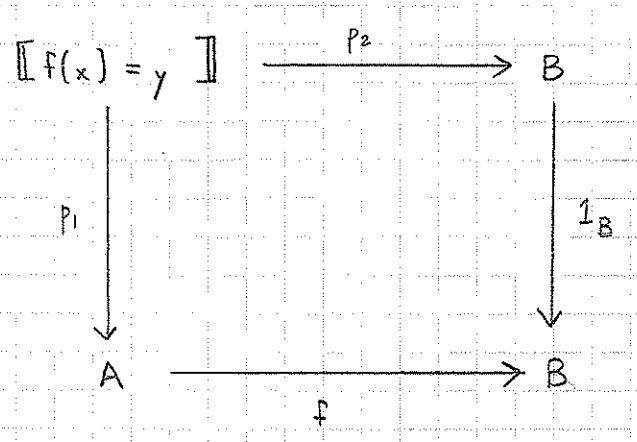
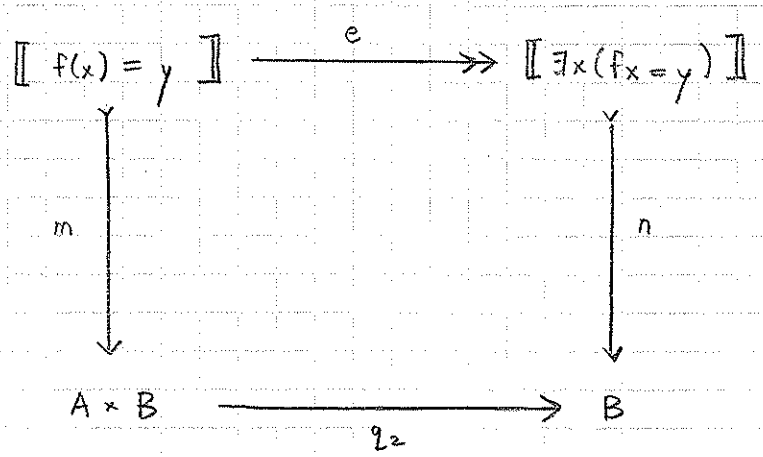
Claim $f : A \longrightarrow B$ is regular epi iff $\vdash_y \exists x (f(x) = y)$ is true.

Proof Recall that $[\![f(x) = y]\!] = [\![f(x) = 1_B(y)]\!]$ is a pullback:



If f is regular epi then p_2 (its pullback) is regular epi. Furthermore, if p_2 is regular epi then $f p_1$ is regular epi, and since p_1 is iso, f is regular epi. Now $[\![\exists x (f(x) = y)]\!]$ is the image of $q_2 m$ in the following diagram



But $q_2 m = p_2$. Hence $[\![\exists x (fx = y)]\!]$ is maximal iff $\mathrm{Im}(p_2) = B$ iff p_2 is regular epi iff f is regular epi. ▨