Mark Budolfson

Proof Strategies

Teaching someone how to do something is often difficult, especially when a complicated procedure is involved, and especially when that procedure is difficult to put into words. For example, imagine having to teach a person who had never played a sport how play basketball. The best strategy in that case would probably involve showing them (over and over) how to shoot, dribble, etc., while explaining to them in words the rules and goals of the game, etc.

Teaching students how to find the best proofs of sequents is difficult in an analogous way. It is easy enough to explain the rules that constrain proofs in general, but that's not the same thing as explaining how to discover particular proofs for sequents.

One thing that we have been doing is providing examples of proofs. But another thing we can do is to outline a strategy that can be followed and internalized. It must be admitted that this will not amount to a procedure that can be applied mechanically, but it constitutes a helpful strategy nonetheless—especially when you are otherwise 'stuck'.

I will assume that you have been given a sequent, and that you are writing a proof on a blank sheet of paper. Essentially, the strategy requires you to work both forward and backward—forward from the top of the page, starting with the 'premises' of the sequent, and backward from the bottom of the page, starting with the 'conclusion' of the sequent. Your proof should be complete if you can connect the top and bottom sections and fill in the details in accord with the rules.

Some of these steps will be obvious, but I've included them for the sake of completeness.

(*See opposite page*.)

1 Write the '**premises**' of the sequent as assumption lines at the top of the page. Go to 2.

2 Look at the '**conclusion**' of the sequent.

If it is not obvious how to finish the proof,
then write the 'conclusion' at the bottom of the page.
The 'conclusion' is then the 'most immediate goal'.
Go to 3.

3 Look at the most immediate goal.

If it is not obvious how to finish the proof, then
examine the **logical form** of the most immediate goal.

f it is neither a conditional
ıor a biconditional, then it is
ıot so obvious what to do.
Гry the things below first;
f they don't help, go to 4.

If it is a **conditional**,
then add the antecedent
as an assumption, and write
the consequent on the line
above the most immediate goal.
By doing this, the consequent
has become your new most immediate goal.
Return to 3.

If it is a **biconditional**,
then write each of the
corresponding conditionals
on the two lines above the
most immediate goal.
By doing this, these conditionals
have become your new immediate
goals. (Choose one to pursue first.)
Return to 3.

irst, see if you can apply any of the rules to what you already have at the top of the page; toward this end, you'll want to xamine the **logical form** of each line you have at the top of the page. This will give you a hint as to what you can do. For xample, if a line is a conditional, then see if you also have or can get the antecedent of that conditional (or the negation of the onsequent) on a line without introducing unwanted dependencies, and apply **MPP** (or **MTT**) if you can. If a line is a onjunction, then apply **&-Elim**. If it is a disjunction, then think about working toward applying **v-Elim**.

If you still don't see a way to get what you want, then if your most immediate goal is a(n)…

..**conjunction**,
vrite each conjunct on the
wo lines above the most
mmediate goal. By doing
his, these formulas have
ecome your new most immediate
oals. (Choose one to pursue
irst.) Return to 3.

…**disjunction**
First, consider whether it
is feasible to get either
disjunct from what you have at the.
top of the page. (If you can, then you
can use v-Intro to get the disjunction.)
If this doesn't look feasible, then
go to 4.

…**negated formula**
If you can't get a negated
formula from what you
have above, then assume
the formula that results
from removing the negation, and make
any standard contradiction your new most
immediate goal.
(The idea here is that you'll use RAA later.)
As an example of this strategy: if your goal
is the negated formula '~A', then add 'A' as
an assumption, and make any contradiction
your new most immediate goal. Return to 3.

…**atomic formula**
Go to 4.

4 Assume the negation of the goal, and make *any* standard contradiction your new most immediate goal. (This is the '**RAA** strategy'.) Return to 3.

After you've discovered a proof, follow these additional steps.

5 Check your work to ensure that your use of the rules is permitted and that the details of your proof are filled out properly (assumption dependency columns, etc.).

6 See if there are any unnecessary lines in your proof, or if you can see any other way of constructing a shorter proof of the sequent.