Homework #5 **PHI 340**

Please read: *Possibilities and Paradox*, pp 95–100, and *Logical Pluralism*, pp 61–71. Please do the following problems.

1. Note to yourself that the following are valid for intuitionistic logic.

   (a) $\vdash (A \wedge B) \leftrightarrow (B \wedge A)$

   (b) $\vdash (A \vee B) \leftrightarrow (B \vee A)$

   (c) $\vdash (A \vee (B \vee C)) \leftrightarrow ((A \vee B) \vee C)$

   (d) $\vdash (A \wedge (B \wedge C)) \leftrightarrow ((A \wedge B) \wedge C)$

   (e) $\vdash (A \wedge (B \vee C)) \leftrightarrow ((A \wedge B) \vee (A \wedge C))$

   (f) $\vdash (A \vee (B \wedge C)) \leftrightarrow ((A \vee B) \wedge (A \vee C))$

   (g) $\vdash (A \rightarrow (B \rightarrow C)) \leftrightarrow ((A \wedge B) \rightarrow C)$

   (There is nothing to turn in for this problem. Just go back and look at your CPL proofs to convince yourself that these proofs do not require $\neg\neg$-Elimination.)

2. Please determine if each of the following sentences is intuitionistically valid. If a sentence (or one direction of a biconditional) is *not* valid, then provide a counterexample. You do not need to turn in tableaux; just the counterexamples.

   (a) $\vdash \neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$

   (b) $\vdash \neg(A \rightarrow B) \leftrightarrow (A \wedge \neg B)$

   (c) $\vdash (A \rightarrow B) \leftrightarrow (\neg A \vee B)$

   (d) $\vdash (A \rightarrow B) \vee (B \rightarrow A)$

3. Please prove that each of the following sentences is intuitionistically valid. You may use either natural deduction ("Lemmon style"), or tableaux. (We use "$\bot$" to denote any sentence of the form $A \wedge \neg A$.)

   (a) $\vdash \neg A \leftrightarrow (A \rightarrow \bot)$

   (b) $\vdash \neg A \leftrightarrow \neg\neg\neg A$

   (c) $\vdash (A \rightarrow B) \rightarrow (\neg\neg A \rightarrow \neg\neg B)$

   (d) $\vdash \neg\neg(A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$

4. Suppose that the atomic sentences are given by $\{p_0, p_1, p_2, \ldots\}$. We define the family $\{G_n\}$ of "Glivenko sentences" as follows:

$$G_n = \bigvee_{i,j=0, i \neq j}^{n} (p_i \leftrightarrow p_j).$$

   So, for example,

$$G_2 = (p_0 \leftrightarrow p_1) \vee (p_0 \leftrightarrow p_2) \vee (p_1 \leftrightarrow p_2).$$

   Prove that for each n, G_n is *not* an intuitionistic tautology. (You will have to use induction on n.)

5. Complete the proof (begun in class, Tuesday, Nov 20) of the fact: if A does not contain $\vee$, and all atoms in A are negated, then $\vdash A \leftrightarrow \neg\neg A$.

6. Compute the Gödel translation A° where

$$A = (q \wedge p) \rightarrow (q \vee \neg r) \ .$$

   (Hint: Work from the inside out.)