Homework 9

A. Symbolize, taking the domain of quantification to be persons, and using:

 $Sx \equiv x$ is a soprano. $Tx \equiv x$ is a tenor. $Lxy \equiv x$ is louder than y. $Rxy \equiv x$ respects y.

- 1. No tenor who is louder than all sopranos respects any soprano.
- 2. A tenor who is louder than some soprano is also louder than some tenor.
- 3. There are sopranos who respect only those tenors who are louder than they.
- 4. If a tenor respects all sopranos who respect him, then that tenor is respected by all sopranos.
- **B.** Symbolize, taking the domain of discourse to be persons, and using *only* the following vocabulary:

 $Pxy \equiv x$ is a parent of y $Mx \equiv x$ is male $Ixy \equiv x$ is identical to y $Txy \equiv x$ is taller than y

- 1. x and y are first cousins.
- 2. x has at most two daughters.
- 3. x has (exactly) two grandfathers.
- 4. x is the tallest child of y.

С.

- 1. Show that there is a sentence of propositional logic that is not logically equivalent to any sentence whose only connective is " \rightarrow ". (Hint: Use proof by induction.)
- 2. Show that the inference rule \lor -Elimination is sound; that is, if line *n* results from lines i, j, k, l, m by \lor -Elimination, and lines i, k, m are "good", then line *n* is good. (Definition: A line is **good** if the sentence to the right of the line number is a semantic consequence of the sentences on the dependency lines. A sentence *B* is a **semantic consequence** of sentences A_1, \ldots, A_n if: for any valuation *v*, if *v* assigns true to A_1, \ldots, A_n then *v* assigns true to *B*. We denote this by $A_1, \ldots, A_n \models B$. A propositional logic **valuation** is an assignment of truth values to sentences that obeys the truth-table relationships.)