
Bubbles and Crashes

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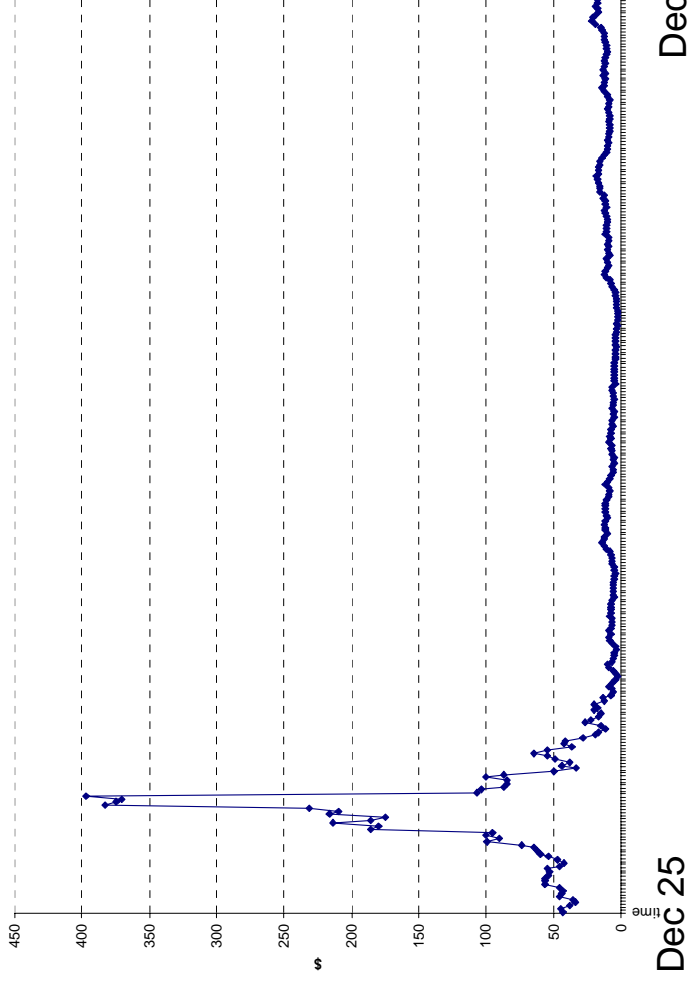
<http://www.princeton.edu/~markus>

Story of a typical technology stock

- Company X introduced a revolutionary wireless communication technology.
- It not only provided support for such a technology but also provided the informational content itself.
- It's IPO price was \$1.50 per share. Six years later it was traded at \$ 85.50 and in the seventh year it hit \$ 114.00.
- The P/E ratio got as high as 73.
- The company never paid dividends.

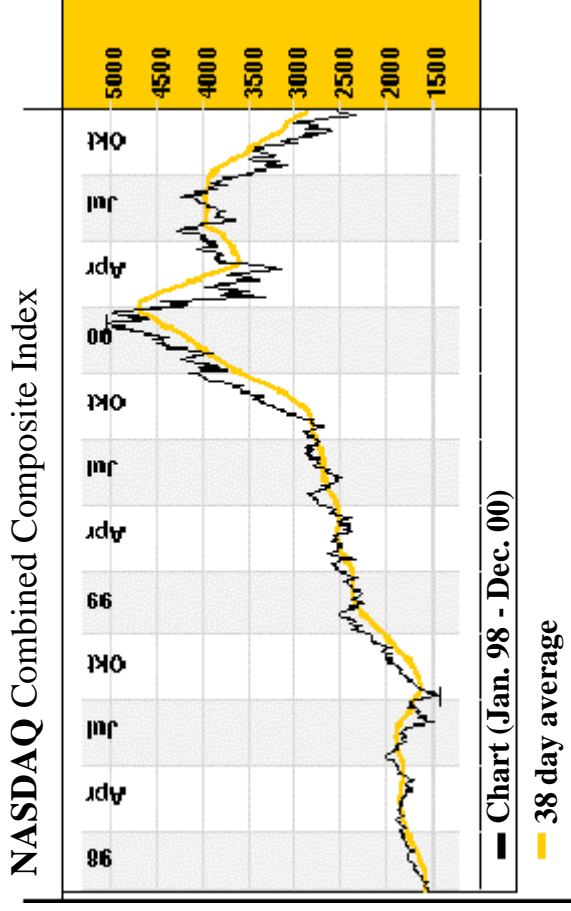
Story of RCA - 1920's

- Company: *Radio Corporation of America (RCA)*
- Technology: *Radio*
- Year: *1920's*



- ▶ It peaked at \$ 397 in Feb. 1929, down to \$ 2.62 in May 1932,

Internet bubble? - 1990's



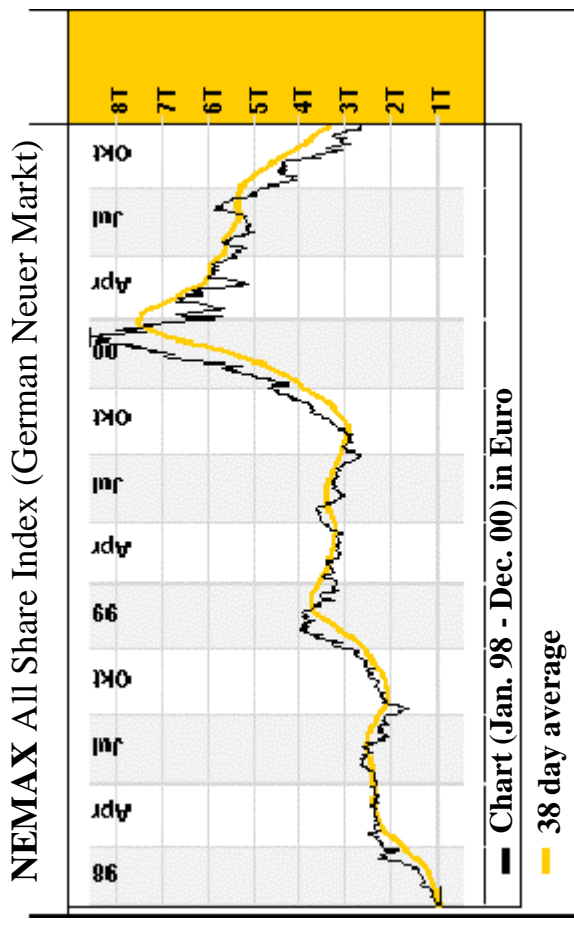
Loss of ca. **60 %**
from high of \$ 5,132

■ Are bubbles *recurrent*?

■ What happened in March 2000?

■ Further evidence of bubble

- ▶ crash was not accompanied by fundamental news.
- ▶ excess volatility



Loss of ca. **85 %**
from high of Euro 8,583

Do (rational) professional ride the bubble?

- South Sea Bubble (1710 - 1720)
 - ▶ *Isaac Newton*
 - 04/20/1720 sold shares at £7,000 profiting £3,500
 - re-entered the market later - ended up losing £20,000
 - “I can calculate the motions of the heavenly bodies, but not the madness of people”
- Internet Bubble (1992 - 2000)
 - ▶ *Druckenmiller* of Soros’ Quantum Fund didn’t think that the party would end so quickly.
 - “We thought it was the eighth inning, and it was the ninth”
 - ▶ *Julian Robertson* of Tiger Fund refused to invest in internet stocks

Pros' dilemma

- ▶ “The moral of this story is that irrational market can kill you ...
- ▶ Julian said ‘This is irrational and I won’t play’ and they carried him out feet first.
- ▶ Druckenmiller said ‘This is irrational and I will play’ and they carried him out feet first.”

Quote of a financial analyst, *New York Times*

April, 29 2000

Classical Question

- ▶ Suppose behavioral trading leads to mispricing.
- **Can mispricings or bubbles persist in the presence of rational arbitrageurs?**
- What type of information can lead to the bursting of bubbles?

Main Literature

- **Keynes (1936)** ⇒ bubble can emerge
 - ▶ “It might have been supposed that *competition between expert professionals*, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.”
- **Friedman (1953), Fama (1965)**
- **Efficient Market Hypothesis** ⇒ no bubbles emerge
 - ▶ “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.”
- **Limits to Arbitrage**
 - ▶ arbitrageurs are myopic/short-lived and risk averse (DeLong et al. [DSSW], 1990a)
 - ▶ fund managers (arbitrageurs) face liquidation risk due to principal-agent problem (Shleifer & Vishny, 1997)
 - ▶ arbitrageurs exploit delayed reaction of feedback traders (DeLong et al. [DSSW], 1990b)

Market timing & Synchronization

- Market timing
 - ① ride the bubble as long as possible (conflicting interest)
 - sell right before others
 - ② bubble only bursts if more than κ arbitrageurs attack
 - CONFLICT ← (common action)
 - ③ arbitrageurs become sequentially aware of the mispricing



Lack of synchronization



“delayed arbitrage”



Bubble persists

introduction

model setup

persistence of **bubbles**

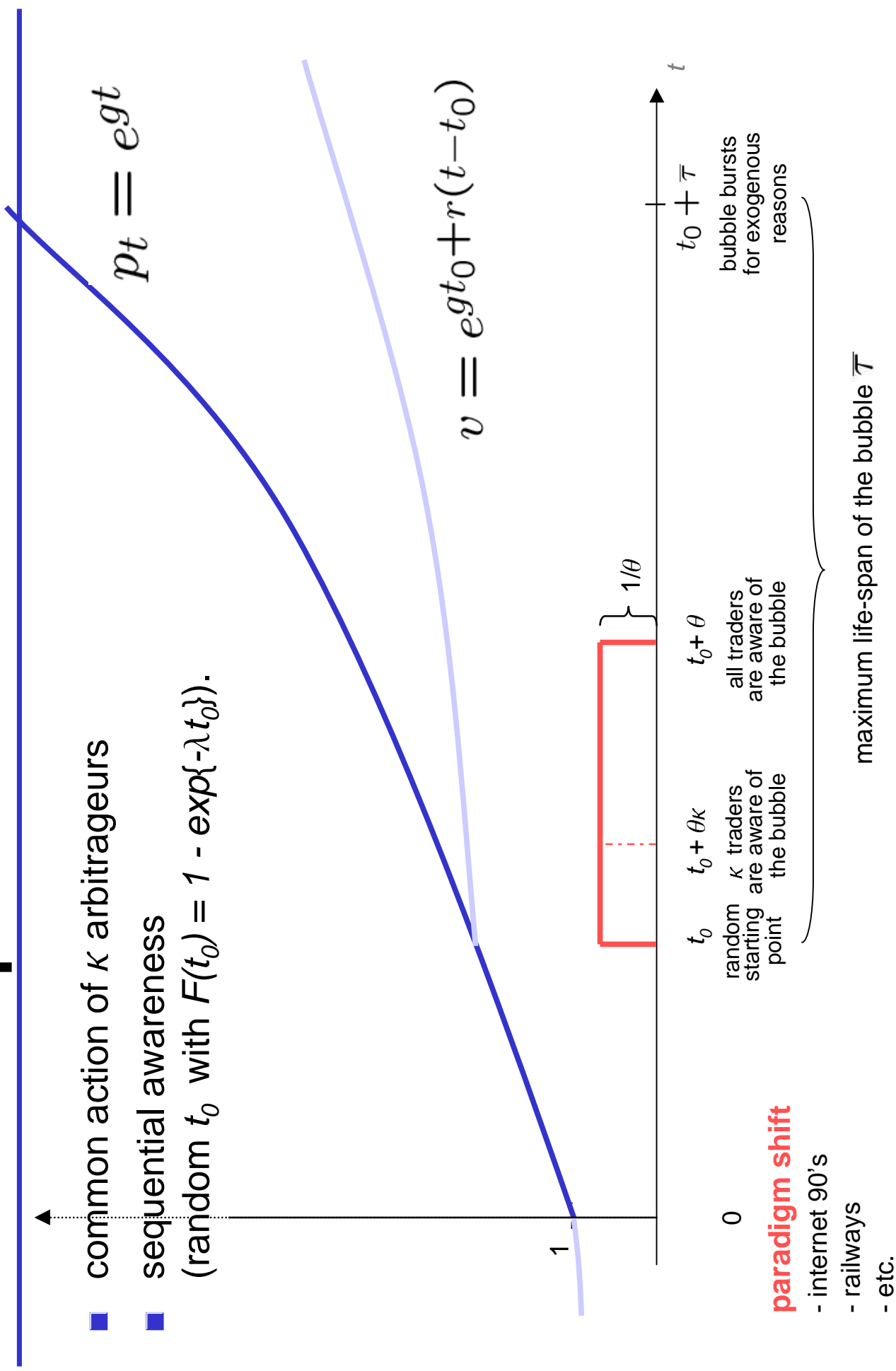
endogenous **crashes**

public events

conclusion

Model setup

- common action of κ arbitrageurs
- sequential awareness
(random t_0 with $F(t_0) = 1 - \exp\{-\lambda t_0\}$).



Payoff structure

- Cash Payoffs (difference)
 - ▶ Sell 'one share' at $t-\Delta$ instead of at t .

$$p_{t-\Delta} e^{r\Delta} - p_t$$

$$\text{where } p_t = \begin{cases} e^{gt} & \text{prior to the crash} \\ e^{gt_0+r(t-t_0)} & \text{after the crash} \end{cases}$$

- ▶ Execution price at the time of bursting.

$$p_t^{\text{burst}} = \begin{cases} e^{gt} & \text{for first random orders up to } \kappa \\ e^{gt_0+r(t-t_0)} & \text{all other orders} \end{cases}$$

Payoff structure (ctd.), Trading

- Reputational penalty cp_t
for attacking if bubble does not burst
 - ▶ relative performance evaluation
 - ▶ draw downs
- Risk-neutrality but max/min stock position
 - ▶ max long position $\sigma = 0$
 - ▶ max short position $\sigma = 1$
 - ▶ due to capital constraints, margin requirements etc.
- **Definition 1: *trading equilibrium***
 - ▶ Perfect Bayesian Nash Equilibrium
 - ▶ Belief restriction: trader who attacks at time t believes that all traders who became aware of the bubble prior to her also attack at t .

introduction

model setup

persistence of bubbles

arbitrageurs never burst bubble

graphical illustration

proof for symmetric strategies

lack of common knowledge

endogenous crashes

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Persistence of Bubbles

- **Proposition 1:** Suppose $\theta_K \geq -\ln\left(1 - \frac{\lambda}{g-r+c}\right)\left(\frac{1}{\lambda}\right)$
 - ▶ existence of a unique trading equilibrium
 - ▶ traders begin attacking after a delay of $\tau^* < \bar{\tau}$ periods.
 - ▶ bubble **never** bursts due to endogenous selling pressure. It only bursts at $t_0 + \bar{\tau}$.

Attack condition for $\Delta \rightarrow 0$ periods

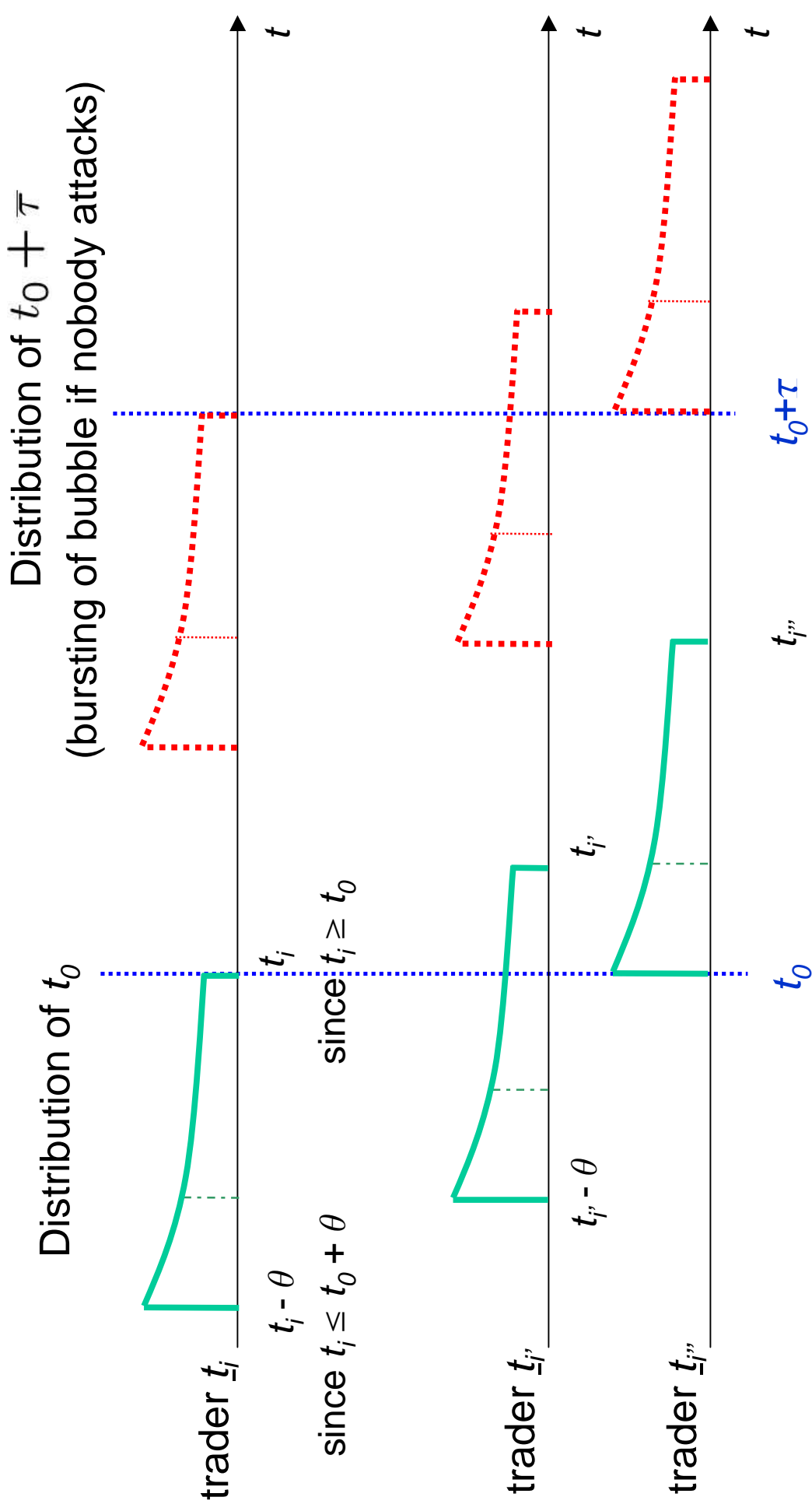
- attack at t
 - ▶ if prob that bubble will burst in next instant > 0
 - ▶ if prob that bubble will burst in next instant $= 0$

$$\underbrace{\Delta h(t|t_i) E_t[\text{bubble}|\cdot]}_{\text{benefit of attacking}} \geq \underbrace{(1 - \Delta h(t|t_i)) [(g - r) + c] p_t \Delta}_{\substack{\text{reputational penalty} \\ \text{cost of attacking}}} \underbrace{\quad}_{\text{appreciation rate}}$$

$$h(t|t_i) \geq \frac{(g-r)+c}{1-E[e^{-(g-r)(t-t_0)}|\cdot]}$$

RHS converges to $\rightarrow [(g-r) + c]$ as $t \rightarrow \infty$

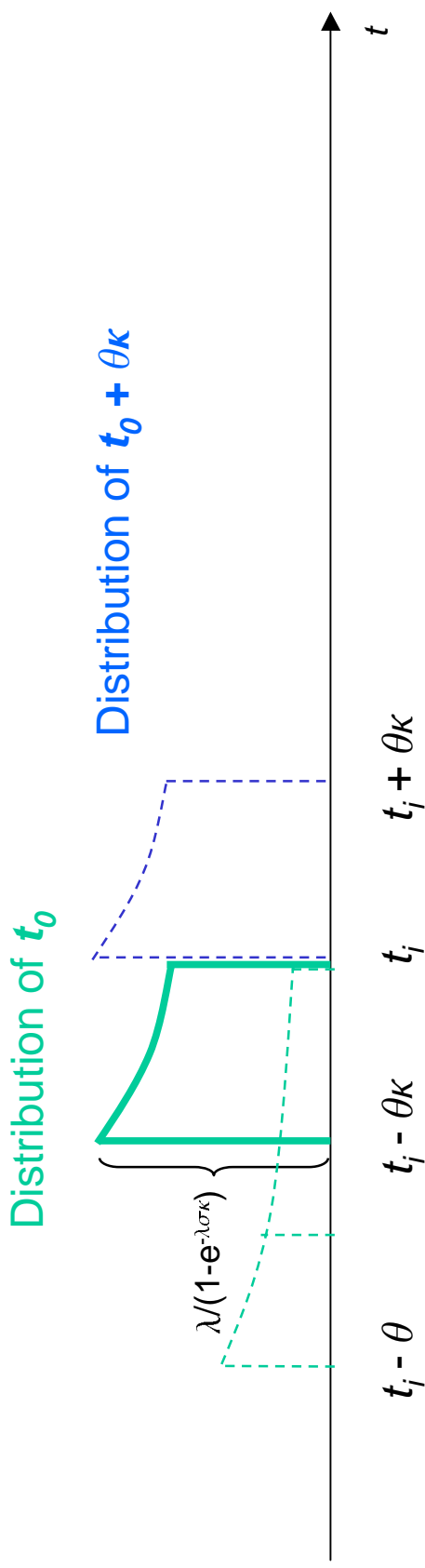
Sequential awareness



Conjecture 1: Immediate attack

⇒ **Bubble bursts at $t_0 + \theta\kappa$**

when κ traders are aware of the bubble

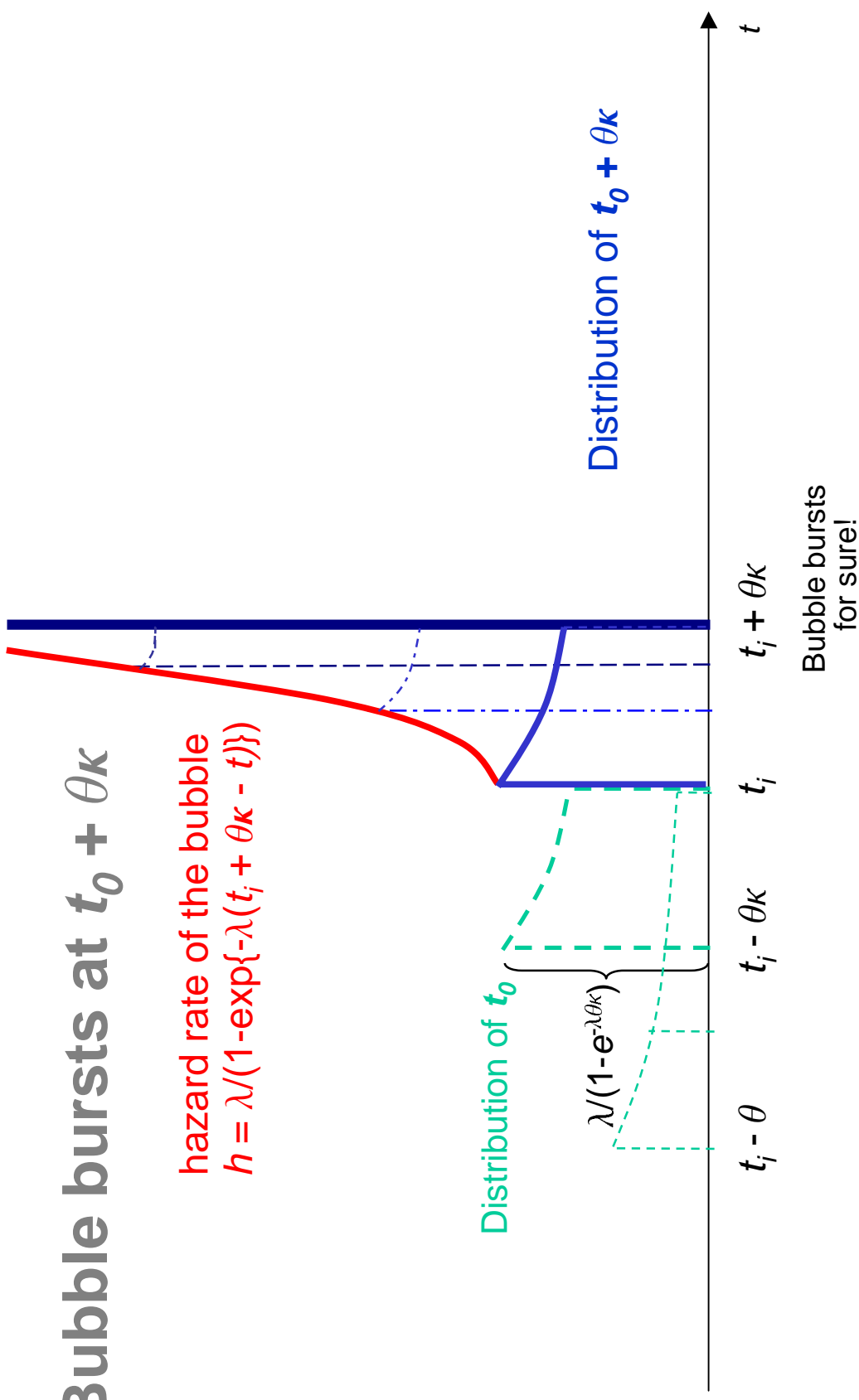


If $t_0 < t_i - \theta\kappa$, the bubble would have burst already.

Conj. 1 (ctd.): Immediate attack

⇒ Bubble bursts at $t_0 + \theta\kappa$

hazard rate of the bubble
 $h = \lambda(1 - \exp\{-\lambda(t_j + \theta\kappa - t)\})$



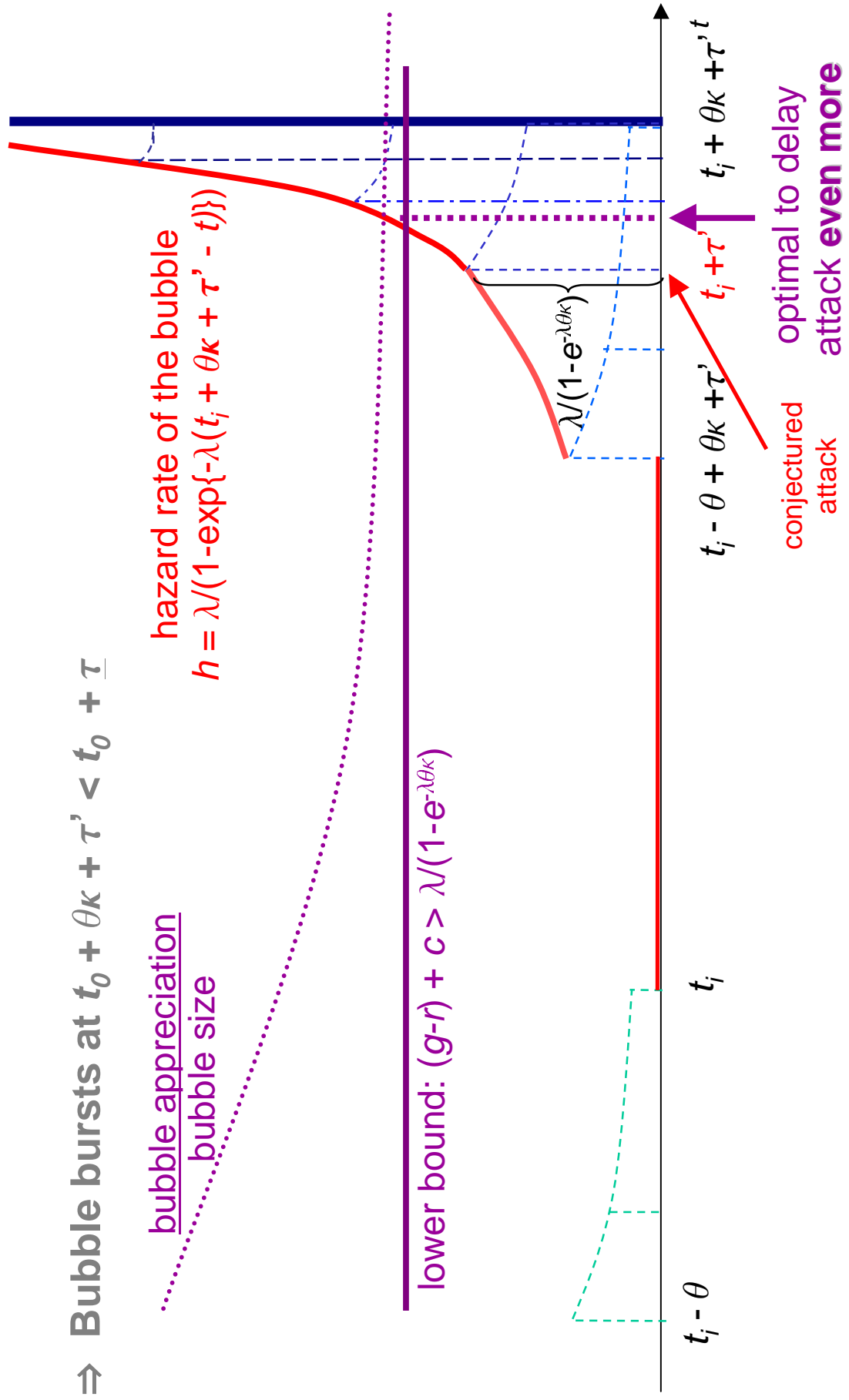
Conj. 2: Delayed attack by arbitrary τ

⇒ Bubble bursts at $t_0 + \theta_K + \tau' < t_0 + \tau$

bubble appreciation
bubble size

hazard rate of the bubble
 $h = \lambda(1 - \exp\{-\lambda(t_i + \theta_K + \tau' - t)\})$

lower bound: $(g-r) + c > \lambda(1 - e^{-\lambda\theta_K})$



- ⇒ attack is never successful
- ⇒ bubble bursts for exogenous reasons at $t_0 + \tau$

Formal analysis for symmetric strategies

- Suppose endogenous selling pressure would burst bubble at $t = t_0 + \hat{\tau}$, where $\hat{\tau} < \bar{\tau}$
 - ▶ for $t < t_i - \theta + \hat{\tau}$ $h(t|t_i) = 0$
 - ▶ for $t \geq t_i - \theta + \hat{\tau}$ $h(t|t_i) = \frac{\lambda}{1 - e^{-\lambda(t_i + \hat{\tau} - t)}}$
- (from attack condition) sell shares at

$$t_i + \hat{\tau} + \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] \left[\frac{1}{\lambda}\right]$$
- mass of arbitrageurs aware of the bubble $\max\{\frac{1}{\theta}(t - t_0), 1\}$
- mass of arbitrageurs attacking (= selling pressure)
 - ▶ $\max\{\frac{1}{\theta}(t - \hat{\tau} - \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right]) - t_0, 1\}$
 - ▶ for $t = t_0 + \hat{\tau}$ $-\frac{1}{\theta\lambda} \ln\left[\frac{(g-r) - \lambda + \lambda e^{-(g-r)\hat{\tau}}}{g-r+c}\right] < \kappa$
 - ▶ **contradiction!**

April 13th - Pre-emption Lemma

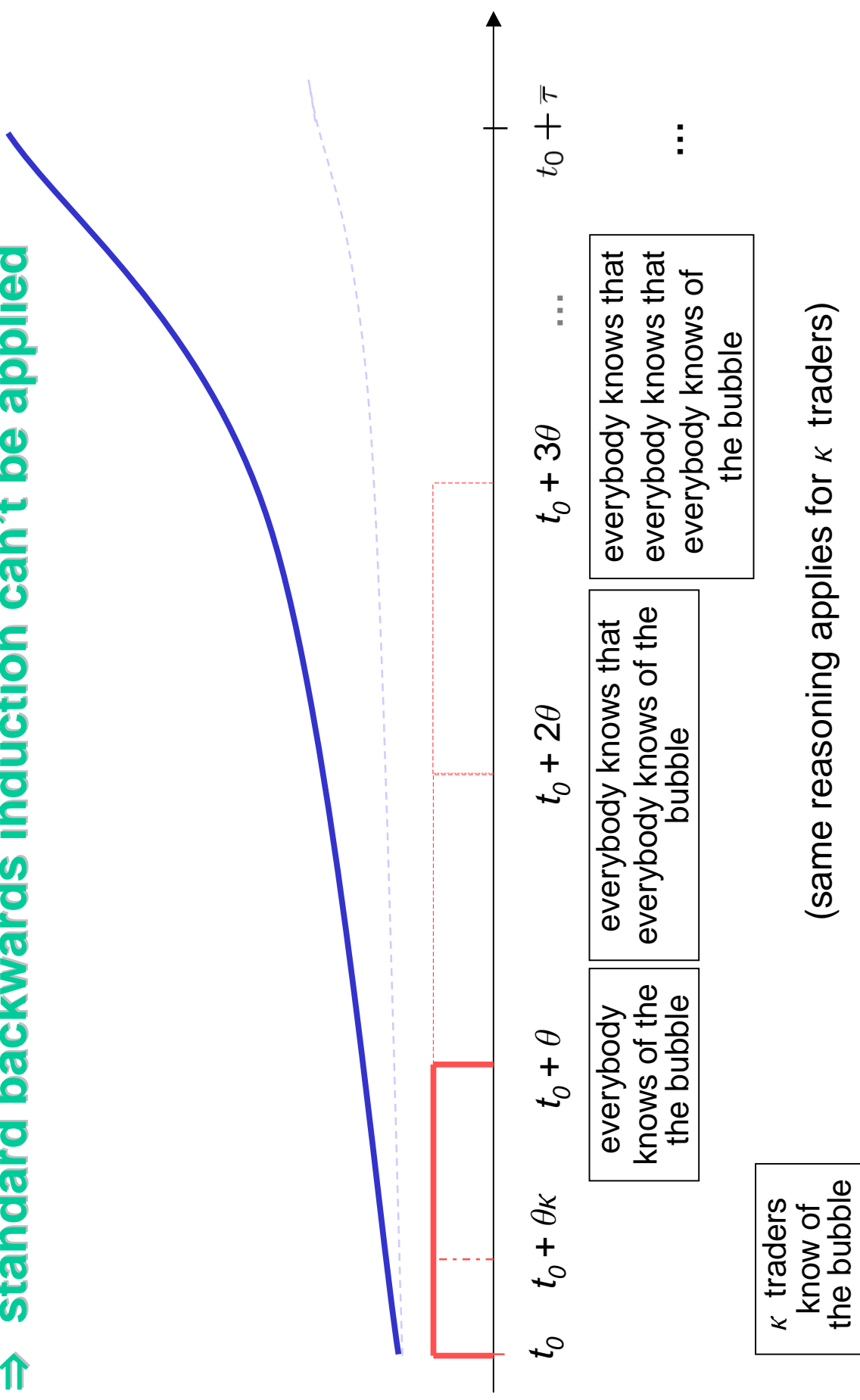
- $\hat{\tau}$ might depend on t_0 in asymmetric equilibria
- ▶ *Example:* all start attacking on Friday, April 13th.

$$\hat{\tau}^{13} = \begin{cases} \hat{\tau} & \text{if } t_0 > t^{13} - \theta\kappa \\ t^{13} - t_0 & \text{if } t_0 \leq t^{13} - \theta\kappa \end{cases}$$

- bubble would burst with strictly positive probability.
- selling pressure $S_{t=13} > \kappa$, $p_{t=13}$ drops already.
- Individual incentive to attack a little bit earlier.
- ▶ **Not an equilibrium!**

Lack of common knowledge

⇒ **standard backwards induction can't be applied**



Related theoretical literature

- **Asynchronized clocks**
 - ▶ Halpern & Moses (1984) [computer science]
 - ▶ Morris (1995)
 - restricted strategy space: condition only on own clock
no conditioning on calendar time, past payoffs, etc.
- **Global Games**
(uniqueness of equilibrium in static games with strategic complementarities)
 - ▶ Carlson & van Damme (1994)
 - ▶ Morris & Shin (1998)

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persistence of **bubbles**

endogenous crashes

endogenous life-span of the bubble

comparative statics

public events

conclusion

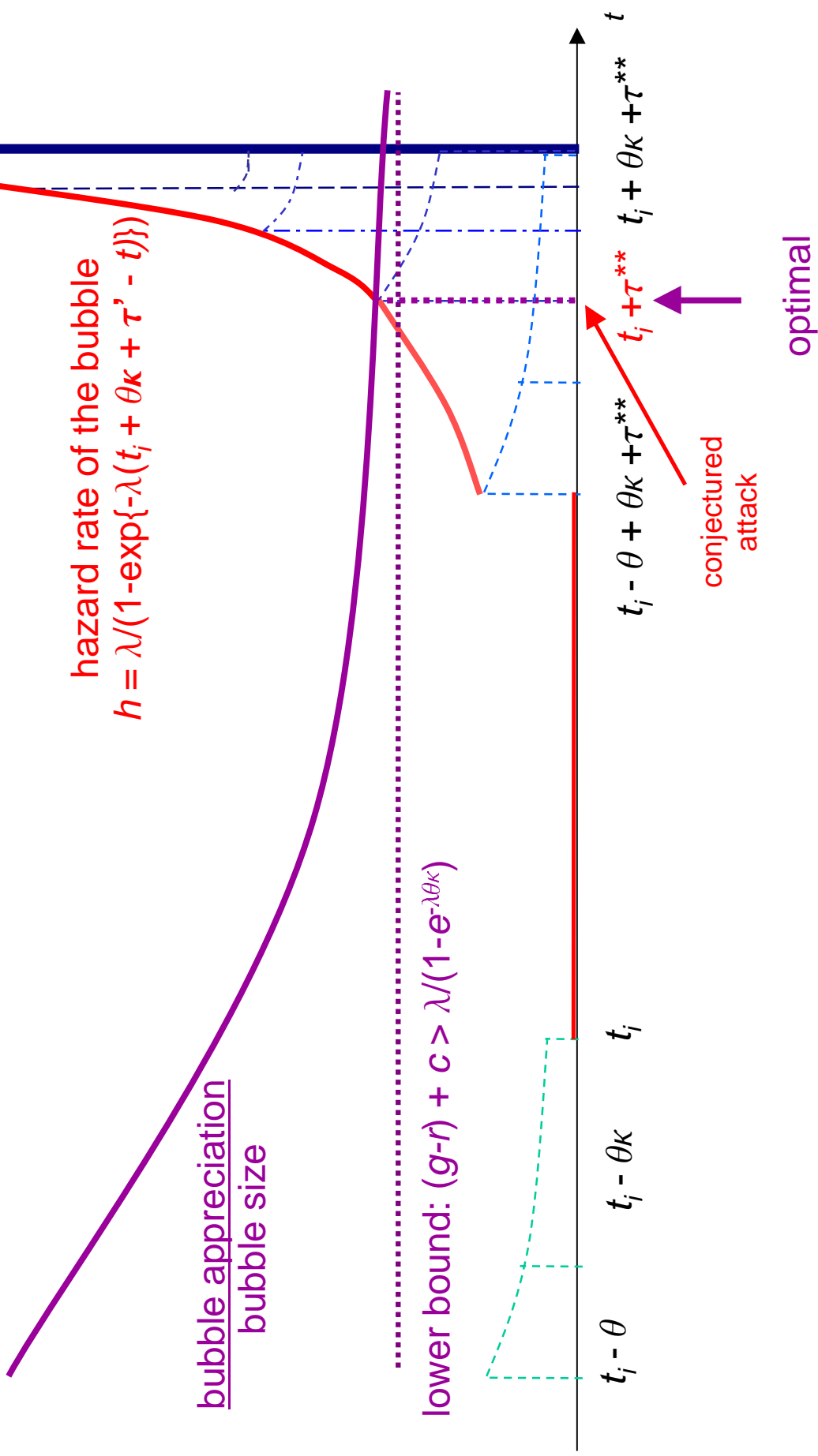
Endogenous crashes

- **Proposition 3:** Suppose $\theta\kappa < -\ln\left(1 - \frac{\lambda - \lambda e^{-(g-r)\bar{\tau}}}{g-r+c}\right)\left(\frac{1}{\lambda}\right)$.
 - ▶ ‘**most aggressive**’ trading equilibrium.
 - ▶ traders begin attacking after a delay of τ^{**} periods.
 - ▶ bubble *bursts* due to endogenous selling pressure at

$$t_0 + \frac{-\ln\left[1 - \frac{g-r+c}{\lambda}(1 - e^{-\lambda\theta\kappa})\right]}{(g-r)}$$

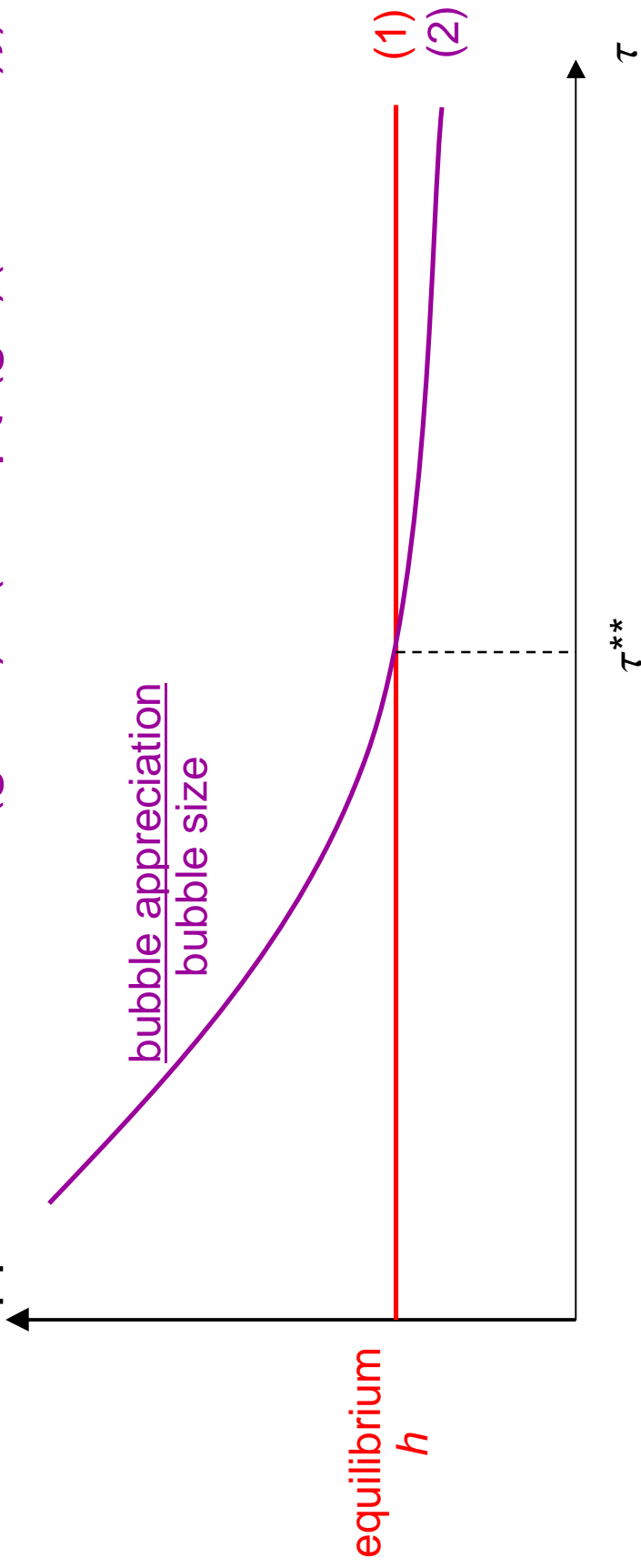
Endogenous crashes

⇒ Bubble bursts at $t_0 + \theta_K + \tau^{**}$



Endogenous crashes - deriving τ^{**}

- In symmetric equilibrium trader $t_i = t_0 + \theta_K$ bursts the bubble.
- When he sells his shares his support of t_0 is $[t_i - \theta_K, t_i]$, hence his hazard rate is $h = \lambda(1 - \exp\{-\lambda\theta_K\})$ (1)
- The bubble bursts at $t_i = t_0 + \theta_K + \tau^{**}$, hence it bursts at a size of $e^{g\tau}(1 - \exp\{-(g-r)(\theta_K + \tau^{**})\})$ bubble appreciation/size = $(g-r+c) / (1 - \exp\{-(g-r)(\theta_K + \tau^{**})\})$ (2)



Comparative statics

- Role of information dispersion λ, θ
- ▶ Prior distribution of t_0 $F(t_0) = 1 - \exp\{-\lambda t_0\}$
 - the smaller λ , the larger the life span of bubble
 - $\lambda \rightarrow \infty \Rightarrow t_0 = 0$, no info dispersion \Rightarrow no bubble
 - $\lambda \rightarrow 0 \Rightarrow$ distributions \rightarrow uniform [lifespan - $\ln\{1 - \theta\kappa(g-r+c)\}/(g-r)$]
- ▶ Dispersion of opinion θ
 - as $\theta \uparrow \Rightarrow$ bubble's life-span \uparrow
 - for $\theta \geq -\frac{1}{\kappa} \ln\left(1 - \frac{\lambda - \lambda e^{-(g-r)\bar{T}}}{g-r+c}\right) \left(\frac{1}{\lambda}\right) \Rightarrow$ exogenous crash
- Role of momentum traders $\kappa \Rightarrow$ same as for θ
- excess growth rate $(g - r) \uparrow$ [2 effects]
 - ▶ instantaneous appreciation effect $\uparrow \Rightarrow$ life span of bubble \uparrow
 - ▶ size of bubble (past appreciation) $\uparrow \Rightarrow$ life span of bubble \downarrow

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synchronizing public events

pre-scheduled versus unanticipated

price cascades and rebounds

conclusion

Pre-scheduled vs. unanticipated public news

- Pre-scheduled public events
 - ▶ news is unknown, but timing is fixed in advance. (FOMC Meetings etc.)
 - ▶ “martingale news”: correctly reflected in the price
 - ▶ ⇒ *pre-scheduled news will only move price by its fundamental content, but not beyond.*
 - Why? It cannot serve as a synchronization device.
 - If it would, then the bubble would burst with strictly positive probability on this date. In this case arbitrageurs have incentive to attack slightly earlier (same as Friday 13th of July)

Pre-scheduled vs. unanticipated public news

- Unanticipated public events
 - ▶ pre-emption argument does not apply!
 - ▶ can serve as synchronization device.
 - ▶ there are millions of public events (weather, etc.)
 - ▶ viewing something as a public event is also a coordination problem in itself.
- ▶ Extended setting
 - focus on news with *no* informational content (sunspots).
 - public event occurs with Poisson density λ_p .
 - Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
 - » only traders who became aware of the mispricing more than τ_p periods ago observe (look out for) public events.

Public events & Market rebounds

■ Proposition 5:

Attack a) always at the time of a public event t_p ,

b) after $t_j + \tau^{***}$ (where $\tau^{***} < \tau^{**}$),

except after a failed attack at t_p , **re-enter** the market

for $t \in (t_p, t_p - \tau_p + \tau^{***}) \cap (t_j + \tau^{p,1})$.

■ Intuition for re-entering the market:

- ▶ for $t_p < t_0 + \theta_K + \tau_p$ attack fails, agents learn $t_0 > t_p - \tau_p - \theta_K$
- ▶ without public event, they would have learnt this only at $t_p + \tau_p - \tau^{***}$.
 - the existence of bubble at t reveals that $t_0 > t - \tau^{**K} - \theta_K$
 - that is, no additional information is revealed till $t_p - \tau_p + \tau^{***}$
 - density that bubble bursts for endogenous reasons is zero.

Role of information

- Only unanticipated public news can burst a bubble.
- News which is considered as important can be more important than real fundamental news.
- Fads and fashions in information.

Price cascades and rebounds

- Price drop as a synchronization device (public event).
 - ▶ through psychological resistance line
 - ▶ by more than, say 5 %
- **Exogenous price drop**
 - ▶ after a price drop
 - if bubble is ripe
 - ⇒ bubble bursts and price drops further.
 - if bubble is not ripe yet
 - ⇒ price bounces back and the bubble is strengthened for some time.

Price cascades and rebounds (ctd.)

Proposition 6:

Attack a) after a price drop if $\tau_i \geq \tau'_p$

b) after $t_i + \tau^{****}$ (where $\tau^{****} < \tau^{**}$),

re-enter the market after a rebound at t'_p

for $t \in (t'_p, t'_p - \tau'_p + \tau^{****}) \cap (t_i + \tau'^p, 1)$.

- ▶ attack is costly, since price might jump back
 - ⇒ only arbitrageurs who became aware of the bubble more than τ'_p periods ago attack the bubble.
- ▶ after a rebound, an endogenous crash can be temporarily ruled out and hence, arbitrageurs re-enter the market.

Conclusion

- Bubbles
 - ▶ Dispersion of opinion among arbitrageurs causes a synchronization problem which makes coordinated price corrections difficult.
 - ▶ Arbitrageurs time the market and ride the bubble.
 - ▶ \Rightarrow Bubbles persist
- Crashes
 - ▶ can be triggered by unanticipated news without any fundamental content, since
 - ▶ it might serve as a synchronization device.
- Rebound
 - ▶ can occur after a failed attack, which temporarily strengthens the bubble.