

(MAT 313 / PHI 323) Category Theory  
*Categories for the Working Mathematician*, Chapter 6.

Homework: CWM p 142, #2. This problem might look more difficult than it is. To show that  $G^T : X^T \rightarrow X$  creates limits, choose a cone  $(\alpha_j : \langle r, h \rangle \rightarrow \langle x_j, h_j \rangle)$  in  $X^T$  that becomes a limiting cone when the structure is forgotten, i.e.  $(\alpha_j : r \rightarrow x_j)$  is a limiting cone in  $X$ . We just need to show that the original cone in  $X^T$  is a limit. To this end, pick another cone  $(\beta_j : \langle s, k \rangle \rightarrow \langle x_j, h_j \rangle)$  in  $X^T$ , and show that there is a unique arrow  $\gamma : \langle r, h \rangle \rightarrow \langle s, k \rangle$  etc.. Since  $(\alpha_j : r \rightarrow x_j)$  is limiting in  $X$ , there is a unique arrow  $\gamma : r \rightarrow s$  etc.. The work comes in showing that  $\gamma$  is in fact a structure-preserving arrow.xz