

Homework 3

- Which of the following sentences is a *substitution instance* of the sentence $P \rightarrow \neg Q$.
 - $Q \rightarrow \neg P$
 - $P \rightarrow R$
 - $(P \rightarrow \neg Q) \rightarrow \neg(P \rightarrow \neg Q)$.
- Write out a full truth table for the following sentence. Highlight in some way (e.g. draw a circle around) the column under the main connective of the sentence.

$$\neg(P \vee R) \& (\neg Q \rightarrow (P \& R))$$

- Use truth tables to determine if the following arguments are valid. You do not need to show all of your work. But if an argument is not valid, give a truth-assignment that witnesses this fact.
 - $(P \rightarrow Q) \vee (Q \rightarrow R)$
 - $\neg R \rightarrow \neg(P \& Q) \quad // \quad Q \rightarrow \neg P$
 - $(P \vee Q) \rightarrow (R \vee S)$
 - $P \leftrightarrow \neg(R \& S)$
 - $Q \leftrightarrow \neg(P \& R) \quad // \quad (S \& P) \rightarrow \neg(P \vee R)$
- Determine whether each of the following sentences is consistent. If a sentence is consistent, give a truth-assignment relative to which the sentence is true.
 - $(P \vee \neg Q) \rightarrow (P \leftrightarrow (Q \& R))$
 - $(\neg P \vee (\neg Q \rightarrow R)) \rightarrow ((P \& R) \rightarrow \neg Q)$

5. For each of the following pairs of sentences, determine whether the first sentence implies the second. If the implication fails to hold, give a truth-assignment that witnesses this fact.

(a) $(P \& Q) \leftrightarrow (Q \& R)$ $P \leftrightarrow Q$

(b) $P \leftrightarrow (Q \vee R)$ $\neg P \rightarrow (Q \leftrightarrow R)$

6. Write out a truth table for the *exclusive* sense of “or”. That is, fill in the following table:

P	Q	$P\#Q$
T	T	
T	F	
F	T	
F	F	

where # denotes the exclusive “or” connective. Now write a sentence containing only $P, Q, \&, \vee,$ and \neg that has the same truth table.