MAT 313, Category Theory

Final take-home exercise

Instructions: You have 72 hours to complete this exam. You are permitted to use only class materials (van Oosten book, Butz book, lecture notes, previous homework). You should *not* gather information from other textbooks, from the internet, or from other students.

Please answer one question from each section (three questions total).

Section A

1. Is there a functor $F : \mathrm{SET} \to \mathrm{SET}$ that *fails* to preserve monomorphisms?
2. Does the identity functor on FSET (finite sets) have a colimit?
3. Does the free group functor $F : \mathrm{SET} \to \mathrm{GRP}$ have a left adjoint?

Section B

1. van Oosten Exercise 98 (uniqueness of adjoints)
2. van Oosten Exercise 102 (if F, G give an equivalence of categories then $F \dashv G$ and $G \dashv F$)
3. A functor $F : \mathcal{C} \to \mathcal{D}$ is said to *create limits* just in case: for every functor $E : \mathcal{E} \to \mathcal{C}$, if FE has a limit (L, μ), then E has a limit (K, ν) that is carried by F to (L, μ). Prove that the forgetful functor $F : C/\mathcal{C} \to \mathcal{C}$ creates limits. (Here $C/\mathcal{C}$, for an object C of $\mathcal{C}$, is the category whose objects are arrows $f : C \to X$, and an arrow between $f : C \to X$ and $f' : C \to X'$ is an arrow $g : X \to X'$ such that $gf = f'$.)

Section C

1. van Oosten Exercise 32 (equivalence of categories)
2. van Oosten Exercise 118 (Kleisli category; uniqueness of the comparison functor L)
3. Let $\mathcal{C}$ be a category, let J be a discrete category (i.e. a set), and let $\mathcal{C}^J$ be the category of functors from J to $\mathcal{C}$. If $\mathcal{C}$ is Cartesian closed then is $\mathcal{C}^J$ also Cartesian closed?