Summary of facts established in Chapter 3.

1. Yoneda lemma: For any functor $F : C \longrightarrow \mathbf{Set}$ there is a bijection from Fc to $\mathrm{Nat}(\hom_C(c, -), F)$. This bijection is natural in F and c.

2. Let C be a category, and let c, d be objects of C. If there is a natural isomorphism $\alpha : C(c, -) \Rightarrow C(d, -)$, then c is isomorphic to d.

3. Every equalizer is a monomorphism. Every coequalizer is an epimorphism.

4. If $e \xrightarrow{k} d \underset{g}{\overset{f}{\rightrightarrows}} c$ is an equalizer diagram, then k is an isomorphism if and only if $f = g$.

5. In **Set** every monomorphism is an equalizer.

6. Let

$$\begin{tikzcd} a \arrow[r, "y"] \arrow[d, "x"'] & b \arrow[d, "f"] \\ c \arrow[r, "g"'] & d \end{tikzcd}$$

   be a pullback diagram. If f is a monomorphism then so is x. If f is an isomorphism then so is x. If f is a regular monomorphism then so is x.

7. Given two commuting squares:

$$\begin{tikzcd} a \arrow[r, "f"] \arrow[d, "g"'] & b \arrow[r, "l"] \arrow[d, "h"] & e \arrow[d, "m"] \\ c \arrow[r, "k"'] & d \arrow[r, "n"'] & r \end{tikzcd}$$

   If both inner squares are pullbacks, then so is the rectangle. If the right hand square and the rectangle are pullbacks, then so is the left hand square.

8. Given an arrow $f : c \longrightarrow d$, the following are equivalent:

(a) f is a monomorphism.

(b) The following square is a pullback:

$$\begin{tikzcd} c \arrow[r, "1_c"] \arrow[d, "1_c"'] & c \arrow[d, "f"] \\ c \arrow[r, "f"'] & d \end{tikzcd}$$

(c) The pullback of f along itself exists:

$$\begin{tikzcd} e \arrow[r, "x"] \arrow[d, "y"'] & c \arrow[d, "f"] \\ c \arrow[r, "f"'] & d \end{tikzcd}$$

and $x = y$.

9. If f is a regular mono and epi, then f is iso.

10. Every split mono is regular.

11. Suppose that the category C has equalizers. If $F : C \longrightarrow D$ preserves equalizers and reflects isomorphisms then F is faithful.

12. If the category C has finite limits then for any object d of C, the slice category C/d has binary products.