PHI 340: Final exam study guide

December 22, 2007

The final exam will be cumulative, but with large emphasis on those topics covered after the midterm exam.

I. A reminder of what we covered before the midterm exam:

- Basics of set theory. Definitions, axioms, how to prove things.
  - e.g. Cartesian products; relations; functions (a special kind of relation); inductively defined sets
- Basics of languages and logics.
  - What are the components of a *language*?
  - What are the valuations of classical propositional logic (**CPL**)?
  - What is the definition of "$X \vDash A$"?
  - What is a *tautology*?
  - What is a *base* for a valuation? Do all valuations have bases?
  - What is a *logic*?
  - Define: *soundness, completeness, argument completeness, strong completeness, compactness.*
  - What is the difference between $\vDash$ and $\vdash$?
  - Calculus of deductive systems. e.g. be able to show that: $Cn(Cn(X)) = Cn(X)$; if $X \subseteq Y$ then $Cn(X) \subseteq Cn(Y)$.
  - (From after midterm) Let reductionist-CPL be the language that has two connectives $\vee, \neg$, and has as inference rules the Rule of Assumptions, and the Intro and Elim rules for these two connectives. Explain the relation between CPL and reductionist-CPL.
- The normal modal logics: K, D, T, S4, S5
  - Models and restrictions on the accessibility relation R.
  - What is the relationship between "models" and "valuations"?

- Tableau methods for the normal modal logics (K, D, T, S4, B, S5).
- Characteristic sentences for the normal modal logics.
- Paradoxes of material implication.
- The rule of necessitation:

  If $A_1, \ldots, A_n \vDash B$ then $\Box A_1, \ldots, \Box A_n \vDash \Box B$.

  For which modal logics is this true?
- (Optional) Natural deduction for S5.
- Explain how, in one sense, CPL is an extension of S5.
- Do the valuations of S5, S4, etc. have bases?

II. The main emphases of the final exam:

Intuitionistic Logic

- What is the semantics for **I**? What is a model for **I**, and what are the truth conditions for sentences?
- Be able to decide whether or not an argument is valid in **I**. If it is valid, be able to prove it with natural deduction. If it is invalid, be able to give a counterexample.
- Be able to sketch the proof that **I** does not permit a finite-valued functional semantics. (How does the proof use the Glivenko sentences?)
- What is the relationship between **I** and **CPL**? In particular, define the Gödel translation, and show that it preserves validity. Compare and contrast the features of the embedding F of **CPL** in **I**, and of the embedding G of **I** in **CPL**. Are these really the same logic?
- You should understand how intuitionistic logic can be thought of as an extension of S4, in particular by mapping $\neg A$ to $\Box{\sim}A$, and $A \rightarrow B$ to $\Box(A \supset B)$. (Question for reflection: Does this mean that intuitionistic negation and conditional are "intensional" connectives?)

- What are some of the most salient **CPL** theorems that are invalid in **I**? Does **I** avoid the paradoxes of material implication?

Relevance Logic

- Know the structural rules, and how they define the logical systems **RW**, **R**, **RM**, and **CPL**.
- Be able to give natural deduction proofs of arguments for the languages **R** and **RM**. e.g. you should be able to prove "$\vdash p \vee \neg p$" in **R**.
- **FDE** tableaux or the **FDE** algorithm for deciding validity. (You should be proficient at one of these two methods.)
- Be able to prove that in **FDE** there are no tautologies or contradictions. (In general, you should be proficient at these sorts of arguments using induction on the construction of sentences.)
- Be able to use Sugihara's semantics to show that an argument is not valid in **RM**.
- You will *not* be expected to create sophisticated $\mathbf{R}_+$ or **R** models from scratch. But you will be expected to understand what **R** models are, and to be comfortable determining whether or not sentences are true in a given model. You should also be able to construct simple **R** models — e.g. to construct a model in which $a \vDash p \wedge \neg p$.

Of course, you should understand all of this material so well that you can answer integrative questions that ask you to compare and contrast the various logics.