Midterm Exam Key (Version W)

A.1. ... there is a truth assignment relative to which both $A_1, \ldots, A_n$ are true and B is false.

A.2. ... there is a truth assignment relative to which A is true, and another truth assignment relative to which A is false.

B.1. $L \rightarrow -D$

B.2. $L\&(M \rightarrow I)$

B.3. $-L\& -R$

C.1.

1	(1) $R \rightarrow -(P \rightarrow Q)$	A
2	(2) $-(-Q \vee -R)$	A
3	(3) $-R$	A
3	(4) $-Q \vee -R$	3 vI
2,3	(5) $(-Q \vee -R)\& -(-Q \vee -R)$	4,2 &I
2	(6) $- -R$	3,5 RAA
2	(7) R	6 DN
1,2	(8) $-(P \rightarrow Q)$	1,7 MPP
9	(9) Q	A
10	(10) P	A
9	(11) $P \rightarrow Q$	10,9 CP
1,2,9	(12) $(P \rightarrow Q)\& -(P \rightarrow Q)$	11,8 &I
1,2	(13) $-Q$	9,12 RAA
1,2	(14) $-Q \vee -R$	13 vI
1,2	(15) $(-Q \vee -R)\& -(-Q \vee -R)$	14,2 &I
1	(16) $- -(-Q \vee -R)$	2,15 RAA
1	(17) $-Q \vee -R$	16 DN

C.2.

1	(1) $(P \rightarrow Q)\&(-P \rightarrow Q)$	A
1	(2) $P \rightarrow Q$	1 &E
1	(3) $-P \rightarrow Q$	1 &E
4	(4) $-Q$	A
1,4	(5) $-P$	2,4 MTT
1,4	(6) Q	3,5 MPP
1,4	(7) $Q\& -Q$	6,4 &I
1	(8) $- -Q$	4,7 RAA
1	(9) Q	8 DN
-	(10) $((P \rightarrow Q)\&(-P \rightarrow Q)) \rightarrow Q$	1,9 CP

D.1. It is invalid. Consider the truth assignment:

$$v(P) = T, v(Q) = F, v(R) = F, v(S) = T$$

This truth assignment makes the premise true and the conclusion false.

D.2. False. For example, let A be the sentence "P" and let B be the sentence "$Q\& - Q$". Then "$P \to (Q\& - Q)$" is contingent although "$Q\& - Q$" is not contingent.

D.3. The sentence "$-(P\&Q)\& - (-P\& - Q)$" is equivalent to "$-(P \leftrightarrow Q)$".

E.1. True. The argument with Line 1 as premise and Line n as conclusion is valid because Line 1 is an inconsistency. (There is no case where Line 1 is true, hence whenever Line 1 is true, so is Line n.) By the completeness of the propositional calculus, it follows that there is a correctly written proof of this form.