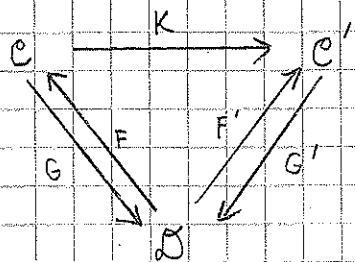


Lemma

Let $\mathcal{C}, \mathcal{C}', \mathcal{B}$ be categories, with functors:



such that $(F \dashv G, \eta, \varepsilon)$ and $(F' \dashv G', \eta', \varepsilon')$ are adjunctions,
also $F' = KF$ and $G = G'K$. If $\eta = \eta'$ then $\varepsilon' * 1_K = 1_K * \varepsilon$.

Proof: By the triangle equalities for $(F \dashv G, \eta, \varepsilon)$ we have

$$1_G = (1_G * \varepsilon) \circ (\eta * 1_G). \quad (*)$$

Then calculate:

$$\begin{aligned} \varepsilon' * 1_K &= (\varepsilon' * 1_K) \circ (1_{F'} * 1_G). \\ &= (\varepsilon' * 1_K) \circ (1_{F'} * 1_G * \varepsilon) \circ (1_{F'} * \eta * 1_G) && \text{insert } (*) \\ &= (\varepsilon' * 1_K) \circ (1_{F'} * 1_{G'} * 1_K * \varepsilon) \circ (1_{F'} * \eta * 1_G) && G = G'K \\ &= (1_K * \varepsilon) \circ (\varepsilon' * 1_K * 1_{F'} * 1_G) \circ (1_{F'} * \eta * 1_G) && \text{interchange} \\ &= (1_K * \varepsilon) \circ (\varepsilon' * 1_{F'} * 1_G) \circ (1_{F'} * \eta * 1_G) && KF = F' \\ &= (1_K * \varepsilon) \circ (1_{F'} * 1_G) && \text{triangle equality for } F', G'. \\ &= 1_K * \varepsilon. \quad \square \end{aligned}$$