$\boxed{\text{Lemma}}$ Let $\mathcal{C}, \mathcal{C}', \mathcal{D}$ be categories, with functors:

$$\begin{tikzcd}
\mathcal{C} \arrow[rr, "K"] \arrow[dr, shift right, "G"'] & & \mathcal{C}' \arrow[dl, shift left, "G'"] \\
& \mathcal{D} \arrow[ul, shift right, "F"'] \arrow[ur, shift left, "F'"] &
\end{tikzcd}$$

such that $(F \dashv G, \eta, \varepsilon)$ and $(F' \dashv G', \eta', \varepsilon')$ are adjunctions, also $F' = KF$ and $G = G'K$. If $\eta = \eta'$ then $\varepsilon' * 1_K = 1_K * \varepsilon$.

<u>Proof:</u> By the triangle equalities for $(F \dashv G, \eta, \varepsilon)$ we have

$$1_G = (1_G * \varepsilon) \circ (\eta * 1_G). \qquad (*)$$

Then calculate:

$$\begin{aligned}
\varepsilon' * 1_K &= (\varepsilon' * 1_K) \circ (1_{F'} * 1_G) \\
&= (\varepsilon' * 1_K) \circ (1_{F'} * 1_G * \varepsilon) \circ (1_{F'} * \eta * 1_G) && \text{insert } (*) \\
&= (\varepsilon' * 1_K) \circ (1_{F'} * 1_{G'} * 1_K * \varepsilon) \circ (1_{F'} * \eta * 1_G) && G = G'K \\
&= (1_K * \varepsilon) \circ (\varepsilon' * 1_K * 1_F * 1_G) \circ (1_{F'} * \eta * 1_G) && \text{interchange} \\
&= (1_K * \varepsilon) \circ (\varepsilon' * 1_{F'} * 1_G) \circ (1_{F'} * \eta * 1_G) && KF = F' \\
&= (1_K * \varepsilon) \circ (1_{F'} * 1_G) && \text{triangle equality for } F', G'! \\
&= 1_K * \varepsilon. \qquad \blacksquare
\end{aligned}$$