Interpretations for Relation Symbols

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Suppose that your domain $\mathcal{D}$ is all living people. Then, to give an interpretation of a two-place relation symbol (say, "R") you need to specify some relation that can hold between people. e.g., you could specify:

$$Rxy \equiv x \text{ is taller than } y.$$

In this case, the "ordered pair" ⟨Shaquille O'Neal, Allen Iverson⟩ is in the extension of R, but the ordered pair ⟨Shirley Tilghman, Shaquille O'Neal⟩ is not in the extension of R. Note that *order matters*. Note also that ⟨Shaq, Shaq⟩ is not in the extension of R, since Shaq is not taller than himself. Unlike "taller than", some natural language relations are symmetric — e.g., "is married to".

Quantifiers with relations: We need to think carefully about how to read sentences with quantifiers and relations. Consider the following.

- $(y)(\exists x)Rxy$ says that for each person y, there is some person x than is taller than y. This sentence is false, because there is some person that is as tall as every other person.
- $(\exists x)Rxx$ says that someone is taller than him/herself. This is obviously false.
- If m is interpreted as naming George Bush, then $(\exists x)Rmx$ says that George Bush is taller than someone.
- $(\exists x)Rxm$ says that someone is taller than George Bush.
- Rmy isn't a sentence, because the y isn't "bound" by a quantifier. (Actually, Rmy should be thought of as a one-place predicate — namely, "y is shorter than George Bush".)

- $(x)(x)Rxx$ says that everyone is taller than him/herself. (The first quantifier is simply redundant. You should save yourself trouble by never placing a quantifier with variable x inside the "scope" of another quantifier with variable x.)

So, to interpret a relation symbol you can:

1. Interpret it as a relation in ordinary language (as we did above), or as some well-known technical relation (e.g., in the domain of natural numbers, Rxy could be interpreted as the relation "x is a prime factor of y".)

2. If your domain is finite, and indeed very small, you can interpret an n-place relation symbol by giving a list of ordered n-tuples of objects in your domain. For example, suppose the domain is $\mathcal{D} = \{\alpha, \beta, \gamma\}$. To give an interpretation for a two-place relation symbol (say, R) you need to specify a set of ordered pairs of objects in $\mathcal{D}$. This can be done in a number of ways.

   List: Give a list of ordered pairs, e.g.:

$$\text{Ext}(R) = \{\langle\alpha, \gamma\rangle, \langle\beta, \beta\rangle, \langle\gamma, \alpha\rangle\}.$$

   Matrices: If the relation is binary (i.e., is followed by *two* variables), you can specify an interpretation with a matrix. The column gives the first entry and the row gives the second entry. A "+" indicates that the pair is in the extension of the relation and a "−" indicates that the pair is not in the extension. e.g., the following table gives the same interpretation of R as the list above:

R	α	β	γ
α	−	−	+
β	−	+	−
γ	+	−	−

   Arrow diagrams: For binary relations, we write a dot for each member of the domain, and we write an arrow between any two elements that bear the relation R to each other. [We will give examples in lecture and precept.]

A ternary (i.e., three place) relation on a domain is a set of ordered triples. So, once again, such a relation may be specified by supplying a natural language ternary relation — such as "x is between y and z". Or, a ternary relation can be specified by listing the elements in its extension.

Do you think that the following argument is valid?

> Princeton is between NYC and Philly.
> Therefore, Princeton is between Philly and NYC.

Translate the argument into quantifier notation, and explain why the resulting argument is not valid.

1 Predicates from Relations

We can build predicates out of relations and quantifiers. For example, if "Lxy" is interpreted as "x loves y", then we have:

$(\exists y)Lxy$ x is a lover.

$(\exists x)Lxy$ y is beloved.

If "Kxy" is interpreted as "x kills y", then "$(\exists y)Kxy$" says that x is a killer. If "Rxy" is interpreted as "x victimizes y", then "$(\exists x)Rxy$" says that y is a victim.