

MAT 313, Category Theory

Midterm take-home exercise

Instructions: These problems are due by 8am on Monday, November 29. You are permitted to use any sources (including Mac Lane's book, lecture notes, internet), and even to talk about problems with other students. But if work is collaborative, please note this fact.

1. Is there a functor $F : \mathbf{Set} \rightarrow \mathbf{Set}$ that *fails* to preserve monomorphisms?
2. Does the identity functor on \mathbf{Set}_f (the category of finite sets) have a colimit?
3. Let $F : D \rightarrow C$ and $G : C \rightarrow D$ be functors, and suppose that GF is naturally isomorphic to 1_C , and FG is naturally isomorphic to 1_D , i.e. F and G give an equivalence of categories. Show that $F \dashv G$, i.e. F is left adjoint to G .
4. An arrow $f : a \rightarrow b$ is said to be a *regular monomorphism* just in case f is an equalizer of a parallel pair $g, h : b \rightarrow c$ of arrows. True or False: the pullback of a regular monomorphism is a regular monomorphism. That is, in a pullback diagram:

$$\begin{array}{ccc} a & \longrightarrow & b \\ \downarrow g & & \downarrow f \\ c & \longrightarrow & d \end{array}$$

if f is a regular mono, then so is g .

5. Show that if a category C has products and equalizers then C has pullbacks.