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NOBUHIRO KIYOTAKI
ALEXANDER MICHAELIDES
KALIN NIKOLOV

Housing, Distribution, and Welfare

Housing is a long-lived asset whose value is sensitive to variations in expectations of long-run growth rates and interest rates. When a large fraction of households has leverage, housing price fluctuations cause large-scale redistribution and consumption volatility. We find that a practical way to insure the young and the poor from the housing market fluctuations is through a well-functioning rental market. In practice, homeownership subsidies keep the rental market small and the housing cycle affects aggregate consumption. Removing homeownership subsidies hurts old homeowners, while leverage limits hurt young homeowners.

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Housing is the most important nonhuman asset for many households. Booms and busts in housing values have been often associated with financial crises throughout recent history (Jorda, Schularick, and Taylor (2015)), especially when accompanied by real estate lending booms (Jorda et al. 2016). Many academics and policymakers are therefore concerned about the vulnerability of household balance sheet conditions, calling for reforming the housing finance market so as to make it more stable.[1]

In this paper, we build a tractable macro-economic model with housing and use it to examine the causes and consequences of housing market volatility and evaluate alternative housing-related policies. We argue that expectations of long-run growth rates and interest rates have been important for explaining the low-frequency movements in housing values for the United States and other developed countries. We show that

Nobuhiro Kiyotaki *is at Princeton University* (*E-mail*: kiyotaki@princeton.edu). Alexander Michaelides *is at Imperial College London* (*E-mail*: a.michaelides@imperial.ac.uk). Kalin Nikolov *is at European Central Bank* (*E-mail*: kalin.nikolov@ecb.int).

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1. Piazzesi and Schneider (2016) contain an excellent discussion of the role of housing in macroeconomics.

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changes in these fundamentals can generate substantial movements in housing prices and consumption. The paper then examines removing homeownership subsidies and restricting the loan-to-value (LTV) ratio as alternative policies that can help to reduce the economy's volatility in response to changes in fundamentals. Both policies hurt some homeowners significantly, even though they benefit renters. The removal of homeownership subsidies hurts old homeowners, while the LTV cap hurts young leveraged homeowners. As older generations tend to be more politically active, our results may explain why reforming the system of homeownership subsidies has been difficult to implement.

We make a modeling contribution by constructing a tractable overlapping generations (OLG) model populated with young households (whose income is expected to grow) and old households (whose income is declining and who face a probability of dying). We assume that the young households are heterogeneous in deriving utility from owning versus renting homes while old households always prefer to own. The resulting framework aggregates into a representative young homeowner household with leverage, a representative young renter household, and a representative old household, yet generates realistic life-cycle profiles for homeownership and household leverage.[2] Our model does not require assumptions of heterogeneous impatience, instead uses well-documented facts about the life-cycle earnings profiles (Gourinchas and Parker (2002)) to calibrate the model's parameters and to motivate borrowing and lending. We use the minimum amount of heterogeneity in order to study the allocation and distribution associated with housing market fluctuations. We will argue that young homeowners are especially vulnerable by virtue of being leveraged while renters are more protected.

To generate large movements in housing prices, we consider "land" with limited elasticity of supply as a key factor in common with Davis and Heathcote (2005), Iacoviello and Neri (2010), Kiyotaki, Michaelides, and Nikolov (2011), and Liu, Wang, and Zha (2013, 2016). Recently, Knoll, Schularick, and Steger (2017) have provided empirical evidence from 14 advanced economies supporting the role of land prices, rather than construction costs, in explaining housing price booms.

As in Mankiw and Weil (1989), Kiyotaki, Michaelides, and Nikolov (2011), and Miles and Sefton (2021), we stress the importance of expectations of long-run aggregate income growth rates and real required rates of return in explaining the fluctuations of housing prices.[3] In steady state with inelastic housing supply, the imputed rental price grows at the rate of aggregate income. Therefore, if house prices equal

2. Our OLG model relies on Gertler (1999). These preferences are a variant of Epstein–Zin–Weil preferences and feature risk neutrality across states of nature but finite intertemporal elasticity of substitution across time periods.

3. Favilukis and Van Nieuwerburgh (2021) consider the impact of an influx of out-of-town buyers on housing prices and welfare of renters and homeowners in local housing markets. Aggregate local income equals the product of population and per capita income. Because population tends to be more persistent than per capita income, their mechanism is consistent with our explanation of emphasizing the expectations of aggregate income growth.

the present value of future rents, then the price-to-rent ratio equals the inverse of the gap between the long-run required return and the expected economic growth rate.[4] To check the empirical relevance of this prediction, we use data from Jorda et al. (2019) to construct measures of expectations of future growth rates and future inflation rates based on the realized average rates of growth and inflation in the preceding 10 years.[5] Comparing the steady-state implication of this partial equilibrium example and the data over the average of 5-year non-overlapping periods, we find that such a simple example is broadly consistent with the low-frequency movements of the housing price-to-rent ratio in the United States from 1880 to 2015, as well as 11 other developed countries since 1960.[6]

There is a large literature emphasizing the role of interest rates and other factors in explaining the boom–bust cycle in U.S. and European housing prices. Miles and Monro (2021) emphasize the large effect of the decline in the real interest rate on the evolution of housing prices in the United Kingdom. Sommer, Sullivan, and Verbrugge (2013), Burnside, Eichenbaum, and Rebelo (2016), Landvoigt (2017), and Kaplan, Mitman, and Violante (2020) emphasize the need for exuberant beliefs about house prices to fully account for the data. Garriga, Manuelli, and Peralta-Alva (2019) stress the role of market segmentation, while Favilukis, Ludvigson, and Van Nieuwerburgh (2017) demonstrate the importance of changes in risk premia caused by movements in the LTV ratio.[7] Even though we also use real interest rates for mortgage borrowers as an explanatory factor, we emphasize beliefs about the long-run growth rate of the economy.[8] The housing financing constraint is the centerpiece in our framework to analyze the homeownership rate, distribution, and welfare, even though it turns out to have a limited impact on housing prices, as in Kiyotaki et al. (2011) and Kaplan, Mitman, and Violante (2020).

Despite the linear period utility in the model, households care about housing market risks because households' marginal utility of wealth is negatively correlated with the rate of return on homeownership. Households value wealth more in states in which consumption is low temporarily and the user cost of housing is low with the arrival

4. The required rate of return equals the sum of the long-run mortgage rate and the tax and maintenance cost of owning a house (as a percentage of housing value).

5. Such an approach is consistent with the evidence on experiential learning (see, e.g., Malmendier and Nagel (2011, 2016) and the survey in Coibion et al. (2018)) showing that households form expectations based on recent personal experiences.

6. Naturally, our example does not explain well the housing price-to-rent ratios during the turbulent periods of the two world wars, the Great Depression and the Great Recession.

7. Justiniano, Primiceri, and Tambalotti (2019) argue that abundant credit supply best explains the house price boom.

8. There are also theories which attribute housing and asset price fluctuations to bubbles. See the survey in Brunnmermeier (2009), Miao, Wang, and Zhou (2015) and Jiang, Miao, and Zhang (2022). See also Adam et al. (2022) for a model with extrapolative expectations leading to housing price volatility. These papers are complementary to our fundamentals-based framework.

of an adverse shock. This is why both households and policymakers in the model are concerned with housing market volatility.[9]

Our main interest is to examine what policies and institutions can help mitigate the problems associated with the housing market. Since the key missing market in the model is the market for state-contingent mortgage debt, either a policy or an institution replicating such a market's insurance properties would be welfare improving. A housing equity contract would be a close substitute, that has been proposed as a way to replace traditional mortgage debt (Shiller (1998), Greenwald, Landvoigt, and Van Nieuwerburgh (2021), Guren, Krishnamurthy, and McQuade (2021)), but a housing equity market has not developed so far. One practical form of housing tenure that delivers net worth insurance is renting, because renting avoids leveraged exposure of a household's net worth to house price fluctuations. For renters, net worth and consumption are just as well shielded from housing price fluctuations, as for homeowners under state-contingent debt markets. Therefore, a policy that increases the size of the rental market makes the consumption of young households more stable by transferring the risk of housing price fluctuations to the old. In this respect, a well-functioning rental market is a good substitute for state-contingent mortgage debt.[10]

In a stochastic version of the model, we show that, when households recognize the risk involved in homeownership, they self-insure by taking on less debt. This moderates the prevalence of homeownership and the level of housing prices relative to rents. The individual household, however, does not internalize the benefit of reducing leverage for aggregate fluctuations. Moreover, the generous homeownership subsidies in the United States and other advanced economies push in the opposite direction. Such subsidies make homeownership with leverage more attractive, boosting housing prices, squeezing the rental market, and moving the economy further away from the full insurance benchmark.

Two types of housing market finance policies have been proposed or used in different countries to mitigate households' vulnerability to housing price fluctuations. We conduct a novel analysis of the preferences of different households (old, young, renters, and owners) over a policy that reduces homeownership subsidies and a policy that imposes LTV limits on borrowers.[11] Reducing homeownership subsidies diminishes the incentive of households to take on housing ownership with high leverage, while LTV limits directly impose constraints on the leverage households can take.

9. Our emphasis on the vulnerability of leveraged homeowners to income and wealth shocks receives empirical support from the work of Campbell and Cocco (2007), Cloyne and Surico (2017), and Cloyne, Ferreira, and Surico (2020), among others.

10. In our framework, renters also face movements in rental prices as demonstrated by Sinai and Souleles (2005). However, when the sources of fluctuations are changes in long-run expected growth rate and required rate of returns rather than changes in local characteristics, we can show that young renters face much less fluctuations of lifetime utility than young homeowners with leverage.

11. Mendicino, Lambertini, and Punzi (2013) and Mendicino et al. (2018) stress the importance of agent heterogeneity (different degree of impatience) in generating winners and losers from macroprudential policy.

Both policies reduce the housing price and the vulnerability of consumption of various households.[12]

We find that some groups are significantly hurt by the introduction of these policies. The LTV cap hurts young homeowners most, because it reduces the housing consumption of leveraged house buyers.[13] In contrast, the removal of homeownership subsidies hurts older homeowners most, in line with Sommer and Sullivan (2018). This analysis illustrates that policies to reduce the economy's vulnerability to housing price fluctuations have substantial redistributive effects. Moreover, since they create prominent losers, such policies would be politically difficult to implement.

1. LONG-TERM HOUSING FUNDAMENTALS AND HOUSING PRICES

1.1 A Simple Example

To build intuition, we start with a simple dividend discount model under perfect foresight.[14] Suppose that housing does not depreciate and its rental price r_t^r grows by a factor G_t. We will later develop a model with a fixed supply of housing/land in which the growth rate G_t is determined in an endowment economy.[15] Let R_t be the gross real interest rate and μ be the maintenance and tax cost as a fraction of housing value. Then, the housing price P_t is given by the following dividend discount formula:

$$P_t = r_t^r - \mu P_t + \frac{P_{t+1}}{R_t}. \tag{1}$$

Defining $\widehat{P}_t \equiv \frac{P_t}{r_t^r}$ as the price-to-rent ratio, we can solve for the evolution of $\widehat{P}_t$ as:

$$\begin{aligned}\widehat{P}_t(1+\mu) &= 1 + \frac{r_{t+1}^r}{r_t^r}\frac{\widehat{P}_{t+1}}{R_t} \\ &= 1 + \frac{G_{t+1}}{R_t}\widehat{P}_{t+1}.\end{aligned}$$

12. Sommer and Sullivan (2018) find, similarly to our model, that the removal of tax deductibility of mortgage leads to a fall in housing prices. On the other hand, their model has a minimum house size for homeowners and therefore the fall in the housing price helps poorer households to buy housing, increasing homeownership. In our model, there is no minimum house size and homeownership declines as marginal homeowners switch to renting when the subsidy is removed.

13. Those who lose include also young households who decide not to buy as a result of the LTV cap.

14. See Mankiw and Weil (1989) for a similar partial equilibrium model of housing.

15. For a production economy setting, see, for example, Kiyotaki et al. (2011).

In the steady state, the price-to-rent ratio is given by

$$\widehat{P} = \frac{R}{R(1+\mu) - G} \simeq \frac{1}{R - G + \mu}, \tag{2}$$

where the variables without time subscripts denote long-term steady-state values. The housing rental yield is given by the inverse of $\widehat{P}$.

The simple dividend discount model presented here has several implications for housing prices which are broadly consistent with the observations in the following subsection. First, a lower value of $R - G + \mu$ leads to a higher price-to-rent ratio $\widehat{P}$ and to a lower housing rental yield. Second, the smaller the gap between the required rate of return (long-term real interest rate plus the maintenance cost) and the growth rate, the larger is the impact of permanent changes in the interest rate and/or the growth rate on the price-to-rent ratio. Loosely speaking, news about long-term interest rates or growth rates have a disproportionately larger impact on housing prices when the gap between the required return and the growth rates is small.

1.2 Observations

Jorda et al. (2019) have compiled a cross-country macrofinancial data set for the 1880–2018 period. We use their data for the U.S. economy to construct simple measures for expected growth rates and real interest rates in Figure 1.

Figure 1, Panel A plots a measure of the long-term real interest rate: the 10-year government bond yield minus a proxy for inflation expectations. Following Malmendier and Nagel (2016), we assume a simple form of expectations based on the recent past. More specifically, the expected inflation rate at year t equals the average CPI inflation rate from year $t - 10$ to year t. As our simple example entirely abstracts from dynamics, we focus on longer periods: each observation in Panel A is a 5-year average of nonoverlapping periods. We can see that our measure of the long-term real interest rate fluctuates considerably over time, especially during periods of volatile inflation and financial repression such as the two world wars and the Great Inflation in the 1970s.

Panel B shows a proxy for expected real GDP growth since 1880. We again assume the expected real GDP growth equals the average real GDP growth over the past 10 years, and then take the average of 5-year nonoverlapping periods. This too has been volatile especially in the periods around the two world wars as well as the Great Depression.

Panel C plots the difference between the real interest rate in Panel A and the long-term expected growth rate in Panel B, adding a constant adjustment (5%) to capture maintenance costs (and possible risk premium). In Panel D, we plot the data on the housing rent-to-price ratio (sometimes referred to as the housing "rental yield") in a solid line and compare it with our simple measure of fundamentals ($R + \mu - G$) in a dotted line. It is clear that our measure of fundamentals has been much more volatile than the housing rental yield before WWII.

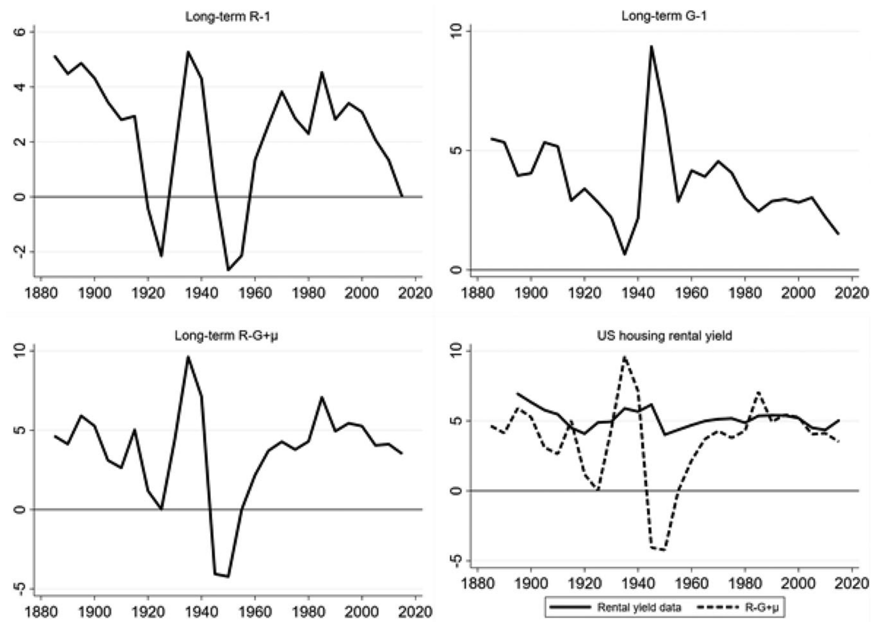


Fig 1. The U.S. Residential Housing Rental Yield and Two Key Determinants: 1880–2018.

NOTES: The housing rental yield is the annual gross residential housing rent divided by purchase price. The data are presented as 5-year nonoverlapping averages. For instance, the data observation for 1980 is the average for the 1976–80 period.
SOURCE: Jorda et al. (2019).

Indeed, the failure of our simple example during the volatile periods of the two world wars and the Great Depression is perhaps not surprising. First, the simple example we use relies on static expectations of inflation and real GDP growth based on the past 10 years' experience which may work poorly during very volatile periods. Second, the model assumes that households can borrow freely at market interest rates which may not have been true in periods of financial repression and credit restrictions. Third, the simple example assumes a constant risk premium which may explain why it fails to generate the increase in the housing rental yield during the recent housing bust in 2007–12. But in periods of relative stability since 1960 (and especially since 1980), the model tracks the low-frequency movements in the U.S. data reasonably well. We take the success of the simple example in recent years as suggestive of the important role of real interest rates and GDP growth rates in the housing market, even though clearly other factors were important, especially in the short run.

The Jorda et al. (2019) data set also contains data on a number of other countries and we focus on the implications of our frictionless example for 12 countries (including the United States) starting from 1960. In Figure 2, we compare the implications of our steady-state example based on the past 10 years' experience and data

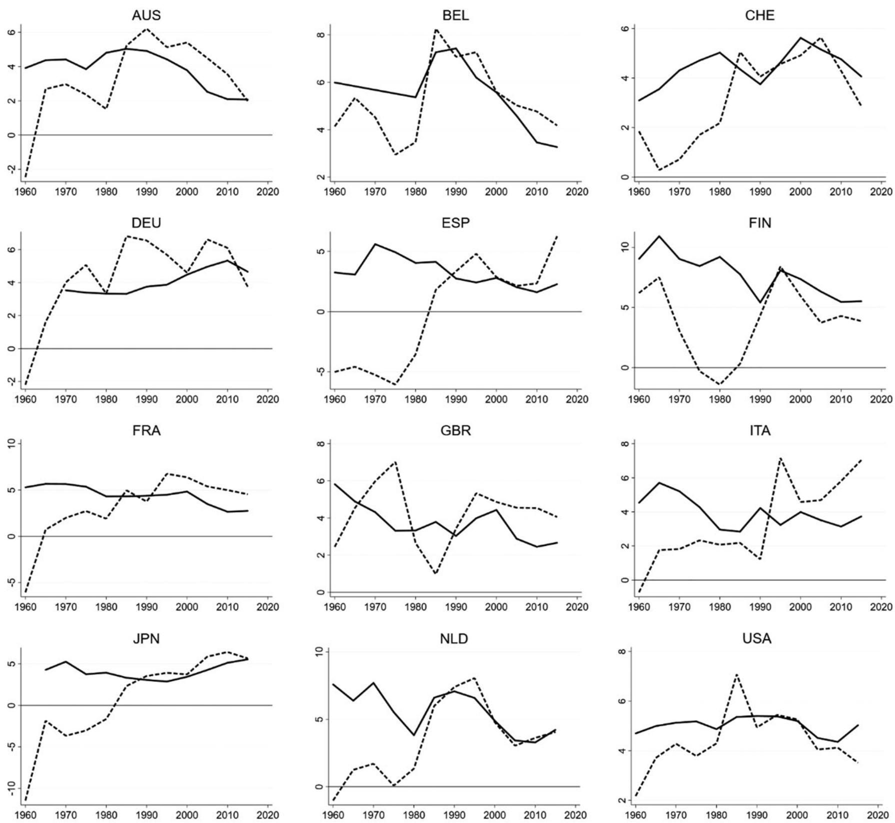


Fig 2. The Housing Rental Yield versus Fundamentals in an International Perspective Since 1960.

Notes: Solid line = data, Dashed line = simple measure of fundamentals ($R - G + \mu$). The labels are as follows: AUS = Australia, GBR = Great Britain, BEL = Belgium, NLD = The Netherlands, FIN = Finland, ESP = Spain, DEU = Germany, FRA = France, CHE = Switzerland, JPN = Japan, ITA = Italy, USA = The United States. The data are presented as 5-year nonoverlapping averages. For instance, the data observation for 1980 is the average for the 1976–80 period).

Notes: The housing rental yield is the annual gross residential housing rent divided by purchase price.

Source: Jorda et al. (2019).

for the housing rental yields using the average of 5-year periods.[16] Here, we see most clearly that our example does best from 1980 onward also for other countries. We think the simple example works well after the 1980s because financial markets were liberalized making the interest rate more important than quantitative restrictions on credit. Moreover, the inflation and growth rates do not undergo violent swings in this

16. Just as in the case of the U.S. economy, our steady-state example does not deliver a close match with the data during the volatile periods spanning the two world wars and the Great Depression. Therefore, we only use the data for the more stable period since 1960.

period (relative to earlier periods) and therefore our proxies for growth and inflation expectations based on past experiences are reasonable approximations.

2. OLG MODEL OF THE HOUSING MARKET

In the previous section, we argued that the influence of a single factor, the difference between the long-run required real return on housing and the growth rate ($R + \mu - G$), has been important in driving the low-frequency evolution of the housing price-to-rent ratio. We now integrate the simple housing market example outlined above into a general equilibrium framework with financial frictions that is suitable for assessing the aggregate and distributional consequences of housing price fluctuations. Our aim is to analyze the macro-economic impact of shocks to real interest rates and trend growth rates as well as the effect on the welfare of different groups in the society.

2.1 Endowment Economy and Demographics

We build a small open endowment economy model consisting of two types of households: "young" and "old" households. There are two main differences between the different types of households: (i) the age-related labor productivity of the young household grows, while that of the old household declines and (ii) the old household faces mortality risk, while the young household faces the risk of becoming old. See Appendix B for the details of population and aggregate income dynamics.

Households are born young with zero assets. They remain young with probability γ and become old with probability $1 - \gamma$ in the following period. When old, agents survive with probability σ and die with probability $1 - \sigma$. The population of young households grows at a rate G_N, which we assume to be constant. The ratio of the population of young (N_t^y) and old (N_t^o) households is constant at

$$N_t^y : N_t^o = G_N - \sigma : 1 - \gamma.$$

We normalize the efficiency unit labor of newborns to be unity as $x_t = 1$. Let G^y and G^o be the age-related labor productivity growth rate (plus one) of young and old agents at date t:

$$\frac{x_t}{x_{t-1}} = G^y > 1, \text{ when young at date } t-1$$

$$\frac{x_t}{x_{t-1}} = G^o < 1, \text{ when old at } t-1 \text{ and survives at } t.$$

Let X_t^y and X_t^o be the aggregate age-related labor of young and old agents. Aggregate efficiency unit labor is

$$X_t = X_t^y + X_t^o.$$

Aggregate income is

$$Y_t = A_t X_t,$$

where A_t is the aggregate labor productivity which is equal to the productivity of the new born and grows at

$$\frac{A_t}{A_{t-1}} = G_{At}.$$

2.2 Households

The preferences of young and old agents ($i = y, o$) are given by

$$V_t^y = \left\{ \left(u_t^y\right)^{\frac{\eta-1}{\eta}} + \beta \left[\gamma E_t V_{t+1}^y + (1-\gamma) E_t V_{t+1}^o \right]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}, \tag{3}$$

and

$$V_t^o = \left[\left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{4}$$

where β is the discount factor. This utility implies that agents are risk-neutral but have finite intertemporal elasticity of substitution of $\eta \in (0, \infty)$.

Period utility is given by

$$u_t^i = \left(\frac{c_t^i}{\phi}\right)^{\phi} \left(\frac{h_t^i}{1-\phi}\right)^{1-\phi},$$

if the household is a homeowner and by

$$u_t^i = \left(\frac{c_t^i}{\phi}\right)^{\phi} \left(\frac{\chi^i h_t^i}{1-\phi}\right)^{1-\phi},$$

if the household is a renter. Here, c_t^i and h_t^i are the consumption of goods and housing by type i agent at date t, and χ^i is an individual-specific parameter that represents the preference for renting over owning. Households with $\chi^i > 1$ would prefer to rent, and those with $\chi^i < 1$ would prefer to own if there were no other frictions. Below we will introduce financing constraints and homeownership subsidies and this will modify this cutoff. However, the principle remains the same: there is a cutoff value for χ^i above which households choose to rent. We assume that, for young households, χ^y is uniformly distributed on $[0, \overline{\chi}]$. For old households, $\chi^o = 0$ meaning that all old households want to own housing. This is a simple and tractable way of capturing the fact that homeownership is increasing over the life cycle, and that the homeownership rate of retirees does not fall with age, perhaps due to "place attachment" motives (see Cocco and Lopes (2020)).

Constraints for young homeowners. The budget constraint of young homeowners is given by

$$c_t + Q_t(1+\mu_t - \upsilon_t)h_t - \frac{b_t}{R_t} = (1-\tau_t)w_t x_t + Q_t h_{t-1} - b_{t-1},$$

where w_t, Q_t, and R_t are the wage rate, housing price, and gross interest rate, all in terms of consumption goods, while x_t and b_{t-1} are age-related labor productivity and debt (negative bond holdings). The variable μ_t is the maintenance (and tax) cost of housing in terms of goods, which the residents have to pay in proportion to the value of housing.[17] The variable υ_t is a homeownership subsidy, while τ_t is a wage tax imposed on all households in order to pay for the subsidy.

Each young homeowner faces the borrowing constraint:

$$E_t Q_{t+1} h_t - b_t \geqslant \omega(Q_t h_{t-1} - b_{t-1}) \tag{5}$$

for $\omega \in (0, 1)$. The collateral constraint (5) is a generalization of the standard specification introduced in Kiyotaki and Moore (1997). It ensures that, in the long run, a household can borrow up to the expected value of the house($b_t \leqslant E_t Q_{t+1} h_t$) and must pay the imputed rent (or user cost) of housing

$$r_t^h = (1+\mu-\upsilon_t)Q_t - \frac{E_t Q_{t+1}}{R_t} \tag{6}$$

as a down payment when it buys a house. In the short run, if unanticipated aggregate shocks drive the household into negative equity ($E_t Q_{t+1} h_t - b_t < 0$), the constraint allows the negative equity position to be closed gradually over time when $\omega > 0$. This is a tractable way to capture the effects of long-term debt in the model.

Defining the nonhuman net worth of agent at date t (i.e., excluding human capital) as

$$a_t = Q_t h_{t-1} - b_{t-1},$$

we can rewrite the budget constraint as

$$c_t + r_t^h h_t + \frac{E_t a_{t+1}}{R_t} = (1-\tau_t)w_t x_t + a_t,$$

where the borrowing constraint is

$$E_t a_{t+1} \geq \omega a_t. \tag{7}$$

17. We think "maintenance costs" include a number of costs of owning property: property taxes and the cost of maintaining the structures. We assume the total maintenance cost is proportional to the housing value. We will calibrate the cost in order to match the average U.S. housing price-to-rent ratio in the 1980–2015 period.

Budget constraint of young renters. The budget constraint for a young renter is given by

$$c_t + r_t^r h_t - \frac{b_t}{R_t} = (1 - \tau_t) w_t x_t - b_{t-1},$$

where r_t^r is the rental price of housing. The borrowing constraint is given by

$$b_t \leq 0,$$

meaning that renters cannot borrow.

Budget constraint of the old. The old agents purchase housing both for own use (h_t) and as an investment (s_t) as landlords. The costs of the two investments differ because owner-occupied housing receives subsidies υ_t. In addition, they typically save ($b_t < 0$) by lending to young homeowners and/or holding an internationally traded bond with an exogenous interest rate of R_t.

$$c_t + Q_t(1 + \mu_t - \upsilon_t) h_t + \left[Q_t(1 + \mu_t) - r_t^r\right] s_t - \frac{b_t}{R_t}$$
$$= (1 - \tau_t) w_t x_t + \frac{Q_t(s_{t-1} + h_{t-1}) - b_{t-1}}{\sigma}. \tag{8}$$

When agents are old at date $t - 1$, we assume a perfect annuity market in which surviving old agents share the assets of dying agents proportionally, which is why the return on assets for survivors is multiplied by $1/\sigma$.

The indifference condition between saving through rental housing and bonds gives us the rental price of housing:

$$r_t^r = (1 + \mu_t) Q_t - \frac{E_t Q_{t+1}}{R_t}. \tag{9}$$

It differs from the imputed rental cost of owner-occupied housing due to homeownership subsidies.

Defining

$$a_t = \frac{1}{\sigma}[Q_t(s_{t-1} + h_{t-1}) - b_{t-1}],$$

we can rewrite the budget constraint as

$$c_t + r_t^h h_t + \frac{\sigma E_t a_{t+1}}{R_t} = (1 - \tau_t) w_t x_t + a_t, \tag{10}$$

where r_t^h is the user cost for homeowners in (6).

3. PERFECT FORESIGHT EQUILIBRIUM

This section describes the general equilibrium of an open economy facing an exogenous real interest rate under perfect foresight regarding the evolution of the aggregate economy. See Appendix C for more details on deriving the perfect foresight equilibrium. The equilibrium under uncertainty is outlined in the Online Appendix.

3.1 The Young Homeowner

The young households expect an upward sloping earnings profile, and they are likely to remain young for a long time. Since they start life with no assets, ($a_t = 0$), we expect the young households to be borrowing-constrained, $a_{t+1} = \omega a_t$, which we will verify later. This implies that the household exhausts its entire available resources in paying for nondurables and for its housing down payment:

$$c_t + r_t^h h_t = (1 - \tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t.$$

It is easy to show that, due to the Cobb–Douglas period utility and due to the absence of uncertainty, the usual consumption and housing demands obtain.[18]

$$c_t = \phi \left[(1 - \tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right],$$
$$h_t = \frac{1 - \phi}{r_t^h} \left[(1 - \tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t \right].$$

In the steady state, $a_{t+1} = 0$ and the household simply consumes its after-tax wage.

We guess and verify that the value function is proportional to period utility:

$$V_t^h = \Delta_t^h u_t^h.$$

Then, the ratio of the value function to period utility is a recursive function that depends on the growth rates of utility over time:

$$\Delta_t^h = \left\{ 1 + \beta \left[\gamma \Delta_{t+1}^h G_{t+1}^{hh} + (1 - \gamma) \Delta_{t+1}^o G_{t+1}^{ho} \right]^{\frac{\eta - 1}{\eta}} \right\}^{\frac{\eta}{\eta - 1}}, \tag{11}$$

18. Once we introduce uncertainty, the expenditure shares will depart from the Cobb–Douglas utility weights of housing and consumption. See Online Appendix for derivations on this case.

where

$$G^{hh}_{t+1} = \frac{G^A_{t+1} G^y (1-\tau_{t+1}) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) \omega a_t}{(1-\tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t} \left(\frac{r^h_t}{r^h_{t+1}}\right)^{1-\phi}$$

is the utility growth rate conditional on remaining young, while

$$G^{ho}_{t+1} = \frac{\nu^o_{t+1}(W^{ho}_{t+1} + \omega a_t)}{(1-\tau_t) w_t x_t + \left(1 - \frac{\omega}{R_t}\right) a_t} \left(\frac{r^h_t}{r^o_{t+1}}\right)^{1-\phi}$$

is the utility growth rate when the household switches from youth to old age. The variable ν^o_{t+1} is the marginal propensity to consume out of human capital W^{ho}_{t+1} and nonhuman wealth when old, which we will explain shortly.

In steady state when $a_{t+1} = 0$, the expenditure of the young homeowner follows her income while her utility grows at the growth rate of expenditure adjusted by the housing inflation rate caused by the upward trend in the user cost of housing r^h_t. Once the household becomes old and its income starts to fall, it loses some of its human wealth but it gains the ability to smooth consumption over time as reflected in the fall in the marginal propensity to consume out of total wealth from unity to ν^o_{t+1}.

3.2 *The Young Renter*

Due to the upward-sloping path of income, young renters also do not wish to save. Borrowing constraints prevent them from borrowing hence they choose $a_{t+1} = 0$. They spend their entire budget on consumption and rent

$$c_t = \phi(1-\tau_t) w_t x_t,$$
$$h_t = \frac{1-\phi}{r^r_t}(1-\tau_t) w_t x_t.$$

Yet again, we can guess and verify that the young renter will have a value function which is proportional to period utility

$$V^r_t = \Delta^r_t u^r_t,$$

where Δ^r_t is given below

$$\Delta^r_t = \left\{1 + \beta\left[\gamma \Delta^r_{t+1} G^{rr}_{t+1} + (1-\gamma) \Delta^o_{t+1} G^{ro}_{t+1}\right]^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}. \tag{12}$$

Here,

$$G^{rr}_{t+1} = \frac{1-\tau_{t+1}}{1-\tau_t} G^A_{t+1} G^y \left(\frac{r^r_t}{r^r_{t+1}}\right)^{1-\phi}$$

is the utility growth rate while the household remains a young renter and

$$G^{ro}_{t+1} = \nu^o_{t+1} \frac{W^{ho}_{t+1} G^y}{(1-\tau_t) w_t} \left(\frac{r^r_t}{\chi r^o_{t+1}} \right)^{1-\phi}$$

is the utility growth rate in the period when the household becomes an old homeowner.

3.3 *Housing Tenure Choice*

Households decide whether to rent or own by comparing the utility of the two types of housing tenure. Since switching tenure is assumed to be costless, households compare period utility rather than value functions. In a perfect foresight equilibrium, the period utility from owning for a household with zero net worth is given by

$$u^h_t = \frac{(1-\tau_t) w_t x_t}{\left(r^h_t\right)^{1-\phi}},$$

where r^h_t is the imputed rent of homeowners. The utility of renting is given by

$$u^r_t = \frac{(1-\tau_t) w_t x_t}{\left(r^r_t / \chi\right)^{1-\phi}},$$

where r^r_t is the rental price. The household chooses to own rather than rent when the following condition is satisfied:

$$r^h_t < \frac{r^r_t}{\chi}. \tag{13}$$

If the user cost of owning is cheaper than the rental price taking into account the preference for renting over owning (χ), the household chooses to own. Since χ is uniformly distributed on $[0, \overline{\chi}]$ at the individual level, the above utility comparison allows us to solve for the indifferent household for whom equation (13) holds with equality. Then, we can compute the aggregate homeownership rate of young households as:

$$\xi_t = \frac{1}{\overline{\chi}} \left(\frac{r^r_t}{r^h_t} \right). \tag{14}$$

We can think of young homeowners as rich hand-to-mouth and young renters as poor hand-to-mouth.

3.4 *The Old*

The old agent chooses consumption and saving to maximize the utility (4) subject to the budget constraint (10). Since the old agent faces the downward sloping earning profile, we expect that he is not borrowing-constrained, which we verify later. Optimal expenditure implies that

$$c_t = \phi e_t,$$
$$h_t = \frac{1-\phi}{r_t^h} e_t,$$
$$u_t^o = \frac{e_t}{\left(r_t^h\right)^{1-\phi}},$$

where

$$e_t = c_t + r_t^h h_t$$

is total expenditure.

The budget constraint for the old household is given by

$$e_t + \frac{\sigma a_{t+1}}{R_t} = (1-\tau_t) w_t x_t + a_t. \tag{15}$$

Then, the first-order condition for savings a_{t+1} implies that the growth rate of consumption basket (period utility) is given by

$$\frac{u_{t+1}}{u_t} = \left(\beta R_t^u\right)^\eta, \tag{16}$$

where

$$R_t^u = \left(\frac{r_t^h}{r_{t+1}^h}\right)^{1-\phi} R_t. \tag{17}$$

We can think of R_t^u as the real interest rate in terms of utility.

Using Gertler (1999), we guess that expenditure is proportional to total wealth

$$e_t = \nu_t^o W_t^o, \tag{18}$$

where total wealth (W_t^o) is sum of nonhuman (a_t) and human wealth ($W_t^{ho} x_t$) as

$$W_t^o = a_t + W_t^{ho} x_t,$$
$$W_t^{ho} = (1-\tau_t) w_t + \frac{\sigma}{R_t} G^o W_{t+1}^{ho}.$$

Then, we can show that:

$$\frac{1}{\nu_t^o} = 1 + \sigma \beta^\eta \left(R_t^u\right)^{\eta-1} \frac{1}{\nu_{t+1}^o}. \tag{19}$$

This verifies the guess that consumption is proportional to total wealth in (18).

We also follow Gertler (1999) to guess $V_t^o = \Delta_t^o u_t^o$. Then, from (4, 16), we get

$$\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} = 1 + \sigma \beta^\eta \left(R_t^u\right)^{\eta-1} \left(\Delta_{t+1}^o\right)^{\frac{\eta-1}{\eta}}.$$

Thus from (19), we verify the guess by setting

$$\Delta_t^o = \left(\nu_t^o\right)^{\frac{\eta}{1-\eta}}. \tag{20}$$

The old agent is not borrowing-constrained if $a_{t+1} > \omega a_t$, and the most likely old agent who becomes constrained is the old agent who was young in the previous period so that there is no initial nonhuman asset. Thus the old households are not constrained if

$$(1-\tau_t) w_t x_t > \nu_t^o W_t^{ho} = \nu_t^o \left[(1-\tau_t) w_t x_t + \frac{\sigma}{R_t} W_{t+1}^{ho} \right].$$

We will check that this inequality holds for our parameters in the neighborhood of the steady state later.

3.5 Aggregate Equilibrium

In our simple open economy, housing supply is constant at $\overline{H}$ but requires maintenance which is proportional to the value of housing. Aggregate output is the sum of aggregate consumption C_t, housing maintenance (and tax) $\mu_t Q_t \overline{H}$, and net exports NX_t as

$$Y_t = C_t + NX_t + \mu_t Q_t \overline{H}.$$

We assume that the wage tax finances the homeownership subsidy, the property tax component of the maintenance cost finances government purchases of goods, and the government balances the budget in every period.[19]

Aggregate demand for consumption and housing by young households are

$$C_t^y = C_t^h + C_t^r = \phi \left[(1-\tau_t) w_t X_t^y + \left(1 - \frac{\omega}{R_t}\right) A_t^h \right], \tag{21}$$

$$H_t^h = \frac{1-\phi}{r_t^h} \left[\xi_t (1-\tau_t) w_t X_t^y + \left(1 - \frac{\omega}{R_t}\right) A_t^h \right], \tag{22}$$

19. We make this assumption to prevent government fiscal policy from affecting the liquidity of different households.

$$H_t^r = \frac{1-\phi}{r_t^r}(1-\xi_t)(1-\tau_t)w_t X_t^y, \tag{23}$$

where

$$A_t^h = \gamma\left(Q_t H_{t-1}^h - B_{t-1}^h\right)$$

is the net worth of young homeowners. In steady state, the borrowing constraint pins down the debt taken on by the young homeowners at

$$B_t^h = E_t Q_{t+1} H_t^h. \tag{24}$$

Aggregate demand for consumption and housing by old households are

$$C_t^o = \phi \nu_t^o \overline{W}_t^o, \tag{25}$$

$$H_t^o = \frac{1-\phi}{r_t^h} \nu_t^o \overline{W}_t^o, \tag{26}$$

where $\overline{W}_t^o$ is aggregate total wealth of old households which equals to the sum of nonhuman and human wealth

$$\begin{aligned} \overline{W}_t^o &= W_t^{ho} X_t^o + Q_t\left(H_{t-1}^o + H_{t-1}^r\right) - B_{t-1}^o + (1-\gamma)\left(Q_t H_{t-1}^h - B_{t-1}^h\right), \\ W_t^{ho} &= (1-\tau_t)w_t + \frac{\sigma G^o}{R_t} W_t^{ho}, \end{aligned} \tag{27}$$

where B_t^o is aggregate net borrowing of the old households at the beginning of date t. The bond market clearing condition at date $t-1$ implies that the sum of net debt of old and young households and the foreign sector must add up to zero as:

$$B_t^o + B_t^h + B_t^* = 0. \tag{28}$$

B_t^* is the net debt of the foreign sector (or the net foreign asset position of the home country) which evolves with net exports as follows:

$$\frac{B_t^*}{R_t} = B_{t-1}^* + NX_t.$$

Combining the goods market and net foreign asset accumulation, we have

$$\frac{B_t^*}{R_t} = B_{t-1}^* + Y_t - C_t - \mu_t Q_t \overline{H}. \tag{29}$$

We assume foreigners do not own home housing. Housing market equilibrium is given by

$$\overline{H} = H_t^h + H_t^r + H_t^o. \tag{30}$$

The endogenous state variables are $(B_{t-1}^*, H_{t-1}^h, B_{t-1}^h)$, and N_t^y, N_t^o, X_t^y, X_t^o, A_t, and R_t follows an exogenous process. Then, 15 endogenous variables Q_t, r_t^h, r_t^r, ξ_t, R_t^u, ν_t^o, C_t^y, C_t^o, H_t^h, H_t^r, H_t^o, $\overline{W}_t^o$, B_t^h, B_t^o, and B_t^* are determined by the 15 equilibrium conditions (6, 9, 14, 17, 19, 21–30) as a function of the state variables.

3.6 Stationary Representation

In the following, we focus on the case of constant population growth and constant age-related labor productivity growth, that is, G_N, G^y, and G^o are all constant. In contrast, aggregate productivity growth rate G_{At} and the real interest rate R_t may have a once-for-all permanent change unexpectedly.

We detrend the following variables by dividing by A_tN_t, because they have the same trend with A_tN_t :

$$Y_t, C_t^h, C_t^r, C_t^o, B_t^h, B_t^o, B_t^*, Q_t, r_t^h, r_t^r, r_t^o.$$

Because w_t has the same trend with A_t, we detrend w_t by dividing by A_t. We detrend the following variables by dividing by N_t, because they have the same trend with N_t

$$X_t^y, X_t^o, N_t^y, N_t^o.$$

We do not need to detrend the following variables:

$$H_t^h, H_t^r, H_t^o, \Delta_t^h, \Delta_t^r, \Delta_t^o, \xi_t, \nu_t^o.$$

4. CALIBRATION

4.1 Baseline Calibration

We calibrate the model to U.S. data at the annual frequency. All data series were obtained either from FRED of the Federal Reserve Bank of St. Louis or from the Lincoln Institute for Real Estate. All data definitions and sources are described in Appendix A.

In Table 1, there are a number of parameters that can be directly calibrated from the data. Starting with the growth rates, we set the population growth rate $G^N - 1$ to 1.4% based on the average annual growth rate of the Civilian Noninstitutional Population in the 1960–2019 period. Over the same period, aggregate real GDP grew at an annual rate of 3.1% implying an annual growth rate of GDP per capita ($G^A - 1$) of approximately 1.7%. We measure the real interest rate available to house buyers

TABLE 1
Baseline Parameter Values

G^N	1.0140	Population growth rate
G^A	1.0170	GDP per capita growth rate
G^y	1.0498	Endowment growth during youth
G^o	0.9476	Endowment growth when old
β	0.9615	Discount factor
γ	0.9638	Probability of remaining young
σ	0.9597	Probability of survival when old
μ	0.0406	Housing maintenance cost factor
υ	0.0082	Homeownership subsidy
η	0.5000	Intertemporal elasticity of substitution
$\overline{\chi}$	2.5536	Upper bound (housing preference)
ϕ	0.9323	Expenditure share nondurables
ω	0.5000	Speed of debt repayment
R	1.0400	Real mortgage rate

$(R - 1)$ using the average 30-year mortgage rate minus the average inflation rate between 1962 and 2019. Full-year data on the 30-year mortgage exists on FRED from 1972 onward. However, the 10-year Treasury bond yield goes back to 1962 and this is extremely highly correlated with the 30-year mortgage rate (see Data Appendix). We therefore impute the 30-year mortgage rate for the 1962–71 period based on the average relationship between the two series for the 1972–2019 period. This procedure gives a net real interest rate of 4.0% per annum.[20]

In our model, each young household stochastically switches from "youth" with rising endowment to "old age" with falling endowment. The average life-cycle profile of earnings is characterized by the endowment growth in youth (G^y) and the endowment growth rate in old age (G^o), together with transition rates from young to old and from old to death. We calibrate these parameters to match two moments from the profile of life-cycle earnings in the United States obtained from Gourinchas and Parker (2002) and to additional moments from the age distribution of the population. Gourinchas and Parker (2002) estimate that earnings at age 50 and 65 are, respectively, 40% and 20% higher than at age 25.

We calibrate the discount factor of households (β) at 0.9615 which is exactly the inverse of the long-term real interest rate in the baseline calibration. This implies a flat consumption profile in retirement. We set the homeownership subsidy (υ) to match the estimates of Poterba and Sinai (2008) who use the 2004 Survey of Consumer Finances and estimate that the homeownership subsidy is approximately 16.7% of the user cost of housing for homeowners (before the subsidy) in the United States.[21]

20. Using only the available 30-year mortgage rate data (1972–19) delivers an almost identical value for the net real interest rate.

21. We use the results in Table 2 for all households. The table shows that removing the tax distortions would increase the user cost of owner-occupied housing from 0.06 to 0.072. This means that under current policies, the user cost of housing is at least 16.7% lower than under no tax distortions.

There remain five additional parameters: the annual costs (including taxes) of owning housing as a fraction of the housing value (μ), the expenditure share on housing (ϕ), the probability of death for old households (σ), the upper bound on the uniform distribution for the rental preference parameter ($\overline{\chi}$) and the probability of switching from "youth" to "old age" (γ). We pick these parameters in order to minimize the sum of squared deviations of six key data moments from the U.S. housing market and their model counterparts. Specifically:

(i) We target the ratio of the value of housing to the value of the aggregate endowment to ensure that the average ratio of housing and consumer durables to GDP is equal to its average value of 1.55 over the 1960–2019 period. This moment is mostly affected by the expenditure share on housing (ϕ).

(ii) The average U.S. housing rent-to-price ratio in the period 1960–2018 is equal to 4.9% according to the updated data set of Davis, Lehnert, and Martin (2008). This helps identify the annual cost of owning housing (μ) which has a strong effect on the housing rent-to-price ratio.

(iii) The average ratio of household debt to GDP is equal to 59.8% in the 1960–2019 period. γ and σ are the most important parameters which affect this moment.

(iv) The net foreign asset position is targeted at the (negative of the) net worth of the Rest of the World vis-à-vis the United States. This has trended down over the past 20 years so we use the 2011–19 average which is around 35% of GDP. γ and σ also affect this moment.

(vi) The size of the population aged 20–54 is approximately 60% of the total population aged over 20. We use again σ and γ to target this moment.

(vii) The average homeownership rate in the United States is equal to 65.2% in the 1965–2019 period. This is matched by the upper bound on the uniform distribution for the rental preference parameter ($\overline{\chi}$).

Whenever we conduct simulations with a stochastic version of the model where the long-term level of growth or real interest rates can undergo probabilistic switches, we calibrate the probability of switches to 5% which makes such events a once-in-20-years occurrence.

We set ω—the adjustment speed with which households in negative equity must go back to positive housing equity—to 0.5. This implies that households eliminate half of the remaining negative equity each year, taking 5–6 years to get back to zero net worth following an adverse shock. Finally, we set η (the intertemporal elasticity of substitution) equal to 0.5.

Tables 1 and 2 show the baseline parameter values and the comparison between the data and the calibrated results. Despite having more moments than parameters, the model matches the data moments very well.

How about the model's implications for untargeted moments? Figure 3, Panel A compares the model-generated age profile for homeownership with the U.S. data

TABLE 2

Model vs. Data

Moment	Data	Model
Housing to GDP ratio	1.550	1.554
Net foreign assets to GDP ratio	−0.350	−0.352
Pop aged 20–54 as % of all aged over 20	0.600	0.600
Housing rent to price ratio	0.049	0.049
Homeownership rate	0.652	0.682
Mortgages to GDP ratio	0.598	0.585
(Homeowner imputed rent)/(rental price)	0.833	0.833
Income at age 55/income at age 25	1.400	1.365
Income at age 65/income at age 25	1.200	1.218

Note: "Data" refer to the U.S. data moments we target in the calibration procedure (for definitions and sources, see Appendix A). "Model" refers to the model implications for these moments as computed using the deterministic steady state.

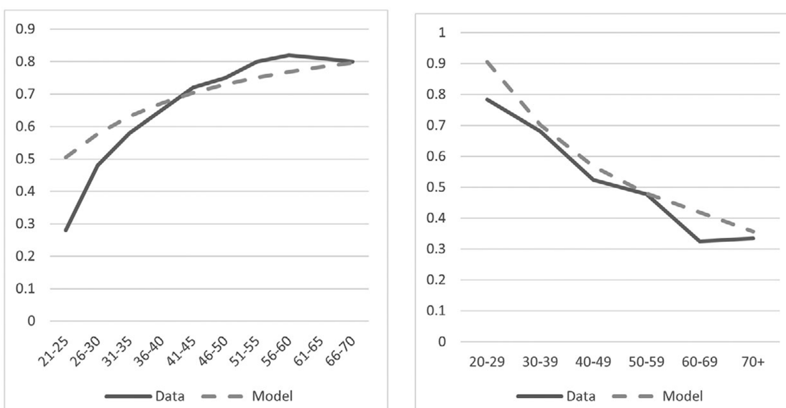


Fig 3. Homeownership over the Life Cycle: Data versus Model.

Notes: The homeownership data are from Halket and Vasudev (2014). The household leverage data are from Yilmazer and Devaney (2005). In both data and model, leverage is defined as debt to total assets. The *x*-axis is different age groups.

reported in Halket and Vasudev (2014).[22] Even though, at the individual level, our model produces a constant fraction of young households and all old households owning a home, the random transition of individuals from youth and old age produces aggregate life-cycle implications which are not so far from the data.[23]

Figure 3, Panel B compares the age profile for household leverage (debt to assets) of the model with the U.S. data reported in Yilmazer and Devaney (2005).[24] Here

22. See figure 3 in Halket and Vasudev (2014) for the data we use.

23. A notable discrepancy is that the model overpredicts homeownership in early youth due to the simplistic assumption that a fixed fraction of all young households buy a home regardless of age.

24. We use the data for Debt/Assets in table 3 in Yilmazer and Devaney (2005) (All Households with Assets below $151,800).

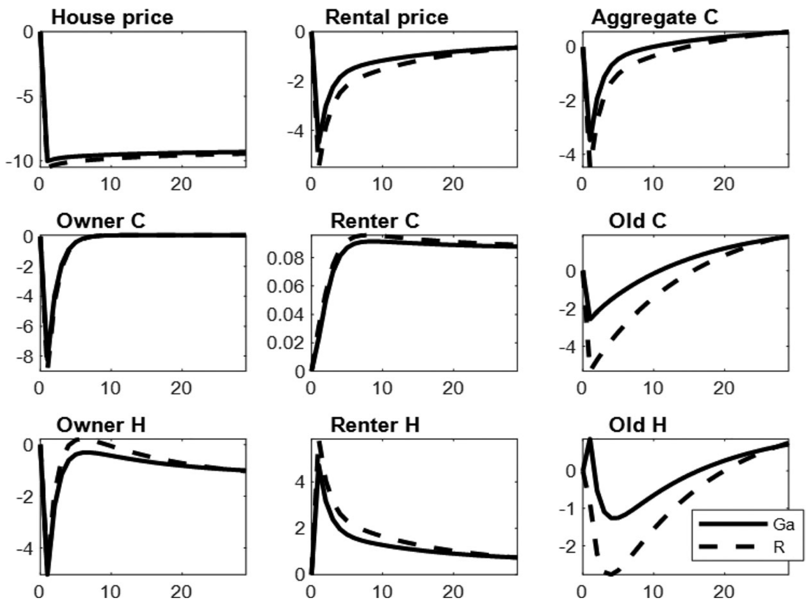


Fig 4. Comparing a 0.5% Increase in R and a 0.5% Decrease in G_A.

NOTES: The solid line is the IRF to a permanent 0.5% fall in the endowment growth rate in the baseline model. The dashed line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model. All IRFs are expressed as a percentage deviation from the baseline steady-state trend. Baseline parameter values are in Table 1.

again, we have a simplistic individual profile for the LTV ratio which is constant and close to unity for young homeowners. In old age, households scale down their expenditure due to the decline in human wealth and begin to save in expectation of declining future incomes. The home downsizing and the increased savings lead to a rapid decline in the LTV ratio at the individual level following a switch from "youth" to "old age." Yet again, the random transitions of individuals from youth to old age smooth the average life-cycle profiles making those relatively close to the data despite these being untargeted moments.

5. HOUSING AND WELFARE

We now use our calibrated OLG model with heterogenous households and financing constraints to analyze the macro-economic and welfare impact of changes in fundamentals. We focus on the role of long-term real interest rates and growth rates.

Figure 4 shows the response to a 0.5% permanent increase in the world real interest rate (the dashed line), and compares it to the response of price and quantities to a 0.5% permanent reduction in the growth rate of the per capita endowment (the solid line).

TABLE 3

Welfare Impact of Shocks

Shock	R	G^A
Old	0.40	−4.95
Young owners	−0.24	−5.21
Young renters	0.78	−4.16

Note: The table computes the percentage change in steady-state expenditure for different groups (Old, Young owners, and Young renters) which are equivalent to each shock in terms of their welfare impact in the baseline model. The R column examines a permanent 0.5% increase in the world real interest rate while the G^A column examines a permanent 0.5% reduction in the endowment growth rate. Baseline parameter values are in Table 1.

All variables use the stationary representation using the new trend of each variable as explained in Section 4.6.[25]

The key message from the graph is that the impact of the two shocks on housing prices and the consumption of the young is similar, even though the effects on the consumption of old households are somewhat different. House prices fall substantially (around 10%), and this leads to a significant decline in the consumption of leveraged owners (down by around 8%). Older homeowners experience smaller falls (5% with interest rate hike and 3% with growth rate fall) since they hold housing without leverage. The consumption of renters is mostly unaffected since they are not exposed to movements in housing prices.

The interest/growth rate shock also leads to a substantial redistribution of housing usage. The housing used by those exposed to housing prices falls (the old and the young homeowners) while the housing consumption of renters goes up, reflecting the decline in the housing rental price.

Table 3 shows the welfare impact of the two shocks on the three groups we are focusing on. The identity of the three groups is defined immediately before the shock occurs. All welfare measures in the table are "consumption equivalents": they show the permanent increase in consumer expenditure that would deliver the same welfare increase to the household as the interest and growth rate shocks we consider. The measures fully take into account the transition following the shock.

The first column shows the impact of a permanent interest rate hike by 0.5%. Young homeowners lose the equivalent of 0.24% of a permanent reduction of consumption, because they suffer from the leveraged loss of net worth associated with a lower housing price. Old homeowners gain from the increase in interest income on their saving but lose from the fall in housing prices. Overall, the impact of the higher interest rate dominates, leading to a 0.4% rise in welfare. Renters' welfare increases significantly (equivalent to a 0.78% permanent increase in consumption). Renters enjoy larger consumption due to the decline in rents associated with a permanently higher interest rate. The renters also gain from a lower housing price because they can buy a house cheaper when they get old in the future.

25. In our OLG model, aggregate saving largely depends upon the life cycle of earnings. Net foreign assets are therefore relatively stable in response to these exogenous permanent changes of the interest rate and the growth rate—unlike the representative household model with infinite lifetimes.

TABLE 4

Welfare Impact of Shocks with a Smaller $R + \mu - G$

Shock	R	G^A
Old	−0.27	−6.36
Young owners	−0.28	−5.91
Young renters	1.19	−4.46

Note: The table computes the percentage change in steady-state expenditure for different groups (Old, Young owners, and Young renters) which are equivalent to each shock in terms of their welfare impact in the model in which the world real interest rate is 1 pp lower than its 4.0% baseline value while the endowment growth rate (and all other parameters) are at baseline values (see Table 1). The R column examines a permanent 0.5% increase in the world real interest rate while the G^A column examines a permanent 0.5% reduction in the endowment growth rate.

The second column of Table 3 shows the welfare impact of a permanent decline in the per capita endowment growth rate by 0.5%. Households lose much more heavily from the permanent fall in the growth rate compared to the interest rate increase. This is because the lower growth rate leads to a fall in lifetime resources rather than merely representing a change in the relative price of future consumption.

The ranking of the welfare impacts is the same as for the real interest rate hike. Young homeowners lose (the equivalent of a 5.21% permanent decrease in consumption) due to leverage. Old homeowners' welfare falls as the housing price falls because they are net sellers of housing. In comparison to homeowners, renters lose less because the rents are cheaper and they can benefit from lower housing prices when they become old and buy a house in future. Still, young renters lose significantly as their permanent income falls.[26]

We argued in Section 1 that the impact of changes in long-term interest and growth rates depends crucially on $R + \mu - G$. In Figure 5, we compare the impact of a 0.5% increase in real interest rates under the baseline calibration and under an alternative calibration where $R + \mu - G$ is lower by 1% because the long-term real interest rate is 3% instead of 4% as in the baseline calibration. We see that the fall in both the housing price and in housing rents is greater when the initial $R + \mu - G$ is lower. The larger movement of the housing price means that leveraged owners are more adversely affected. The larger decline in rents induce renters to increase their consumption of housing and nondurables to a greater extent.

Table 4 displays the welfare effect of a 0.5% interest rate hike as well as a 0.5% growth rate fall at the lower value of $R + \mu - G$. The table shows that the level of $R + \mu - G$ also matters for the welfare effect of the shock. Compared to Table 3, there are important differences. Old homeowners now experience a decline in welfare under both shocks because, as net sellers of housing, they lose more from the fall in housing values. In contrast, young homeowners remain net buyers of housing, which

26. Of course when there is a favorable shock (such as a fall in interest rate or a rise in growth rate), the leveraged homeowners benefit more than renters. But the leveraged homeowners expand their houses when the house price and their net worth are high, and contract when the house price and their net worth are low. In other words, the rate of return on homeownership is negatively correlated with their marginal utility of consumption. Therefore, we want to emphasize that the leveraged owners are the most exposed to housing market risks.

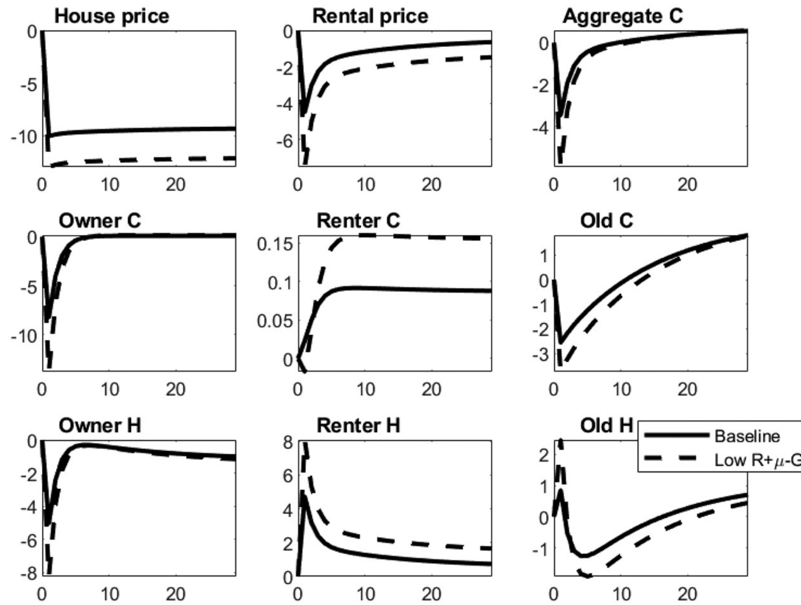


Fig 5. The Impact of a 0.5% Increase in R under the Baseline and under a Lower Value of $R + \mu - G$.

Notes: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate under the baseline calibration. The dashed line is the IRF to the same permanent 0.5% increase in the world interest rate in the model in which the world real interest rate is 1 pp lower than the baseline value of 4.0% while the endowment growth rate (and all other parameter values) are at their baseline values (see Table 1). All IRFs are expressed as a percentage deviation from the baseline steady-state trend.

helps to temper the welfare loss due to the net worth decline. Still, young homeowners are also more adversely affected compared to the baseline case because the larger fall in the housing price hurts their net worth to a greater extent.

The results in this section are largely unchanged even when the arrival of shocks to the interest rate and/or growth rate is anticipated. In the Online Appendix, we solve for the stochastic steady state of our economy under the assumption that households know that permanent shocks to the world real interest rate or the per capita endowment growth rate could hit with a certain arrival rate. The differences between stochastic and deterministic steady states are discussed in more detail in the Online Appendix. Intuitively, the anticipation of shocks leads to some precautionary behavior. The homeownership rate declines and the remaining young homeowners choose slightly smaller housing. The old save more and reduce the share of housing in their portfolios. However, the qualitative nature of the way the economy responds to R and G^A shocks does not change very much. The welfare impact is also similar. Hence, we continue with the analysis of policy measures in the next section using the simpler deterministic version of the model.

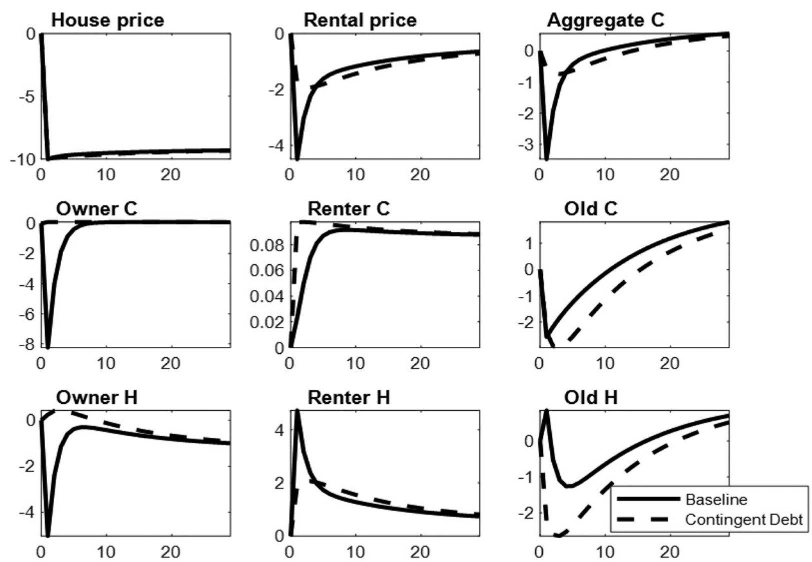


Fig 6. The Impact of a 0.5% increase in *R* under Simple Debt (the Baseline) and under State-Contingent Debt.

NOTES: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model with uncontingent debt. The dashed line is the IRF to the same permanent 0.5% increase in the world real interest rate in the model in which debt repayments are contingent upon the housing price realization. All IRFs are expressed as a percentage change from the baseline steady state. All parameter values are at baseline values (see Table 1).

6. (MIS)MANAGING THE HOUSING MARKET

In this section, we use our model to evaluate a number of housing policies that are either widely used, or that have been proposed.

6.1 State-Contingent Debt

We start by analyzing the role of the key missing market in our economy—the market for state-contingent mortgage debt. The value of state-contingent debt adjusts in line with the house price, insulating the net worth of leveraged young households from house price fluctuations. Figure 6 compares the reaction to a 0.5% permanent real interest rate increase in an economy with simple and with state-contingent mortgage debt (respectively, the solid and the dashed lines).

We can see that state-contingent debt mostly shields the consumption of borrower households from the effects of the shock. Their housing usage actually increases under state-contingent debt due to the decline in the user cost of housing following the shock. The consumption and housing usage of the old decline by more under state-contingent debt because they absorb all the losses from lower housing prices and they are net sellers of houses in future.[27] Preventing the large-scale redistribution away from young leveraged owners also helps to stabilize the rental market. Rents are smoother under state-contingent debt as there is no need for the size of the economy's rental sector to grow following negative shocks to long-term interest rates.

Despite the fact that fully state-contingent contracts do not exist, there are practical alternatives that offer net worth protection to young credit constrained people.[28] Notice in Figure 6 that the evolution of renters' consumption and housing usage under simple debt contracts is extremely similar to that of leveraged homeowners under state-contingent debt markets. The net worth of tenants is shielded from housing price fluctuations. Even though they are exposed to rental price fluctuations, the rental price fluctuations are considerably smaller than housing price fluctuations and tend to help renters absorb the effect of the aggregate shocks to interest rates and growth rates. In this sense, renting housing stabilizes the utility of young credit-constrained agents better than owning it.

The main reason why the rental market is not a full substitute for state-contingent debt or a housing equity market is because it cannot deal with different households' preference for owning versus renting. In our model, some households have a strong preference for owning and the rental market would not be useful for them. Nevertheless, with a healthy rental market, at least some of the credit-constrained young households choose to rent and become shielded from the consumption volatility induced by housing price fluctuations.[29]

6.2 *The Distortionary Effect of Homeownership Subsidies*

Despite the stabilizing risk-sharing properties of the rental market, U.S. public policy encourages homeownership via the tax system in three main ways. First, the interest on debt secured by the household's main residence is deductible for income tax. Second, the imputed rental income from owned houses is untaxed. Third, owner-occupied dwellings are exempt from capital gains tax. Residential property held for rental purposes does not enjoy any of these tax advantages although depreciation and property taxes can be deducted from landlords' tax bills. Sinai and Gyourko (2004) and Poterba and Sinai (2008) compute the size of the subsidies that are handed out to homeowners. Using the 2004 Survey of Consumer Finances, they find that for the average household age and income, implicit subsidies through the tax system reduce the user cost of owner-occupied housing by around 16.7%.

27. If young people can adjust labor supply more than old people, then young people would absorb some fraction of the loses from lower housing prices.

28. Equity contracts could implement a considerable amount of risk sharing but, despite efforts to introduce them, these are not yet widespread in the housing market.

29. If the taste and amenity shocks are causes for fluctuations of housing prices and rental prices, then homeownership may insure better than renting because their net worth tends to fluctuate in the same direction with the marginal utility of consumption (see Sinai and Souleles (2005) for empirical evidence). But these shocks tend to be more important for local housing markets, which are not our focus.

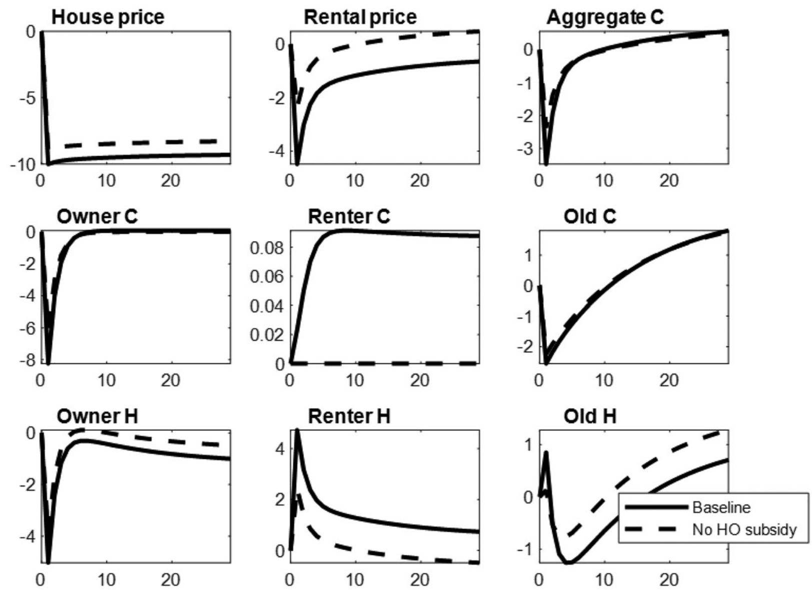


Fig 7. The Impact of a 0.5% Increase in R with a Homeownership Subsidy (the Baseline) and under no Homeownership Subsidy.

NOTES: The solid line is the IRF to a 0.5% permanent increase in the world real interest rate in the baseline model with a homeownership subsidy which in line with the estimates in Poterba and Sinai (2008). The dashed line is the IRF to the same permanent 0.5% increase in the world real interest rate without a homeownership subsidy. All IRFs are expressed as a percentage change from the baseline steady state. All other parameter values are at baseline values (see Table 1).

In line with this evidence, we set the homeownership subsidy at 0.82% of the housing value in the baseline calibration, implying a 16.7% decline in the user cost of housing for owner-occupied relative to landlord-owned rental properties. The effect of this policy on the model's steady state is to increase the homeownership rate and boost the price of housing, leading to higher leverage among mortgagors.

In addition to changing the model's steady state, the homeownership subsidy also alters the way the economy reacts to shocks. Figure 7 compares how the economy without a homeownership subsidy and with the baseline 0.82% homeownership subsidy reacts to a permanent 0.5% increase in the world real interest rate R.

We can see that the economy in which homeownership is subsidized experiences larger housing price and rent fluctuations, more consumption volatility and more wealth and housing redistribution between different groups. This happens for three main reasons. First, homeownership subsidies reduce the effective cost of owning a home and this has a similar effect on housing prices to a reduction in $R + \mu - G$. The higher price-to-rent ratio makes the housing price more responsive to shocks to R or G. Second, the higher homeownership rate means that there are more leveraged households who experience a hit to their net worth when housing prices fall. Third,

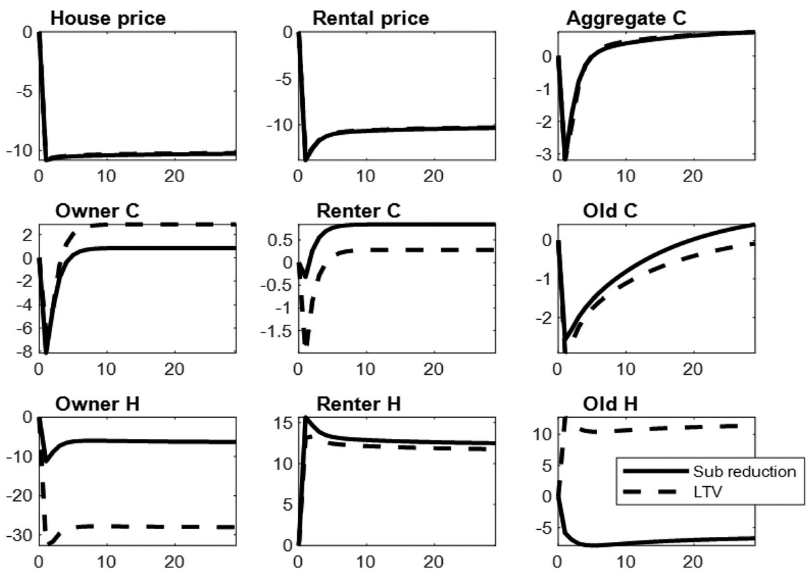


Fig 8. Comparing the Effect of a Homeownership Subsidy Reduction and an LTV Cap Imposition.

NOTES: The solid line is the IRF to a permanent reduction in the homeownership subsidy in the baseline model. The dashed line is the IRF to a permanent LTV cap for young borrowers. For comparability, both policy interventions have been chosen so as to generate a decline of housing prices of approximately 10%. All IRFs are expressed as a percentage change from the baseline steady state. All other parameter values are at baseline values (see Table 1).

the subsidy allows young borrowers to become even more leveraged. All these factors make aggregate consumption more volatile.

6.3 Policy Options

So far we have seen that the combination of volatile housing prices, noncontingent debt, and high leverage for young homeowners leads to large-scale redistribution and consumption volatility following shocks to long-term real interest rates and income growth rates. This is compounded by policies that encourage homeownership.

How can we reduce the vulnerability of the economy with noncontingent debt and homeownership subsidy? We consider two policy options. One is the reduction of homeownership subsidies, the other is the imposition of constraints on borrowers' LTV ratios. Here, we evaluate these policies taking into account the transition from the equilibrium of the baseline economy (with homeownership subsidies and no leverage restrictions) to an equilibrium with reduced subsidies or reduced household leverage.

Figure 8 compares the impact of reducing the homeownership subsidy from 0.82% to zero with that of an LTV cap. The latter policy is implemented in the model via

TABLE 5

Welfare Impact of Policies

Shock	Sub	LTV
Old	−2.87	−2.01
Young owners	−0.39	−2.07
Young renters	1.60	1.26

Note: The table computes the percentage change in steady-state expenditure for different groups (Old, Young owners, and Young renters) which are equivalent to each shock in terms of their welfare impact in the baseline model (see Table 1 for parameter values). The "Sub" column examines a permanent removal of the homeownership subsidy; the "LTV" column examines the imposition of an LTV cap on young borrowers. For comparability, the size of the LTV cap has been chosen to generate approximately the same 10% housing price decline as the subsidy removal.

a borrowing tax, which is imposed on young borrowers when its proceeds are then transferred back to the young homeowners lump sum.[30] The tax is 2.5% and its size is chosen so as to implement the same 10% fall in the housing price as the subsidy removal.

In many respects, the two policies have similar effects on aggregate variables. Housing prices decline and nondurable consumption falls for all households. The homeownership rate falls and housing usage rises for the renters who absorb a lot of the housing vacated by owners. Consumption also falls for renters despite the fact that their net worth is insulated from housing price fluctuations, because the removal of the subsidy to homeowners reduces rents and leads renters to substitute from nondurables into housing.

The impact of the two policy options differs considerably in one respect. LTV caps curtail severely the housing usage of young owners while boosting the housing usage of the old. In contrast, the removal of the subsidy is more "democratic" in the sense that it crowds out all owners (young and old).

Table 5 compares the welfare impact of removing homeownership subsidies (the first column) and imposing an LTV cap (the second column). Several observations stand out from the table. First, only renters benefit from these two policies. The renters gain from a lower housing price because they enjoy lower rents and can buy housing cheaper in the future.[31]

30. This combination of tax and transfer produces the same effect as an LTV constraint because it keeps the resources available to the household unchanged while inducing it to choose a lower level of leverage. We implement the LTV constraint in this way because it is analytically more tractable while still capturing the essence of the way a maximum LTV cap affects the macroeconomy as well as young homebuyers.

31. The reason why renters marginally prefer the removal of homeownership subsidies lies in the interplay of two opposing forces. Renters will become old homeowners in the future and would therefore gain from a policy that preserves homeownership subsidies. However, they are currently tax payers and would like a policy that reduces the taxes they have to pay. Which of these effects dominates depends on the probability of becoming old and on the renters' discount factor. For our baseline calibration, renters marginally prefer the policy that reduces their taxes today at the cost of lower housing subsidies in the future.

Second, homeowners suffer differently from these two policies. Old homeowners lose more from the removal of homeownership subsidies, equivalent to a 2.87% reduction of permanent consumption. Young leveraged homeowners are hurt the most by the LTV cap, equivalent to a 2.07% reduction in permanent consumption. That these two policies create such prominent losers explains why it is so difficult to implement them in practice.

7. CONCLUSIONS

The fluctuations of housing prices largely reflect the value of the land (or location) on which housing is built. Land is also difficult to steal and does not depreciate if transferred to a different owner. These two features make housing good collateral, affording housing buyers substantial leverage.

However, the long-lived nature of housing also makes its price highly sensitive to changes in the expectations of the long-run required real rate of return and the growth rate of aggregate income. Hence, housing is risky and its riskiness grows as the difference between long-term real interest rates and growth rates diminishes as it has done over the past few decades. Leveraged young homeowners are particularly vulnerable to housing price fluctuations because their net worth is highly exposed to changes in home prices.

We build a tractable OLG model that captures the above narrative and use it to examine the impact of housing market policies. The model produces large redistributions following unanticipated shocks to long-run real interest rates and growth rates. When the real interest rate rises persistently, leveraged homeowners suffer from a prolonged period of negative equity that they clear by enduring lower consumption of both housing and nondurables. Old savers gain from higher interest rates because they enjoy a higher rate of return on savings, and the young renters gain because they benefit from lower housing prices when they buy in the future. Lower growth rates hurt all households because they reduce the value of human wealth. Young homeowners lose out especially due to leveraged losses on housing wealth. The impact of such shocks is magnified in a nonlinear fashion when the required return on housing is close to the economy's growth rate. We use these adverse shocks to illustrate the vulnerability of young leveraged homeowners. Although these young homeowners gain from favorable shocks, our framework illustrates the large redistribution working through the housing market.

The redistribution between different groups would be moderated by state-contingent debt although in practice such debt does not exist yet. Outside equity for housing is also not used and equity-like products such as "Shared Appreciation Mortgages" have not proved popular. However, we argue that the rental market has many features that make its impact similar to that of state-contingent debt or housing equity. In particular, it limits the tendency of negative shocks to redistribute from leveraged and credit-constrained young households toward unconstrained older households.

The rental market is therefore a practical way to insulate vulnerable households from fluctuations in housing prices.

Unfortunately, many countries penalize the rental market through a policy of homeownership subsidies. We show that homeownership subsidies amplify the impact of shocks on housing prices and increase the number of leveraged young households who are most affected by fluctuations in home values. We examine the welfare effect of removing homeownership subsidies and compare it to the welfare effect of imposing borrowing limits on young homeowners. On impact, both policies hurt homeowners and benefit renters through their negative impact on housing prices. Young owners are hurt the most by borrowing limits, while old households are hurt the most by lower homeownership subsidies. That both policies create very substantial losers explains why the tax treatment of owner-occupied housing is hard to reform and the use of LTV or loan-to-income caps remains politically controversial.

APPENDIX A: DATA APPENDIX

(i) GDP (1960–2019): average real GDP growth of 3.1% (calibrate $G^A G^N$)—Data source: National Income and Product Accounts (NIPA).

(ii) Civilian noninstitutional population (1960–2019): average growth of 1.4% (calibrate G^N)—Data source: Current Population Survey (Bureau of Labor Statistics).

(iii) U.S. housing rent-price ratio (1960–2018): average of 4.9% (calibrate μ)—Data source: Davis, Lehnert, and Martin (2008), "The Rent-Price Ratio for the Aggregate Stock of Owner-Occupied Housing." *Review of Income and Wealth*, 54(2), 279–84; data located at Land and Property Values in the United States, Lincoln Institute of Land Policy.

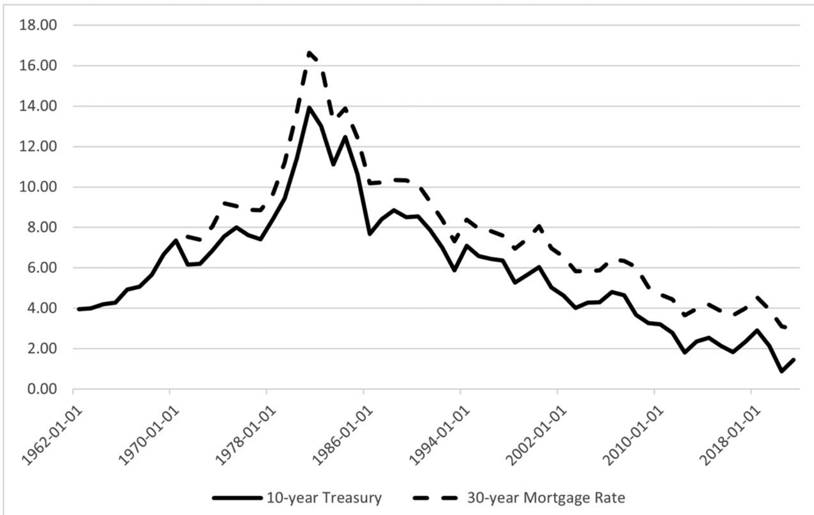
(iv) Value of housing + durables to GDP (1960–2019): average of 1.55—Data source: U.S. Flow of Funds (Balance Sheets of Households and Nonprofit Organizations - Real estate at market value divided by annual GDP).

(v) Real mortgage interest rate (1962–2019): average of 4.0%. 1972–2019, 30-year mortgage rate minus the current rate of inflation (Data source: FRED). For 1962–71, the 30-year mortgage rate is imputed using the 10-year nominal Treasury yield (Data source: FRED). See Figure 9, Panels A and B for a visual demonstration of the close relationship between the 30-year mortgage rate and the 10-year Treasury yield. The regression in Figure 9, Panel B shows the coefficients and specification used to impute the mortgage rate in the 1962–71 period.

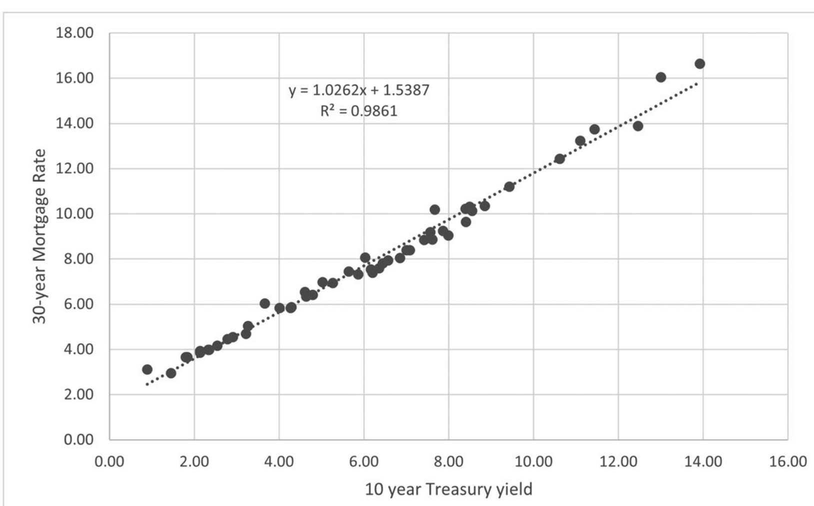
(vi) Life-cycle income process (1): ratio of income aged 50 to income aged 25: 1.4 (Gourinchas and Parker 2002).

(vii) Life-cycle income process (2): ratio of income aged 65 to income aged 25: 1.2 (Gourinchas and Parker 2002).

(viii) Demographic mix: Population aged 20–54/(Population aged over 20) = 0.6 (2019 value)—Data source: Current Population Survey (Bureau of Labor Statistics).



(a) 30-year nominal mortgage rate vs 10-year nominal Treasury yield since 1962



(b) 30-year mortgage rate vs 10-year Treasury yield scatter plot

Fig 9. Scatterplot: 30-Year Nominal Mortgage Rate versus 10-Year Nominal Treasury Yield.

(ix) Homeownership rate for the United States (1965–2019): 65.2%.
(x) Mortgage debt to GDP (1960–2019): average since 1980 = 59.8%—Data source: U.S. Flow of Funds (Balance Sheets of Households and Nonprofit Organizations).
(xi) Net foreign liabilities to GDP (2011–19): average net foreign liabilities of approximately 35% of annual GDP—Data source: U.S. Flow of Funds (Balance Sheets of Foreign Sector: Net worth divided by annual GDP).

APPENDIX B: POPULATION AND AGGREGATE INCOME DYNAMICS

Let N_t^y and N_t^o be population of young and old households. Superscript y denotes young and o old. The population of young and old evolve as follows:

$$N_t^y = \gamma N_{t-1}^y + (G_N - \gamma)N_{t-1}^y = G_N N_{t-1}^y,$$
$$N_t^o = \sigma N_{t-1}^o + (1-\gamma)N_{t-1}^y.$$

The number of young households grows at rate G_N and consists of γN_{t-1}^y households who continue to be young and $(G_N - \gamma)N_{t-1}^y$ newborns.

We normalize the efficiency unit labor of newborns to be unity as $x_t = 1$. Let G^y and G^o be the age-related labor productivity growth rate (plus one) of young and old agents. Let X_t^y and X_t^o be the aggregate age-related (efficiency unit) labor of young and old agents. Then we have:

$$X_t^y = \gamma G^y X_{t-1}^y + 1 \cdot (G_N - \gamma)N_{t-1}^y,$$
$$X_t^o = \sigma G^o X_{t-1}^o + (1-\gamma)G^y X_{t-1}^y.$$

APPENDIX C: PERFECT FORESIGHT EQUILIBRIUM: THE CONSUMPTION-SAVINGS PROBLEM OF THE OLD

After solving the static choice between housing usage and nondurable consumption, the problem of the old households can be reduced to the problem of choosing current expenditure (e_t) versus savings (a_{t+1}):

$$V_t^o = \max_{e_t, a_{t+1}} \left[\left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{C1}$$

subject to the budget constraint:

$$e_t + \frac{\sigma a_{t+1}}{R_t} = (1-\tau_t)w_t x_t + a_t,$$
$$u_t^o = \frac{e_t}{\left(r_t^h\right)^{1-\phi}}.$$

The envelope theorem implies:

$$\frac{\partial V_t^o}{\partial a_t} = \frac{1}{\left(r_t^h\right)^{1-\phi}} \left(u_t^o\right)^{\frac{-1}{\eta}} V_t^{\frac{1}{\eta}}.$$

The first-order condition with respect to savings (a_{t+1}) is given by

$$\begin{aligned} 0 &= -\frac{\sigma}{R_t} \frac{1}{\left(r_t^h\right)^{1-\phi}} \left(u_t^o\right)^{\frac{-1}{\eta}} \left(V_t^o\right)^{\frac{1}{\eta}} + \beta\sigma \left(E_t V_{t+1}^o\right)^{\frac{-1}{\eta}} \left(V_t^o\right)^{\frac{1}{\eta}} \frac{\partial V_{t+1}^o}{\partial a_{t+1}} \\ &= -\frac{1}{\left(r_t^h\right)^{1-\phi}} \left(u_t^o\right)^{\frac{-1}{\eta}} + \beta R_t \left(E_t V_{t+1}^o\right)^{\frac{-1}{\eta}} \frac{1}{\left(r_{t+1}^h\right)^{1-\phi}} \left(u_{t+1}^o\right)^{\frac{-1}{\eta}} \left(E_t V_{t+1}^o\right)^{\frac{1}{\eta}} \\ &= -\left(u_{t+1}^o\right)^{\frac{1}{\eta}} + \beta R_t \left(\frac{r_t^h}{r_{t+1}^h}\right)^{1-\phi} \left(u_t^o\right)^{\frac{1}{\eta}}. \end{aligned}$$

This implies equation (16) in the main text.

We now verify the guess that expenditure is proportional to total (human and non-human) wealth

$$e_t = \nu_t^o W_t^o,$$

where

$$\begin{aligned} W_t^o &= a_t + W_t^{ho} x_t, \\ W_t^{ho} &= (1-\tau_t) w_t + \frac{\sigma}{R_t} G^o W_{t+1}^{ho}. \end{aligned}$$

Under our guess, the budget constraint becomes

$$\nu_t^o W_t^o + \frac{\sigma}{R_t} W_{t+1}^o = W_t^o, \text{ or}$$

$$\frac{W_{t+1}^o}{W_t^o} = \frac{R_t}{\sigma} \left(1 - \nu_t^o\right). \tag{C2}$$

Since $u_t^o = \frac{e_t}{(r_t^h)^{1-\phi}}$, we know that the guess above implies that:

$$u_t^o = \frac{\nu_t^o W_t^o}{\left(r_t^h\right)^{1-\phi}}.$$

Substituting into equation (16), we have:

$$\frac{\nu_{t+1}^{o} W_{t+1}^{o}}{\left(r_{t+1}^{h}\right)^{1-\phi}} = \left(\beta R_t^u\right)^{\eta} \frac{\nu_t^o W_t^o}{\left(r_t^h\right)^{1-\phi}}. \tag{C3}$$

Putting together this and the budget constraint and rearranging we get implies equation (19) in the text. This confirms the initial guess.

Finally, we verify the guess that the value function is proportional to current utility

$$V_t^o = \Delta_t^o u_t^o.$$

Then because

$$V_t^o = \left[\left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma\left(E_t V_{t+1}^o\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

we have

$$\begin{aligned}
\left(\Delta_t^o u_t^o\right)^{\frac{\eta-1}{\eta}} &= \left(u_t^o\right)^{\frac{\eta-1}{\eta}} + \beta\sigma\left(\Delta_{t+1}^o u_{t+1}^o\right)^{\frac{\eta-1}{\eta}}, \text{ or} \\
\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} &= 1 + \beta\sigma\left[\Delta_{t+1}^o\left(\beta R_t^u\right)^{\eta}\right]^{\frac{\eta-1}{\eta}}, \text{ or} \\
\left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}} &= 1 + \sigma\beta^{\eta}\left(R_t^u\right)^{\eta-1}\left(\Delta_{t+1}^o\right)^{\frac{\eta-1}{\eta}}.
\end{aligned}$$

This confirms the initial guess about the value function. Comparing the above equation with (19), we can see that:

$$\frac{1}{\nu_t^o} = \left(\Delta_t^o\right)^{\frac{\eta-1}{\eta}}.$$

This confirms (20) in the main.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Table A1: Comparing the Deterministic and Stochastic Steady States

Table A2: Comparing the Baseline Stochastic Steady State with a Stochastic Steady State where the volatility of R^* changes is doubled

Table A3: Welfare impact of shocks

Figure A1: Comparing a 0.5% increase in R starting from the stochastic and from the deterministic steady state

Data S1