

# Practice Proofs

## 1 Proofs that use only Stage 1 rules

1. Associativity of &

$$P \& (Q \& R) \dashv\vdash (P \& Q) \& R$$

2. Commutativity of &

$$P \& Q \vdash Q \& P$$

3. Idempotence of &

$$P \dashv\vdash P \& P$$

## 2 Proofs that use CP but not $\vee$ -Elimination or RAA

1. Weakening the Consequent

$$P \rightarrow (Q \& R) \vdash P \rightarrow Q$$

$$P \rightarrow Q \vdash P \rightarrow (Q \vee R)$$

2. Strengthening the Antecedent

$$(P \vee Q) \rightarrow R \vdash P \rightarrow R$$

$$P \rightarrow R \vdash (P \& Q) \rightarrow R$$

3. Transitivity

$$P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$$

4. Contrapositive

$$P \rightarrow Q \vdash \neg Q \rightarrow \neg P$$

5. Prefixing

$$P \rightarrow Q \vdash (R \rightarrow P) \rightarrow (R \rightarrow Q)$$

6. Suffixing

$$P \rightarrow Q \vdash (Q \rightarrow R) \rightarrow (P \rightarrow R)$$

7. Contraction

$$P \rightarrow (P \rightarrow Q) \vdash P \rightarrow Q$$

8. Permutation

$$P \rightarrow (Q \rightarrow R) \vdash Q \rightarrow (P \rightarrow R)$$

9. Fusion

$$P \rightarrow (Q \rightarrow R) \dashv\vdash (P \& Q) \rightarrow R$$

10. Positive Paradox

$$P \vdash Q \rightarrow P$$

11. [No name]

$$\neg(P \& Q) \rightarrow \neg(R \vee S) \vdash R \rightarrow Q$$

12. [No name]

$$(P \& Q) \rightarrow R, P \rightarrow Q \vdash P \rightarrow R$$

13. [No name]

$$P \vdash Q \rightarrow (P \& Q)$$

14. [No name]

$$P \rightarrow Q, P \rightarrow R \vdash P \rightarrow (Q \& R)$$

15. [No name]

$$P \rightarrow (Q \& R) \rightarrow (P \rightarrow Q) \& (P \rightarrow R)$$

16. [No name]

$$P \vdash (P \rightarrow Q) \rightarrow Q$$

17. [No name]

$$P \rightarrow (Q \rightarrow R) \vdash (P \rightarrow Q) \rightarrow (P \rightarrow R)$$

18. [No name]

$$\vdash P \rightarrow P$$

### 3 Proofs that use $\vee E$ but not RAA

1. Distribution

$$P \& (Q \vee R) \dashv\vdash (P \& Q) \vee (P \& R)$$

$$P \vee (Q \& R) \dashv\vdash (P \vee Q) \& (P \vee R)$$

2. [No name]

$$P \rightarrow Q, R \rightarrow S \vdash (P \vee R) \rightarrow (Q \vee S)$$

3. Commutativity of  $\vee$

$$P \vee Q \vdash Q \vee P$$

4. Associativity of  $\vee$

$$P \vee (Q \vee R) \dashv\vdash (P \vee Q) \vee R$$

5. Idempotence of  $\vee$

$$P \dashv\vdash P \vee P$$

6. [No name]

$$(P \rightarrow Q) \vee (P \rightarrow R) \vdash P \rightarrow (Q \vee R)$$

## 4 Proofs that use RAA

*RAA can actually be derived from the other rules. So, technically, anything that can be proven with RAA can be proven without it.*

1. Ex Falso Quodlibet

$$P, \neg P \vdash Q$$

2. [No name]

$$P \rightarrow \neg P \vdash \neg P$$

3. Negative Paradox

$$\neg P \vdash P \rightarrow Q$$

4. Chain Order

$$\vdash (P \rightarrow Q) \vee (Q \rightarrow P)$$

5. [No name]

$$\vdash (P \rightarrow Q) \vee (Q \rightarrow R)$$

6. Material Implication

$$P \rightarrow Q \dashv\vdash \neg P \vee Q$$

$$\neg(P \rightarrow Q) \dashv\vdash P \& \neg Q$$

7. De Morgan's Laws

$$\neg(P \vee Q) \dashv\vdash \neg P \& \neg Q$$

$$\neg(P \& Q) \dashv\vdash \neg P \vee \neg Q$$

8. Modus Tollendo Ponens (Disjunctive Syllogism)

$$P \vee Q, \neg P \vdash Q$$

9. Modus Ponendo Tollens

$$\neg(P \& Q), P \vdash \neg Q$$

10. [No name]

$$(P \rightarrow Q) \rightarrow Q \vdash P \vee Q$$

11. Peirce's Law

$$\vdash ((P \rightarrow Q) \rightarrow P) \rightarrow P$$

12. Excluded Middle

$$\vdash P \vee \neg P$$

13. Law of Non-Contradiction

$$\vdash \neg(P \& \neg P)$$

14. [No name]

$$\neg(P \leftrightarrow Q) \dashv\vdash (\neg P \leftrightarrow Q)$$

15. [No name]

$$\neg(P \leftrightarrow Q) \dashv\vdash (P \vee Q) \& (\neg P \vee \neg Q)$$

16. Positive Paradox

$$P \vdash Q \rightarrow P$$