

HOMEWORK #3 SOLUTIONS

1. $P \rightarrow \neg Q$

a) $\mathcal{Q} \rightarrow \neg P$

This is a substitution instance by replacing the variable ‘ P ’ by ‘ Q ’ and the variable ‘ Q ’ by ‘ P ’

b) $P \rightarrow R$

This is not a substitution instance because substitution can only be performed on propositional variables, and not, for example, on negated variables. Thus to make this a substitution instance it would have to be changed to $P \rightarrow \neg \neg R$ by replacing the variable ‘ Q ’ by ‘ $\neg R$ ’ and leaving P .

c) $(P \rightarrow \neg Q) \rightarrow \neg(P \rightarrow \neg Q)$

This is a substitution instance by replacing the variable ‘ P ’ by the wff ‘ $(P \rightarrow \neg Q)$ ’ and the variable ‘ Q ’ by the wff ‘ $(P \rightarrow \neg Q)$ ’

2.

P	Q	R	$\neg(P \vee R)$	$\&$	$(\neg Q \rightarrow (P \& R))$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	F
F	T	T	F	F	T
F	T	F	T	F	F
F	F	T	F	F	T
F	F	F	T	F	F

3.

a) The argument is invalid as the following truth assignment shows

P	Q	R	$(P \rightarrow Q)$	\vee	$(Q \rightarrow R)$	$\neg R$	\rightarrow	$\neg(P \& Q)$	Q	\rightarrow	$\neg P$
T	T	T	T	T	T	T	T	F	T	F	T

b) The argument is invalid as the following truth assignment shows

P	Q	R	S	$(P \vee Q) \rightarrow (R \vee S)$	$P \leftrightarrow \neg(R \& S)$	$Q \leftrightarrow \neg(P \& R)$	$(S \& P) \rightarrow \neg(P \vee R)$
T	T	F	T	T	T	F	F

4.

a)

P	Q	R	$(P \vee \neg Q)$	\rightarrow	$(P \leftrightarrow (Q \& R))$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	T	F	F

b)

P	Q	R	$(\neg P \vee (\neg Q \rightarrow R))$	\rightarrow	$((P \& R) \rightarrow \neg Q)$
T	T	T	F T T	F	T T T F F T
T	T	F	F T T	T	T F F T F T
T	F	T	F T T T F T T	T	T T T T T F
T	F	F	F T F T F F	T	T F F T T T F
F	T	T	T F T F T T	T	F F T T F T
F	T	F	T F T F T F	T	F F F T F T
F	F	T	T F T T T T	T	F F T T T F
F	F	F	T F T T F F	T	F F F T T T F

5.

- a) The first sentence does not imply the second as the third row of the truth table shows:

P	Q	R	$(P \& Q)$	\leftrightarrow	$(Q \& R)$	P	\leftrightarrow	Q
T	T	T	T T T	T	T T T	T	T	T
T	T	F	T T T	F	T F F	T	T	T
T	F	T	T F F	T	F F T	T	F	F
T	F	F	T F F	T	F F F	T	F	F
F	T	T	F F T	F	T T T	F	F	T
F	T	F	F F T	T	T F F	F	F	T
F	F	T	F F F	T	F F T	F	T	F
F	F	F	F F F	T	F F F	F	T	F

- b) The first sentence does indeed imply the second as the truth table shows:

P	Q	R	P	\leftrightarrow	$(Q \vee R)$	$\neg P$	\rightarrow	$(Q \leftrightarrow R)$
T	T	T	T	T	T T T	F	T	T T T
T	T	F	T	T	T T F	F	T	T F F
T	F	T	T	T	F T T	F	T	F F T
T	F	F	T	F	F F F	F	T	F T F
F	T	T	F	F	T T T	T	F	T T T
F	T	F	F	F	T T F	T	F	T F F
F	F	T	F	F	F T T	T	F	F F T
F	F	F	F	T	F F F	T	F	F T F

6.

P	Q	P	#	Q	$(\neg P \& Q)$	v	$(P \& \neg Q)$
T	T	T	F	T	F T F T	F	T F F T
T	F	T	T	F	F T F F	T	T T T F
F	T	F	T	T	T F T T	T	F F F T
F	F	F	F	F	T F F F	F	F F T F