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A GENERAL LOGIC

John Slaney

This is an essay in logic in the traditional sense: the formal theory of inference. In it I attempt a unified account of a fairly wide range of logical systems, some very well known and others less so. These systems include classical logic, relevant logics such as Anderson and Belnap's R, close relatives of fuzzy logic, some modal logics and many weaker, but still interesting, nonstandard systems. These diverse logics, quite different in their philosophical underpinnings, are displayed as variations on a single and simple theme. Although its concern is with formal logic this paper is designed to be accessible to non-specialists. It only assumes familiarity with a natural-deduction formulation of elementary logic such as that of Lemmon's [6]. Nonclassical logics much as presented here have been introduced successfully to students with no formal logical training beyond a standard one-semester first course.

Little of the formal material of this paper is really new. In particular, following the work of Dunn, Meyer, Sylvan etc, relevant logicians have been using the most important technical device of my presentation at least since 1973 (see J. M. Dunn's ' "Gentzen system" for Positive Relevant Implication' in [1]). Recent work by logical intuitionists on realizability semantics for their theories is also very close both formally and motivationally to my account. What is slightly more original to this paper is (I hope) a maximally smooth version of relevant logics and many of their rivals intended partly to dispose of the complaint that such systems are difficult, highly contrived and esoteric. I have also tried to emphasise the unified theory of so wide a spread of logics, to provide a framework within which they can be compared and contrasted with each other and which renders their extensive common content clearly visible. It has, at any rate, long seemed desirable that these scattered facts, in some form familiar to many of us, should somehow be gathered together in one place.

PART ONE: A MODEST PROPOSAL

Bodies of information may be used in more than one way. The difference between two of these ways in particular seems to me to be of great importance to logical theory of the standard sort and to have been neglected by a majority of logical theorists of the standard sort. There is intuitively a difference between using information to tell you how the world stands and using it

to tell you how you may reason. That is, you may treat it as a guide to the facts or as a source of inference tickets. A good analogy comes from computation theory: as von Neumann pointed out, one and the same series of symbols can function as program or as data to suit the occasion; as program it operates on input much as a proof operates on its premises, while as data itself it may be the input to or output from other operations. The distinction being sketched is not quite the traditional one between 'laws' and 'facts', for the difference resides neither in the propositions themselves nor in the information they convey but in the use to which they are put in one context or another. I shall not, for now, attempt to make the distinction any more precise, but proceed without further analysis to ask what effect it might have on the formalisation of logic.

Let us adopt some notation. Where X and Y are things of the right sort to warrant assertions, things I shall call *bodies of information,* the object

$$X\,(Y)$$

shall be the result of taking X as the determinant of available inference and applying it to Y. The main presumption of this paper is that some such notion makes sense as one of the ways in which premise-like things can be put together for the sake of argument. Another way of combining bodies of information is to *pool* them. Pooling, we may suppose, is very much like forming set unions, so let us write

$$X \cup Y$$

for the result of pooling information X with information Y. While we are setting up notational conventions, let it be stipulated that

$$X \models S$$

means that information X warrants or renders correct the statement S. Perhaps application and pooling will turn out coincident in the sense that

$$X\,(Y) \models S \Leftrightarrow X \cup Y \models S$$

and then again perhaps not. If you think that truth-bearers are not statements but sentences, propositions, assertions or what-not, please feel free to interpret my utterances with a pinch of charity. On a related point, note that the concept of a body of information is being left similarly rough.

It could be a set of statements, some of a person's beliefs, a theory, a FORTRAN program, a file of questionnaire returns or whatever. Formal logic is robust enough to work in pretty much the same way independently of such ontological issues. I am, though, allowing the concept of information to include *mis*information, so what is warranted by some body of information need not thereby be true.

To get closer to logic proper, consider two inferential principles:

$$\frac{A \qquad B}{A \,\&\, B}\ (\text{ADJ}) \qquad\qquad \frac{A \to B \qquad A}{B}\ (\text{DET})$$

On an intuitive level there is a difference between the ways in which the pairs of premises come together in the two cases. For adjunction both premises are *used* to get the conclusion—well, it doesn't follow from just one of them—but they simply occur side by side, whereas in the case of detachment they interact to produce *B*. Part of the explanation is that DET is an elimination rule while ADJ is an introduction rule, but I believe the difference goes deeper. There are two stories to be told as to why the two premises imply the conclusion. On the first, the inference is valid because the two premises cannot both be true unless the conclusion is. On the second, one premise gives a licence to infer the conclusion from the other. Each story *can* be told with some degree of plausibility about each of the above inferences, but with a difference in conceptual priority. Why is it that given *A*, *B* is all you need for *A & B*? Because if *A* is true, and *B* is also true, then *A&B* just has to be true. Why is the joint truth of $A \to B$ and *A* sufficient for the truth of *B*? Because the conditional records the availability of an inference from its antecedent to its consequent. To anticipate a little, the fundamental principle underlying conjunction is

$$X \models A\&B \Leftrightarrow \exists Y \exists Z_{(Y \cup Z = X)} (Y \models A \wedge Z \models B)$$

while the basis for the conditional is

$$X \models A \to B \Leftrightarrow \forall Y_{(Y \models A)}\ X(Y) \models B$$

Thus, roughly speaking, conjunction is associated with 'pooling' of information and conditionals rather with 'application'.

It seems likely that there is more than one use of conditionals in English and like languages. While personally rather drawn to unified theories—witness the present enterprise—I see little reason to think that the full range of constructions using 'if', from conditional probabilities to conditional instructions, from 'even if' to 'if only', will be brought fruitfully under a common description. Nonetheless, I think that what logicians have generally construed as conditionality does correspond to an important way in which 'if' can be used and to one whose logic is a fit and proper subject of inquiry.[1] The truth about it is this: to assert that if *P* then *Q* is to claim to be in a position to assert *Q* given *P*. One is warranted in asserting a conditional just when one has enough extra information to deduce that *Q* from the assumption that *P*. 'Deduce' here means *logically* deduce, so the conditions of correct use of conditionals are tied to those of correct inference.

[1] Richard Sylvan reminds me that this notion is precisely *implication* as distinct both from entailment and from more context-bound sorts of conditionality.

Conditionals may of course be *contingently* true, if the extra information is contingent. This applies to lawlike conditionals such as

If Timmy is a tiger then Timmy is a carnivore

where the inference is warranted by biological theory, not by Pure Reason, as well as to 'one-off' conditionals like

If white exchanges knights on d5 he will lose a pawn

or (as a prediction)

If you say 'Oh reely' once more I shall wring your neck

where the inferences are warranted by facts about the state of a game or about my state of mind.

These remarks on conditionals are not particularly deviant. They are what we usually say to motivate the 'deduction equivalence':

$$\Sigma, A \vdash B \Leftrightarrow \Sigma \vdash A \to B$$

The presentation is based on that offered by intuitionists, though it makes excellent sense from other Logical Points of View including the classical one. What makes the logic of this paper *general* is that I have not granted the expression

$$\Sigma, A$$

the reading usually stipulated for it, namely the set

$$\Sigma \ \cup \{\mathbf{A}\}$$

If the deduction equivalence is to hold then to take premise combination invariably to be set union is to write very strong claims into the foundations of logic. For one thing, set union is commutative, and by the deduction equivalence this yields a permutation rule for conditionals:

$$A \vdash B \to C \Leftrightarrow B \vdash A \to C$$

But many desirable conditionals do not satisfy permutation. Modal ones do not, for example. It may well be, in the end, that we shall want to formulate logic in the usual way with a basic conditional devoid of any modal or quasi-modal force and with stricter connectives defined in terms of it, but that we should is far too strong a claim to be written into the primal story as to what is an A-grade conditional at all. For it is not *obvious* that our fundamental conditional should carry no counterfactual load. Other combinatory properties of set union also result in strong logical claims. For example, idempotence gives us contraction

$$A \vdash A \rightarrow B \Rightarrow A \vdash B$$

—roughly that to use a premise twice you need only assume it once—which is deeply implicated in all sorts of paradoxical reasonings, from the sorites to Epimenides and from Russell's antinomy to Cantor's theorem.

By way of amplification, let us briefly pursue the point that arose a moment ago with respect to modal logic. Suppose that the conditional arrow were to emerge as the strict implication of a normal modal logic while satisfying the deduction equivalence as suggested above. What would 'application' come to in such a case? We begin an answer by introducing a new connective $\dot{\Diamond}$. Recall that the usual semantic story about possibility is that for any world w

$$w \models \Diamond A \Leftrightarrow \exists x_{(Rwx)}\ x \models A$$

Now the new connective is given the reading

$$w \models \dot{\Diamond} A \Leftrightarrow \exists x_{(Rxw)}\ x \models A$$

There are such connectives in natural modal-like contexts. In tense logic, for instance, if $\Diamond A$ means that A will sometime be the case then $\dot{\Diamond} A$ means that A was sometime the case and *vice versa*. Now we use $\dot{\Diamond}$ to define a two-place connective $\odot$:

$$A \odot B = df\ \dot{\Diamond} A \& B$$

Note that for any formulae A and B we have

$$A \vdash \Box B \Leftrightarrow \dot{\Diamond} A \vdash B$$

The two halves of this assertion are equally easy to prove:

> Suppose that A entails $\Box B$ and that $\dot{\Diamond} A$ holds at a world w. Then there is some x such that Rxw where A holds. By the hypothesis then $\Box B$ holds at x, so B holds at w.

> Suppose that $\dot{\Diamond} A$ entails B and that A holds at a world w. Then for any x such that Rwx, $\dot{\Diamond} A$ holds at x. By the hypothesis then B holds at any such x, so $\Box B$ holds at w.

Now as a special case

$$A \vdash B \rightarrow C \Leftrightarrow \dot{\Diamond} A \vdash B \supset C$$

so by the ordinary classical deduction equivalence for the material hook

$$A \vdash B \rightarrow C \Leftrightarrow A \odot B \vdash C$$

Hence the operation symbolised by the premise-combining comma should be fixed with properties corresponding to $\odot$ as those of set union correspond to &. Thus it will not be commutative, since $A \odot B$ and $B \odot A$ are not generally equivalent, but it might or might not be associative, for instance, depending on the strength of the modal logic countenanced. A suitable candidate reading for *X(Y)* might be the *list* whose head is Y and whose body is X. The account of $\odot$ gives the conditions under which a list is available in a world.

If with C.I. Lewis we see the strict arrow of modal logic as *expressing* rather than merely as *indicating* entailment then the deduction equivalence becomes

> A body of information X warrants the assertion of ≪P entails Q≫ iff any body of information got by assuming P in the context of X warrants the assertion of Q.

But of course *assuming* P in the context of X is not just simple-mindedly *adding* P speculatively to X. It also involves *subtracting* certain things from the given. Typically it involves subtracting not-P. Indeed if the modal account is correct it involves subtracting enough to convert X as it were into $\dot{\Diamond} X$, a much weaker assertion.

The point of this digression into modal logic was to indicate a sense of 'taken together' or 'assumed in the context of' different from set union, allowing some connectives other than truth functional ones to emerge as conditionals of the best sort. The moral is that for the purposes of a decent theory of conditionality the operation of combining premises might well be divested of some of its usual properties such as associativity or commutativity. For adjunction and the like, however, all of those normal properties are wanted, to secure the semilattice character of conjunction. What forces itself upon us, therefore, is the option of formulating logic with *two* operations for putting premises together: an extensional one to go with extensional connectives like 'and' and an intensional one for *prima facie* intensional ones like 'if'. The opening reflections suggest that such an intuitive distinction is to hand. The present Modest Proposal is that it be incorporated into our formal logic.

PART TWO: HONEST TOIL

So much for preamble. Preambulation has its place, but there comes a time to stop ambling and start delivering. The bones of an account having been loosely assembled, the next task is to clothe this speculative skeleton in formal flesh. I have chosen to base the system on Lemmon's presentation of elementary logic partly because his logical rules are capable of extremely natural motivation, partly because the changes to be made can be simple and visible since the system wears its sequent calculus reading on its face, but mostly because *Beginning Logic* is the brand leader among first logic texts and a prime desideratum is familiarity. The difference between Lemmon's account and mine is that I allow assumptions to be combined in two ways symbolised by commas and semicolons. This idea is not at all original, being similar to that used in Giambrone's sophisticated cut-free formulations of weak logics,[2] in Meyer's earlier investigation of relevant arithmetic[3] and as long ago as 1969 by Dunn in his work on relevant implication. To the charge of unoriginality I naturally plead Pascal's dictum that to have arranged the same words differently is not always to have said the same thing.[4]

The formal language is very much as usual, with the basic connectives

$$- \quad \& \quad \vee \quad \rightarrow$$

I follow Lemmon in using *arbitrary names* instead of free variables, avoiding such things as vacuous quantifiers, again for familiarity. The notation

$$A^{\alpha}_{\beta}$$

is used to represent the result of substituting α for β throughout formula A. Assumption numbers are bunched in two ways. Where X and Y are two such bunches we write

$$X,Y$$

for the result of pooling the assumptions they represent, and

$$X;Y$$

for the result of applying X to Y. Since these two compounding operations may be nested to any finite depth, parentheses will be used to disambiguate as usual. The assumption of association to be left removes the need for most such parentheses. In writing out the rules I shall use the notation

$$\Gamma(\varphi)$$

to represent a bunch in which φ occurs in some particular place as a sub-branch. The expression

$$\varphi \Leftarrow \psi$$

[2] See [5].

[3] See especially [9] which dates from the early 1970s. Mention should also be made of the seminal [10], which is directly ancestral to the present paper.

[4] [12], 944 (Lafuma's numbering).

is used to mean that a bunch of the form φ may be replaced anywhere in the left side of a sequent by the corresponding ψ. That is, it abbreviates

$$\frac{\Gamma(\varphi) : A}{\Gamma(\psi) : A}$$

The basic structural rules governing the comma are left tacit by Lemmon, but here we are paying great attention to such things, so we specify:

SR1 $\quad X, (Y, Z) \Leftarrow X, Y, Z$

SR2 $\quad Y, X \Leftarrow X, Y$

SR3 $\quad X, X \Leftarrow X$

SR4 $\quad X \Leftarrow X, Y$

Actually SR4 is not a primitive rule in Lemmon's account, though its effect (for finite sequents) is secured by means of non-normalisable proofs. It seems better to come clean about it. It is also useful to admit zero as a special assumption number to symbolise not the empty set but 'logic' or the set of logical truths. Now what follows from X when we apply Logic to it is just what follows (logically) from X. That is, we are justified in adding

SR5 $\quad 0; X \Leftrightarrow X$

Notice that SR5 is two-way. The converses of SR1–SR3 are obviously also valid. In writing proofs we may usually elide appeals to these basic rules with uses of the logical rules as Lemmon does. This removes a good deal of tedious business.

The rule of assumptions is exactly as in Lemmon:

$$\frac{}{A : A}\ \textbf{(A)}$$

That is, any formula may be introduced on any line of any proof, resting only upon itself as assumption. The rest of the rules are the logical rules for introduction and elimination of connectives. These are just Lemmon's,[5] with due allowance made for the difference between commas and semicolons. Conjunction goes with pooling of assumptions, so:

$$\frac{A \,\&\, B}{A}\ (\&\,\mathbf{E}) \qquad\qquad \frac{A \,\&\, B}{B}\ (\&\,\mathbf{E})$$

$$\frac{X : A \qquad Y : B}{X, Y : A \,\&\, B}\ (\&\,\mathbf{I})$$

[5] The exception is his RAA which is derivable from the rest anyway in the classical context but which does not fit the present general scheme so well as DN and MTT.

The conditional, on the other hand, goes naturally with application:

$$\frac{X : A \rightarrow B \qquad Y : A}{X;\, Y : B}\ \textbf{(MPP)}$$

$$\frac{X;\, A\ :\ B}{X\ :\ A \rightarrow B}\ \textbf{(CP)}$$

These are the most important cases in which the distinction between extensional and intensional combination matters. *Modus Tollens* being just *Modus Ponens* in reverse, negation also goes with intensional combination:

$$\frac{X : A \rightarrow B \qquad Y : -B}{X;\, Y :\ -A}\ \textbf{(MTT)}$$

$$\frac{A}{--A}\ \textbf{(DNI)} \qquad\qquad \frac{--A}{A}\ \textbf{(DNE)}$$

The rules for disjunction are as expected:

$$\frac{A}{A \vee B}\ (\vee\textbf{I}) \qquad\qquad \frac{A}{A \vee B}\ (\vee\textbf{I})$$

$$\frac{X\ :\ A \vee B \qquad \Gamma(A)\ :\ C \qquad \Gamma(B)\ :\ C}{\Gamma(X)\ :\ C}\ (\vee\textbf{E})$$

Quantifiers are treated exactly as in Lemmon, except that we need to take account of arbitrary contexts in the statement of EE:

$$\frac{X\ :\ A}{X\ :\ (v)A^v_n}\ \textbf{(UI)}$$ where n is an arbitrary name which occurs in A but which does not occur in any formula in X

$$\frac{(v)A}{A^t_v}\ \textbf{(UE)}$$

$$\frac{A^t_v}{(\exists v)A}\ (EI)$$

$$\frac{X\ :\ (\exists v)A^v_n \qquad \Gamma(A)\ :\ B}{\Gamma\,(X)\ :\ B}\ \textbf{(EE)}$$ where n is an arbitrary name in A but not in B or elsewhere in $\Gamma(\ )$

The easiest way to explain the operation of the new rules is to lay out a couple of sample proofs. These are in Lemmon's linear format, lifted directly

from *Beginning Logic*, whereby a proof (officially a finite tree of sequents appropriately connected up by the rules of inference) is written as a list of lines of proof, numbered sequentially, each of which records a sequent. To the left of the line number are the 'assumption numbers' which represent the premises of the sequent, while to the right the conclusion of the sequent is written out. Some annotation is added on the extreme right recording the rules used and their input line numbers.

$$P,\ \ Q \vee R \vdash (P \,\&\, Q) \vee R$$

1	(1) P	A
2	(2) $Q \vee R$	A
3	(3) Q	A
1, 3	(4) $P \,\&\, Q$	1, 3 & I
1, 3	(5) $(P \,\&\, Q) \vee R$	4, ∨I
6	(6) R	A
1, 6	(7) R	6, SR4
1, 6	(8) $(P \,\&\, Q) \vee R$	7, ∨I
1, 2	(9) $(P \,\&\, Q) \vee R$	2, 3, 5, 6, 8, ∨E

$$P \rightarrow Q,\ P \rightarrow R \vdash P \rightarrow (Q \,\&\, R)$$

1	(1) $P \rightarrow Q$	A
2	(2) $P \rightarrow R$	A
3	(3) P	A
1, 2	(4) $P \rightarrow Q$	1, SR4
1, 2	(5) $P \rightarrow R$	2, SR4, SR2
1, 2; 3	(6) Q	3, 4, MPP
1, 2; 3	(7) R	3, 5, MPP
1, 2; 3	(8) $Q \,\&\, R$	6, 7, &I, SR3
1, 2	(9) $P \rightarrow (Q \,\&\, R)$	3, 8, CP

$$\vdash (-P \rightarrow Q) \rightarrow (-Q \rightarrow P)$$

1	(1) $-P \rightarrow Q$	A
2	(2) $-Q$	A
1; 2	(3) $--P$	1, 2, MTT
1; 2	(4) P	3, DNE
1	(5) $-Q \rightarrow P$	2, 4, CP
0; 1	(6) $-Q \rightarrow P$	5, SR5
0	(7) $(-P \rightarrow Q) \rightarrow (-Q \rightarrow P)$	1, 6, CP

Several points are worth noting. Firstly, in order to apply ∨E in the first proof we need the two disjuncts to be embedded in the same context Γ, which necessitates the SR4 move at line 7. The SR4 and SR2 moves in the second proof are also to unify the left sides of lines 4 and 5 so that

SR3 can apply after the &I leaves the bunch (1, 2; 3), (1, 2; 3). Secondly, CP can only apply where the discharged antecedent is the right half of an intensional bunch, the left half of which may not be empty though it may be '0'. Thirdly, where this system is used for more elaborate proofs much of the surface fussiness will be suppressed by omitting mention of the basic structural rules. Lemmon's meta-rule of Sequent Introduction (a form of 'Cut') can also be used to the usual effect, allowing sequents already proved to be used as macros for the more advanced cases. Extension to first order theories, adding function symbols, function and relation constants (e.g. '+', '=', 'ε'), etc. is straightforward.

The logic defined above is known (for uninteresting historical reasons) as **DW**. Although not free of contentious principles, of which the most obvious surround the intuitionistically invalid rule DNE, **DW** is a weak system in the sense that almost every logic ever seriously proposed validates some inferences not derivable in it. Part of what makes it interesting is the possibility of strengthening it by adding structural rules governing the semicolon while keeping the logical rules fixed. The logics thus obtainable can be seen as giving the same readings to the logical constants while disagreeing as to what entails what because of differences over how inference in the abstract is structured.

One of the stronger systems obtainable in this way is the logic **C**. This is one of the 'relevant' logics and will also be found in the literature under the name of 'RW' or 'R-W'. To convert **DW** into **C**, just add as new structural rules the intensional analogues of SR1 and SR2:

SR6 $X; (Y; Z) \Leftarrow X; Y; Z$

SR7 $Y; X \Leftarrow X; Y$

What these come to is the stipulation that for purposes of 'application' we may abstract from order and association among the bodies of information. All that is relevant to the C-type reasoner is *which* bodies of information are given and *how often* each of them occurs. In other words, just as classically we give application the properties of set union, so in **C** we give it those of *multiset* union. Is **C** right where it differs from **DW**? Is it right where it differs from the classical logic we all know and love? It is not the purpose of this paper to speculate. Still, the **C** perspective on reasoning is an interesting one, and one which looks set to repay a little formal investigation.[6]

C blurs many of the distinctions which **DW** draws. This is in keeping with the pragmatic logical maxim, learned from Quine, to avoid distinctions unless they make a needed difference. One distinction which *is* drawn sharply in **C** but which many have felt makes no worthwhile difference is that between two assumptions of a premise and just one. We might at least want to allow re-cycling of used premises at no extra cost. That is, we might consider adding the intensional analogue of SR3, the rule of 'Contraction':

[6] Such formal investigation is begun in the unpublished doctoral dissertations of the author and of Giambrone. [5] is the best published introduction.

SR8 $X; X \Leftarrow X$

The result of so doing is the relevant logic **R** of Anderson and Belnap. About **R** a good deal has been written.[7] Little will be added here, except to note the effect of the Modest Proposal on the motivation for **R** and its relevant kin. I have always found it particularly easy to show Beginning Logic students why relevant logic is appealing by laying out the simplest Lemmon-style proofs of relevantly unprovable sequents. Consider this specimen, for example.

$P, -P \vdash -Q$

1	(1) P	A
2	(2) $-P$	A
3	(3) Q	A
1, 2	(4) $P \& -P$	1, 2, &I
1, 2, 3	(5) $P \& -P$	4, SR4
1 ,2	(6) $-Q$	3, 5, RAA

In Lemmon's official version of the logic there is no SR4, so its effect must be secured in some such way as this:

1, 2, 3	(4.5) $(P \& -P) \& Q$	3, 4, &I
1, 2, 3	(5) $P \& -P$	4, 5, &E

This is a legitimate proof within the Meaning of the Act, provided the Act is classical. Incidentally, insofar as Lemmon's rules are intuitively well-motivated it is also an *independent* proof in the sense of Lewis.[8] Uncontaminated Logic 1 students, however, have no difficulty in seeing that proofs like this involve, in Fogelin's immortal phrase, 'funny business'. The SR4 manoeuver, or its abnormal &I–&E analogue, is performed solely in order to blame the contradiction on an innocent bystander Q. The Modest Proposal in this context amounts to the claim that it is not beyond the wit of man to discern a sense in which that sort of dodge is no fair. While Q is *used* in the derivation of line 5, it is not involved intimately enough to warrant the claim that the conclusion came *from* it as RAA and CP require. Funny business lies not in using any of the logical rules, nor in stringing them together into proofs, but in deliberately conflating the way in which SR4 and &I join assumptions with the way in which CP and RAA need them to be joined. The relevant logician as I present him Modestly Proposes that we abjure it.

The intensional counterpart of SR4 is obviously

SR9 $X \Leftarrow X; Y$

[7] See Dunn's essay in [4] for a good entry to the literature. [1] is the classic reference, though it is centred not on **R** but on the weaker and less important system **E**.

[8] [7], pp. 250–251.

The result of adding this to **R** would be to give the two modes of premise-combination the same combinatory properties, thus collapsing them into each other and producing classical logic. Interestingly, though, SR9 can be added to **C** without this collapse. **C** plus SR9 I shall dub **CK**.[9] **CK** is quite a strong system, in many ways close to classical logic but lacking contraction. It is thus a good tool for investigating the effects of doing without SR8 and cognate rules. Some of these effects are very interesting indeed.

In the first place, it is well known that contraction is the key move in making trouble from the logical and semantic antinomies. This is most clearly seen in the case of Curry's paradox arising from the naïve set N_Q:

$$N_Q = \{x : x \quad \varepsilon \ x \ \rightarrow Q\}$$

Plainly there is, according to the naïve comprehension axiom, such a set for every Q. Now N_Q is a badly-behaved set and gives rise to a badly-behaved proposition, the proposition that $N_Q \ \varepsilon \ N_Q$, which is a P such that

$$P \leftrightarrow (P \rightarrow Q)$$

whence

$$P \rightarrow (P \rightarrow Q)$$

so by the deduction equivalence and SR8

$$P \rightarrow Q$$

But from this and the biconditional

$$P$$

and hence by MPP

$$Q$$

This reasoning can be repeated for any chosen Q, rendering the naïve theories trivial provided they are closed under detachment and contraction. It is known that a truly naïve set theory based on **DW** is simply consistent—and indeed that this result extends to systems considerably stronger than **DW**.[10] It is also known that naïve comprehension, though without extensionality, remains consistent in the context of the Łukasiewicz infinite-valued logics which are properly stronger than **CK**.[11] The consistency problem for full naïve set theory in **C** and in **CK** remains open. It is also not known whether the naïve theories based on those logics are sufficiently strong to serve foundational purposes. At any rate, the Modest Proposal as solution to the antinomies locates the error in arguing for instance from

[9] **CK** is slightly stronger than the closely related system often called **BCK**. For an account of work on **BCK**, see [11].

[10] See [2] for the proof.

[11] See [14].

$$P \rightarrow (P \rightarrow Q)$$

to

$$P \rightarrow Q$$

—i.e. in confusing the harmless *P,P* (which amounts merely to *P*) with the much stronger *P;P*.

In the second place, **CK** offers a potential solution to the sorites paradox. This is a tangled problem and the issues are too complex for the confines of the present paper, but the main deduction-theoretic point can be put very simply. The sorites argument can be made to work by *Modus Ponens* applied many times to an initital claim (that fresh blood is red, that a zillion grains of sand can be formed into a heap or whatever) and a lot of conditionals saying that small changes are not significant (if a thousand grains then 999; if 999 then 998 . . .). In the clearest version these conditionals all result by UE from a generalisation *G*:

$$(x)\,(Hx \rightarrow H\,(x-1))$$

which is intuitively true, or at least well within the range of what is ordinarily assertible. If we try to formulate the sorites argument in **CK**, we find that every use of MPP involves us in another appeal to *G*, so that by the end we have deduced the unacceptable conclusion not from *G simpliciter* but from *G* taken together with itself many times. Lacking contraction, we cannot collapse these many assumptions into one. The Modest Proposal as solution to the sorites takes the confusion to be between the real MPP, based on the correct principle

$$X \models A \rightarrow B \Leftrightarrow \forall Y_{(Y \models A)} \;\; X(Y) \models B$$

and an invalid 'pseudo *Modus Ponens*' based on the incorrect

$$X \models A \rightarrow B \Leftrightarrow \forall Y_{(Y \models A)} \; X \cup Y \models B$$

Thus it meets Dummett's challenge[12] by keeping MPP as constitutive of the meaning of 'if' while blocking the sorites argument by weakening logic.

The systems briefly exposed above are but a few of the many reachable by changing the structural postulates governing the semicolon. For a more extensive list see [13]. What is most striking from the present perspective is how much there is in common, and at how deep a level, between logics whose original motivations and philosophical underpinnings are so disparate as to suggest that any formal similarities would be superficial. I have restricted attention to systems with the same logical rules, but of course we are not constrained to do so. Some views do require changes at that level. For instance, the dispute between classical and intuitionist logicians, or *realists* and

[12] M. Dummett, [3], p. 252.

antirealists as they are often styled nowadays, issues in a disagreement about the logic of negation. The negation rules used in this paper are the classical ones; a different range of logics results from the adoption of an intuitionist-style negation, the Modest Proposal being independent of the realist-antirealist debate. Certain further systems in the realist tradition, such as modal systems in the vicinity of **S4**, the Anderson-Belnap systems **E** and **T**, etc. also require minor changes to the logic of negation. However, there is a time for Honest Toil and a time for Honest Rest. Logic having been generalised enough for one paper, the moment seems ripe for the latter.[13]

The Australian National University

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[13] This paper has been a long time in the making. An earlier version was written while the author was a visiting professor at Dartmouth College, NH, USA in 1987. Thanks are due to Dartmouth College for providing the opportunity to get it down on paper. A still earlier version was produced for the Edinburgh-St. Andrews joint logic seminar in 1984, revised in 1985 and 1987. A more technical exposition was delivered to the ASL European Summer Meeting in 1986.

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CHALMERS ON UNREPRESENTATIVE REALISM AND OBJECTIVISM

K. H. Sievers

Alan Chalmers' book *What is this thing called Science?* is widely read inside and outside the world of academic philosophy. The first part of the book gives a lucid account of current debates in the philosophy of science, and in the second part Chalmers presents his own perspective on science. In spite of its popularity, there seems to be little discussion of the two positions Chalmers outlines in the second part of the book: unrepresentative realism and his version of objectivism.[1] I would like to examine these positions as a part of a coherent account of science and also look at them in a broader philosophical context.

I. Unrepresentative Realism

Let us begin with unrepresentative realism, Chalmers' alternative to the traditional dichotomy between realism and instrumentalism. His main objection to naive instrumentalism is that it forces us to distinguish between observational concepts and non-observational concepts. According to naive instrumentalism, observational concepts get at things that really do exist, while theoretical concepts do not, and it is necessary to use them only because they help us predict what can be observed.[2] Chalmers devotes one whole chapter to arguing that all observation statements are theory-laden, and this 'strikes at the strong distinction that the instrumentalist makes between observational and theoretical entities.'[3] Chalmers' criticism is that if all observations are theory-dependent, then quarks are in the same position as ducks. Either they both really exist or they are both useful fictions.

Chalmers believes that realism is unacceptable for a number of reasons, but the most important seems to be the incommensurability of successive theories. He follows Feyerabend's interpretation of the history of physics. In the light of Einstein, all the consequences of Newton's theory are false if they are interpreted as attempts to describe what the world is really like. From this it follows that one cannot say that Einstein is closer to the truth than Newton, since Newton's physics is simply false.[4]

Chalmers' own position begins with the realist claim that the world exists

[1] A. F. Chalmers, *What is this thing called Science?* 2nd ed., (St. Lucia: University of Queensland Press 1982). This is particularly true of the second edition. In spite of the extensive revision of the last few chapters, the *Philosopher's Index* shows only one review of the revised work.

[2] *Ibid.*, pp. 147-49.

[3] *Ibid.*, p. 148.

[4] *Ibid.*, pp. 156-59.