The goal of formal analysis

1. Example: When are two theories **equivalent**?

   Definition (Tarski & Glymour)**.** Theories T_1 and T_2 are *definitionally equivalent* just in case there are definitional extensions T_i' of T_i such that T_1' is logically equivalent to T_2'.

   (a) Larry Sklar (1982) *Philosophical Topics*

   T_1 All lions have stripes.

   T_2 All tigers have stripes.

   "Whatever this structural 'isomorphism' is to be, it cannot be a purely formal notion. It cannot be, that is, an interrelationship which can be determined to hold solely on the basis of the logical form of the theories in question." (p 93)

   "The meanings of the terms in the theories, however construed, are crucial to questions of equivalence." (p 93)

   (b) Ted Sider (2020) *The Tools of Metaphysics and the Metaphysics of Science*

   "Purely formal accounts [of equivalence] fail because they entirely neglect meaning." (p 181)

   (c) Trevor Teitel (2021) *Philosophical Studies*

   "My tentative conclusion is that formal criteria are of limited non-mathematical interest." (p 4120)

   "... my arguments will show that formal criteria fail even to secure metaphysically necessary equivalence." (p 4121)

   "Formal criteria of equivalence between two representational vehicles A and B cannot tell us something about the semantic properties of A and B simpliciter, absent information about how A and B are being used to represent the world." (p 4125)

   "These critiques rightly conclude that no purely formal relation can illuminate semantic equivalence absolutely; rather, a relation can do so only if it is sensitive to the interpretation or semantic content being associated with the representational vehicles at issue." (p 4127)

   (d) Jeremy Butterfield (2021) *Philosophy Beyond Spacetime*

   "... logical equivalence is too weak an explication of theoretical equivalence — as is, therefore, any of the recently proposed weakenings of logical equivalence." (p 43)

   "... formal mathematical methods can only 'discern structure', and so cannot 'cut finer' than isomorphism, i.e. cannot distinguish isomorphic copies. The classification of an Example as [equivalent theories] therefore goes beyond formal matters such as isomorphism, and depends on interpretation." (p 43)

"... a verdict that two theories are equivalent depends on interpretation." (p 44)

"... the logical framework deliberately sets aside intended meanings." (p 70)

"... we have a striking illustration of how interpretation moulds verdicts of theoretical equivalence." (p 71)

"And the Implication still holds for the now-familiar reason: the definitions of theoretical equivalence ... are formal — they set aside intended meanings." (p 72)

"Its verdict 'these theories are equivalent' misses the fact that the theories, as interpreted, disagree with each other." (p 72)

2. Example: When is a theory **deterministic**?

   Definition (Halvorson & Manchak)**.** Let T be a dynamical theory. We say that T is *deterministic* just in case for any two models M and N of T, and any two isomorphisms $f, g : M \to N$, if $f_i = g_i$ for all i in some initial segment U, then $f = g$.

   (a) Menon and Read (2023)

   "... determinism is at base a *metaphysical* issue" (p 14)

   "... whatever the alternative characterisation [of determinism] at which one arrives, it is crucial that ... it make explicit the importance of both formal as well as representational commitments." (p 14)

   "Here, one might charge Halvorson and Manchak with not engaging sufficiently with the *metaphysical* issue of determinism ... by defining the notion in terms of the mathematical property of `Dynamical rigidity`." (p 15)

3. Historical motivation for formal analysis

   (a) Nineteenth century mathematics

      i. Ex: continuous, differentiable, equinumerous

      ii. Where intuition fails

         A. An everywhere continuous, nowhere differentiable function

         B. Even numbers

   (b) Frege

   "Es ist das Psychologische von dem Logischen, das Subjective von dem Objectiven scharf zu trennen." (p 9)

   (c) Carnap, *Aufbau*

   "... the most fundamental aim of the *Aufbau* [is] the articulation and defense of a radically new conception of objectivity" (Friedman 1987)

   "For science wants to speak about what is objective, and whatever does not belong to structure but to the material ... is, in the final analysis, subjective." (Aufbau §16)

"Carnap argues that only the logical form or structure of a relation is objectively or scientifically communicable" (Friedman 1987)

"The primary problem is to account for the objectivity of scientific knowledge, and the method of solution is based on a form/content distinction." (Friedman 1987)

"We are motivated to pursue a problem of complete formalization by a conception of scientific objectivity that seeks to disengage objective meaning entirely from ostentation." (Friedman 1987)

"...there is absolutely no question remaining concerning 'content' or 'interpretation'." (Friedman 1987)

(d) Carnap, *Logische Syntax*

52a. Jeder Vorgang ist durch seine Ursachen eindeutig bestimmt.

52b. Zu jedem singulären physikalischen Satz $\mathfrak{S}_1$ gibt es für eine beliebige Zeitkoordinate $\mathfrak{A}_1$, die einen kleineren Wert hat als die in $\mathfrak{S}_1$ vorkommende Zeitkoordinate, eine Klasse $\mathfrak{K}_1$ singulärer Sätze mit $\mathfrak{A}_1$ als Zeitkoordinate, derart, daß $\mathfrak{S}_1$ eine P-Folge von $\mathfrak{K}_1$ ist.

53a. Ort und Geschwindigkeit einer Partikel ist durch eine frühere Partikelkonstellation nicht eindeutig bestimmt, sondern nur mit Wahrscheinlichkeit.

53b. Ist $\mathfrak{S}_1$ ein singulärer Partikelsatz und $\mathfrak{A}_1$ eine Zeitkoordinate von kleinerem Wert als die in $\mathfrak{S}_1$ vorkommende, so ist $\mathfrak{S}_1$ nicht P-Folge einer noch so umfassenden Klasse solcher Sätze mit $\mathfrak{A}_1$ als Zeitkoordinate, sondern nur Wahrscheinlichkeitsfolge einer solchen Klasse mit einem Wahrscheinlichkeitskoeffizienten kleiner als 1.

4. Good arguments or changing fashions?

   (a) What Quine's critique of the analytic-synthetic distinction show about formal explication (rational reconstruction)?

   (b) Or is there a different diagnosis of "the misconception at the root of logical syntax"?

5. Syntactic properties as objective properties

6. Two views of analysis/explication

   The paradox of analysis

   (a) Discovery

   (b) Creation

      i. Carnap: elimination of ambiguities

      ii. More or less natural explications
      Ex. translating quantifier phrases

7. Dangers of the material mode: formal semantics

   (a) Metaphysicians' proposal: Theories T_1 and T_2 are *equivalent* just in case they are true in the same possible worlds

      i. If "the class of possible worlds" is explicated in a formal sense, then then the proposal will be written off as purely formal

      ii. If "the class of possible possible worlds" is taken in an unrestricted metaphysical sense, then the proposal provides no practical guidance. Individual researchers will have different intuitions about whether two theories are true in the same possible worlds.
      Ex: Are Lagrangian and Hamiltonian mechanics true in the same possible worlds?
      Ex: Are Newtonian and Cartan gravity true in the same possible worlds?

   (b) Metaphysicians' proposal: A theory T is *fully deterministic* just in case for any two worlds W_1 and W_2 that are possible according to T, if W_1 and W_2 agree on all intrinsic properties (including haecceitistic properties) up to a certain time, then W_1 and W_2 agree on intrinsic properties at all times.

   (c) A warning from Frederic Fitch

   "Semantics, in spite of its far-reaching usefulness for analysis, no more enables one to step outside the magic circle of linguistic forms than does syntax."

   "Semantics may treat adequately enough of the relation of a syntax language to a subject-matter language, employing, in so doing, an 'extended syntax language.' It may even satisfactorily treat of whole hierarchies of languages, where each language sufficiently high in the hierarchy is an extended syntax language for those below it; but semantics still will not present a completely adequate framework for an empirical science, much less for a synthesis of several sciences or for a metaphysical (or 'anti-metaphysical') world-view."

   "However far we analyze existing languages or construct new languages and 'logics,' there still always remains the need to connect these symbol structures with

parts of nature which they are intended to represent or explain. Such connections can not themselves be merely linguistic. At some point they must refer to entities which are not parts of any language. At some point nonlinguistic concepts or entities are relevant." (Fitch 1938)

(d) Fruitfulness

8. What formal analysis can and cannot do

   (a) Shiftability of the form-content divide (or "the receding horizon of content")

      i. Ex: According to propositional logic, "some ravens are black" is equivalent to "all ravens are black"

      ii. Ex: What is $\mathbb{R}$?

      iii. Ex: What are propositional theories?

   (b) Revisiting Sklar's lions and tigers

   A: "Why do you consider these two theories to be inequivalent?"

   B: "Because lions and tigers are different."

   A: "How are they different?"

   B: "Male lions have manes."

   Diagnosis: Whether T_1 and T_2 are inequivalent depends, of course, on the background context. In our background context, the word "lion" plays a different role than the word "tiger", and that is why we consider T_1 and T_2 to be inequivalent.[1]

   But wait, am I not making the mistake of semantic holism?

   I'm merely pointing out that an adequate formal representation of a sentence ϕ can depend on some of our other background beliefs. E.g. "For every natural number, there is a number greater than it" presupposes that the relation "greater than" is a partial order.

   (c) There is never a single "correct" explication, and there is usually more than one acceptable explication (as we emphasize different aspects of our usage/intuitions, or different goals of inquiry)

   (d) From implicit to explicit

   (e) Handholds on the slippery slope

   (f) Formal analysis and couples therapy

[1] Claim: There is a set of sentences K such that $K \cup T_1$ is inequivalent to $K \cup T_2$, even though T_1 and T_2 are equivalent. Proof: Let $K = \{\neg\exists x\, \mathsf{lion}(x)\}$.