Homework 7

A. Let $\mathscr{I}$ denote the interpretation given by:

$$\text{DoQ} = \{1,2,3,4,5\}$$
$$\underline{\text{Ext}}(Fx) = \{1,2,3\} \quad \underline{\text{Ext}}(Gx) = \{4\} \quad \underline{\text{Ext}}(Hx) = \{5\}$$
$$\underline{\text{Ref}}(m) = 2 \qquad\qquad \underline{\text{Ref}}(n) = 4$$

Determine if the following sentences are true or false relative to $\mathscr{I}$.

1. $(x)(-Fx \vee -Gx)$
2. $(x)((Gx\&Fx) \rightarrow Hx)$
3. $-(x)Fx \rightarrow (\exists x)(Fx\&Gx)$
4. $-Gm\&(\exists x)(Fx\&Gn)$
5. $(\exists x)(-Fx\& - Gx)$
6. $(\exists x)(Fx \rightarrow Gx)$

B. Give counterexamples to the following invalid argument forms. You do not need to show your work, but you do need to present an interpretation, and explain how it solves the problem.

1. $(x)(Fx \rightarrow Hx) \vdash (x)((Fx \vee Gx) \rightarrow Hx)$
2. $(\exists x)(Fx \rightarrow Gx)$, $(\exists x)Fx \vee (\exists x)Gx \vdash (\exists x)Fx \rightarrow (\exists x)Gx$
3. $(x)Fx \leftrightarrow (x)Gx \vdash (x)(Fx \leftrightarrow Gx)$

C. Determine if the following arguments are valid. If you say that an argument is valid, explain how you know that fact. If you say that an argument is invalid, provide a counterexample.

1. $(x)(Fx \rightarrow Gx)$, $(\exists x)(Gx\&Hx) \vdash (\exists x)(Fx\&Hx)$
2. $(x)((Fx\&Gx) \rightarrow Hx)$, $(\exists x)Fx \vdash -(x)Gx \vee (\exists x)Hx$
3. $(x)Fx \leftrightarrow (\exists x)Gx \vdash (x)Fx \vee (x) - Gx$