var[\delta^B]$ . For  $Var[\delta^A] = Var[\delta^B]$  and  $Var[\varepsilon] \rightarrow 0$  the inequality to  $1 > \bar{b}_1^B - \tilde{b}_1^B$ . This is always true.

### Prior to trading in $t = 2$

If  $\tilde{\beta}_1^B > \bar{\beta}_1^B$  then  $\tilde{S}^{p_1} = \delta^B + \frac{1}{\tilde{\beta}_1^B} u_1$  is more informative than  $\bar{S}^{p_1} = \delta^B + \frac{1}{\bar{\beta}_1^B} u_1$ , even if  $Var[\varepsilon] = 0$ .

In the  $(\beta_1^B, \lambda_1)$ -space the equilibrium is determined by the intersection of

$$\lambda_1 = \frac{1/2}{\beta_1^B + \frac{3}{4} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \sqrt{(1 - \phi_{S_2^{p_1}}^{\delta^B}) \frac{Var[u_2]}{Var[\delta^B]}}} \quad (1) \text{ with } \tilde{\lambda}_1 = \frac{\tilde{\beta}_1^B Var[\delta^B]}{(\tilde{\beta}_1^B)^2 Var[\delta^B] + Var[u_1]} \quad (2) \text{ in the case where no}$$

information leaks and with  $\bar{\lambda}_1 = \frac{\frac{1}{2\lambda_1} Var[\delta^A] + \bar{\beta}_1^B Var[\delta^B]}{(\frac{1}{2\lambda_1})^2 + (\bar{\beta}_1^B)^2 Var[\delta^B] + Var[u_1]} \quad (3) \text{ in the case of information}$

leakage. (3) can be simplified to

$$\bar{\lambda}_1 = \frac{\bar{\beta}_1^B Var[\delta^B] + \sqrt{(\bar{\beta}_1^B)^2 Var[\delta^B]^2 + (\bar{\beta}_1^B)^2 Var[\delta^B] Var[\delta^A] + Var[u_1] Var[\delta^A]}}{2\{(\bar{\beta}_1^B)^2 Var[\delta^B] + Var[u_1]\}}. \text{ Note that we can restrict}$$

our attention to the positive root only because of the second order condition.

*Claim 1:*  $\bar{\lambda}_1(\bar{\beta}_1^B) > \tilde{\lambda}_1(\tilde{\beta}_1^B)$  for all  $\bar{\beta}_1^B = \tilde{\beta}_1^B$  follows immediately.

*Claim 2:*  $\lambda_1(\beta_1^B) = \frac{1/2}{\beta_1^B + \frac{3}{4} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \sqrt{(1 - \phi_{S_2^{p_1}}^{\delta^B}) \frac{Var[u_2]}{Var[\delta^B]}}} \quad (1) \text{ is strictly decreasing in } \beta_1^B \text{ as long as}$

$$Var[u_1] > \frac{6}{25} \sqrt{\frac{2}{5} Var[\delta^B] Var[u_2]}.$$

Its derivative is negative if the denominators' derivative is positive. The denominator can be rewritten as

$$\beta_1^B + \frac{3}{4} (\beta_1^B)^{-2} \left( Var[\delta^B] + (\beta_1^B)^{-2} Var[u_1] \right)^{-1.5} (Var[\delta^B] Var[u_1] Var[u_2])^{0.5}. \text{ Its derivative}$$

w.r.t.  $\beta_1^B$  is  $1 + \frac{3}{4} \frac{\sqrt{Var[\delta^B] Var[u_2]}}{Var[u_1]} \left(1 - \phi_{S_2^{p_1}}^{\delta^B}\right)^{1.5} \left(2 - 3 \left(1 - \phi_{S_2^{p_1}}^{\delta^B}\right)\right)$ . The global minimum for

$\left(1 - \phi_{S_2^{p_1}}^{\delta^B}\right)^{1.5} \left(2 - 3 \left(1 - \phi_{S_2^{p_1}}^{\delta^B}\right)\right)$  at  $\phi_{S_2^{p_1}}^{\delta^B} = \frac{2}{5}$  is  $-\frac{2}{5} \sqrt{\frac{2}{5} \frac{4}{5}}$ . From this it follows immediately

that for  $Var[u_1] > \frac{6}{25} \sqrt{\frac{2}{5} Var[\delta^B] Var[u_2]}$ ,  $\frac{\partial \lambda_1}{\partial \beta_1^B} < 0$ .

*Claim 3:*  $\tilde{\lambda}_1(\tilde{\beta}_1^B)$  is weakly increasing in  $\tilde{\beta}_1^B$ , i.e.  $\frac{\partial \tilde{\lambda}_1}{\partial \tilde{\beta}_1^B} \geq 0$ .

$$\frac{\partial \tilde{\lambda}_1}{\partial \tilde{\beta}_1^B} = \frac{Var[\delta^B] \left( Var[u_1] - (\tilde{\beta}_1^B)^2 Var[\delta^B] \right)}{\left( (\tilde{\beta}_1^B)^2 Var[\delta^B] + Var[u_1] \right)^2}. \quad \frac{\partial \tilde{\lambda}_1}{\partial \tilde{\beta}_1^B} \geq 0 \text{ if } Var[u_1] \geq \left( \tilde{\beta}_1^B \right)^2 Var[\delta^B]. \text{ Replacing}$$

$\tilde{\beta}_1^B$  with  $\frac{1}{2} \sqrt{\frac{Var[u_1]}{Var[\delta^B] \frac{1}{2} \tilde{b}_1^B (1 - \frac{1}{2} \tilde{b}_1^B)}} \tilde{b}_1^B$  the condition simplifies to  $\tilde{b}_2^A \leq 1$ . This is always the case in equilibrium.

From Claim 1 to 3 it follows that  $\bar{\lambda}_1 > \tilde{\lambda}_1$  and  $\bar{\beta}_1^B < \tilde{\beta}_1^B$  in the corresponding equilibria.

### After trading in $t = 2$

The continuation game in  $t = 2$  corresponds to a static Kyle (1985) model with a risky asset  $w$ . Since the variance  $Var[w] = Var[\delta^B | S_2^{p_1}]$  is higher for lower  $\beta_1^B$ , the price process  $\{p_1, p_2\}$  reveals less information. ■

## A.5 Proof of Proposition 5

In any mixed strategy equilibrium, player 1 has to be indifferent between any  $x_1^A$ , i.e.  $\lambda_1 = \lambda_2 (\gamma_2^A)^2$ . In addition, the second order condition of trader  $B$ ,  $\lambda_1 \geq \lambda_2 (\gamma_2^B)^2$  must hold. Thus, a necessary condition for a mixed strategy equilibrium is

$$\begin{aligned} \gamma_2^B &\leq \gamma_2^A \\ \frac{\frac{1}{2} \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B}}{4 - \phi_{S_2^A, w}^w \phi_w^{S_2^A, w}} \frac{3 - \phi_{S_2^A, w}^w \phi_w^{S_2^A, w}}{\phi_w^{S_2^A, w}} &\leq \frac{1}{\beta_1^B} \phi_{S_2^{p_1}}^{\delta^B} \frac{1}{2\lambda_2} \frac{\frac{1}{2}}{1 - \frac{1}{4} \phi_{S_2^A, w}^w \phi_w^{S_2^A, w}} \\ \lambda_2 &\leq \frac{2}{3 - \phi_{S_2^A, w}^w \phi_w^{S_2^A, w}} < 1, \end{aligned}$$

$$\text{where } \phi_{S_2^A, w}^w \phi_w^{S_2^A, w} = \frac{(\beta_1^A)^2 Var[\varepsilon] + (\gamma_1^A)^2 Var[\zeta]}{(\beta_1^A)^2 Var[\varepsilon] + Var[u_1] + (\gamma_1^A)^2 Var[\zeta]} \frac{Var[\delta^B]}{Var[\delta^B] + \left(\frac{1}{\beta_1^B}\right)^2 Var[u_1]} < 1.$$

Since  $\lambda_2$  is strictly decreasing in  $Var[u_2]$  with  $Var[u_2] \rightarrow 0 \Rightarrow \lambda_2 \rightarrow \infty$ , there exists no mixed strategy equilibrium for the case where  $Var[u_2]$  is strictly smaller than the constant  $C_{Var[u_2]}^*$ . ■

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