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Book Review

Interpreting the Quantum World. By Jeffrey Bub. Cambridge University Press, Cambridge, United Kingdom, 1997, xiv+ 298 pp., $49.95 (hardcover).

Jeffrey Bub himself once mentioned that whenever someone arrives at a position in the philosophy of quantum mechanics s/he seems to lose momentum. Here we have a clear counterexample: since his first book on the subject in 1974, which pioneered many of the subsequent development, Bub has gained a great deal of momentum. His position today has the additional virtue of providing sufficient conditions for the tenability of a whole range of alternative interpretations, thus locating quite clearly many of the diverse ideas found in recent literature. Moreover, he can demonstrate optimal desirable properties for his own favorite interpretation within that range.

For physics students and for much of the general public, interpretation of quantum theory begins with certain mystifying pronouncements by the great scientists of Copenhagen. Familiarity and authority have dulled the sense of mystery, and left us with remnants delicately balanced between vacuity and inconsistency. For the specialist, however, the story of interpretation begins a few years after the Copenhagen breakthrough with von Neumann's unification of matrix and wave mechanics in the Hilbert space formalism. Not that this had its meaning written on its face. Von Neumann strove mightily to add some, through his discussion of measurement ("collapse of the wave packet," "projection postulate") and of propositions representable by projection operators. Fundamental to his effort was what is now commonly called the "eigenstate–eigenvalue link." That is the semantic rule which says that an observable pertaining to a given system has a value if and only if the system is in the corresponding eigenstate of that observable. If that is so, then not only do incompatible observables never have simultaneous values—but in fact most of time most observables have no definite value at all. Accordingly, to assure that at least the outcome of a measurement (and the measured observable at that time) have a definite value, that "collapse" postulate had to be added.

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That is the beginning of the story; the latest developments and perhaps the *denouement* (but that is debatable) come to us in Jeffrey Bub's new book *Interpreting the Quantum World*.

To begin Bub shows clearly how the (in)famous measurement problem and its unsolvability derive directly from the eigenstate–eigenvalue link. (Let us abbreviate to "EE-link.") The options are: either to change the theory or to reject that interpretative principle. Such versions as von Neumann's (or the more recent GRW) which postulate collapse, as interruption to the reign of the Schrödinger equation, are changes to quantum theory, in Bub's view. His book is devoted to the second option, originally mooted by Henry Margenau in response to the Einstein–Podolsky–Rosen paradox, but only recently much explored. Bub presents a fundamental result (the Bub–Clifton uniqueness theorem) which selects a precise spectrum of interpretations that escape the unsolvable measurement problem under certain "natural" constraints.

Quick aside: what seem like a few natural constraints to one person may seem intolerable to another. In the last part of this review I'll discuss how much is left out from Bub's range of admissible interpretations.

So what is Bub's approach? Bub follows von Neumann in the representions of such propositions as "Observable m has value k" by the k-eigenspace of m (the set of pure eigenstates of m corresponding to value k, a subspace of the relevant Hilbert space). Undoubtedly this identification was originally linked to the EE-link. But there is separate motivation for this. The following more basic principle was implicit in much of the early discussion:

Identity of Observables: If the probabilities for measurement outcomes for observables m and m' are the same for every QM state of any system to which m and m' both pertain [if m, m' are "statistically equivalent"], then $m = m'$.

Equivalently: If observables m and m' are represented by the same (Hermitian) operator, then $m = m'$.

Given this conviction there is a one-to-one relation between observables and their representing Hermitian operators. Hence it is possible to represent the propositions which assign values to observables by the eigenspaces of the corresponding operators. There are also interpretations which run counter to this principle, but Bub accepts it.

So there we have the propositions whose truth value was settled by the quantum state itself as long as the EE-link was assumed. At this point (after rejecting the EE-link) all we can say is that some of these propositions are true, some are false (=contraries to true propositions), and some are not truth-valued at all (when the observable has no definite value).

Without the EE-link we need to lay down new conditions on which propositions can be true or false when the quantum state is given. What is the set of truth-valued propositions like? Logic requires us to say that if one proposition entails another, then the second is true if the first is. Moreover, if sets of propositions have greatest lower and least upper bounds—as is indeed the case for subspaces—then other familiar bits of logic appear. These correspond to the familiar rules governing "and" and "or." If we assume this (and not all interpretations go along with it), then the truth-valued propositions form a lattice, a sublattice of the lattice of subspaces of Hilbert space. But they cannot form the entire lattice of subspaces—it is not possible to assign truth values to all the propositions, i.e., definite values to all the observables, while respecting logic in this way. That follows from the famous "no-go" theorems about hidden variables.

So, how large can the lattice of truth-valued propositions be? It is certainly possible to assign a value to one observable, hence to assign truth values to all the propositions "R has value k" for a given observable R. These propositions form a Boolean sublattice. Can such a sublattice be expanded; and if so, how far?

Here is Bub's idea. Supposes that one observable R has privileged status, in that the lattice of truth-valued propositions is a function $D(e, R)$ of the quantum state e and the observable R. Presumably at least some of the eigenspaces of R belong to it—or all of them perhaps. What more would we like? Why not impose a few more desiderata, and then see if we can identify what that function must be like?

Crucial to the empirical content of quantum theory are the answers to questions of form: if observable S is measured, what is the probability that value k is found? These answers come from Born's rule for calculating probabilities. But they are notoriously hard to interpret, and we cannot think of them all together as simply constituting a measure of our ignorance. But what if we look only at $D(e, R)$, the truth-valued propositions? Can we construe those questions as just asking "what is the probability that 'S has value k' is true" (when the k-eigenspace of observable S is in $D(e, R)$), and interpret the answer as a mere measure of ignorance? After all, if we fix what the lattice of truth-valued propositions is, we are still ignorant of how truth and falsity are distributed in it. That ignorance can have a measure, and it seems natural to identify it with the Born probability.

How natural? Well, it would certainly follow at once if we assume that measurement of S will reveal the truth if S already has a definite value. That is not so easy an assumption to motivate when the EE-link has been given up. True: a good measurement of S will not change the quantum state if that state was already an eigenstate of S. But in the present context we are not assuming that S had a definite value only if it was in an eigenstate! In the absence of a motivating argument we should expect to see interpretations that do not agree. Nevertheless, it would be a "nice" feature of the interpretation if Born's calculations can be so simply construed.

Now we come to the Bub–Clifton theorem. Its proof is by a beautiful symmetry argument, on the additional premise that $D(e, R)$ is preserved under the automorphisms that preserve e and R. (To put it another way: the identity of $D(e, R)$ is assumed to be a function of e and R alone, and no other factors are relevant to the question of which propositions have a truth value.) The result proved is then that the above desiderata determine $D(e, R)$ uniquely.

So now all we need to ask is: what is $D(e, R)$ like then; i.e., what is the set of propositions with definite truth values like, on the suppositions of this theorem? That is also really quite simple.

Let me first state the answer in precise, technical form, and then give it a more intuitive gloss. $D(e, R)$ is generated from a set of rays (1-dimensional subspaces). First take all the projections of state e onto the eigenspaces of R to which e is not orthogonal. Let's call the set of these $D1$. Next take the set of all rays that are orthogonal to all the members of $D1$. That second set (call it $D2$) is a subspace, namely the orthocomplement of the subspace spanned by $D1$. Obviously it includes all the eigenspaces of R to which e is orthogonal. Now $D(e, R)$ is the lattice generated by $D1$ and $D2$ together (i.e., $D(e, R)$ is the smallest lattice of subspaces that contains both $D1$ and $D2$).

Let us put this in more intuitive dress. Those rays are very informative propositions. Each member f of $D1$ is an eigenstate of R corresponding to some eigenvalue Ef. Clearly the Ef-eigenspace belongs to $D(e, R)$ in that case. So then the proposition "R has value Ef" is truth-valued. Notice that these are precisely the cases which receive a positive Born probability in state e (i.e., there is a positive probability that if R is measured on a system in state e, the value Ef will be found). Let us call such propositions about R "*allowed by e*." If the Born probability for eigenvalue r of R is 0 in state e, let's call the proposition that R has value r "*disallowed by e*." Since all the r-eigenstates of R for which that is the case belong to $D2$, all those propositions about R that are disallowed by e are also truth-valued.

So far so good; but of course this lattice $D(e, R)$ contains many more similar propositions about other observables. For example, the projection of e onto the 1-eigenspace of R may also be an eigenstate of a quite different observable S, corresponding to eigenvalue 2 of S, say. In that case the proposition that S has value 2 will also be truth-valued. We can divide the truth-valued propositions about any observable S into those allowed by e and those disallowed by e in a similar way (but not apply these labels to propositions which are not truth-valued).

Thinking of it this way, it is understandable how the Born probabilities can be recovered as a possible measure of ignorance about how the truth values True and False are distributed in $D(e, R)$. Given that the system is in state e, we may take it that all disallowed propositions are definitely false. Thus the whole of $D2$ should be treated as a region with zero probability. We can then add that one of the rays in $D1$ is a true proposition, though we don't know which, and we can propose (or interpret) the Born probability for the eigenvalue Ef of R as the measure of our ignorance about whether member f of $D1$ is true. Starting in this way, the probability assignment can be extended to the whole of $D(e, R)$ by additivity, without running into trouble with such "no-go" theorems as the Kochen–Specker result.

An interpretation along these lines results when a particular observable is taken to have privileged status. This specification might be once and for all or it might depend on the state. Bohm chose *position* for this role, once and for all. Von Neumann's EE-link chose a privileged observable very tightly linked to the state, namely the projection on the ray containing that state. Bub has a different suggestion: given a proper selection of observables for such privilege, the measurement problem will disappear.

Here we find the most important conceptual gain of this approach. To explain how the values of the privileged observable can change with time, Bub adapts a proposal by John Bell (as elaborated by Vink). This account of the dynamics of values, together with the above construal of the Born probabilities, provides a very satisfactory corollary about what happens in a measurement. The measurement outcome and the value of the measured observable are both definite, and correspond to each other, at the end of measurement.

I am passing rapidly over some difficult passages here. In a measurement we deal with a composite system of measured object and measuring apparatus. In the above discussion I focused on the total system, assumed to be in a pure state. The two component parts will be in mixed states, and the observables pertaining to the components will be functions of those which pertain to the total system. Obviously the nice corollary about measurement is forthcoming only when the "right" observable is privileged. The "pointer observable" pertaining to the apparatus and the measured observable must be properly related to the privileged observable. Because of the entanglement of the two systems during the measurement interaction, it will suffice if either the measured observable or the pointer observable receives the privileged status. If we assume that either will take on a definite value at the end of measurement, then so will the other.

How compliant will nature be? Is it a question of fact, which observables are privileged in nature in this way? Can we assume that nature will privilege just those observables that we have a special interest in? But perhaps that is the wrong question to ask here. Quantum mechanics works; the task is to propose an interpretation that makes sense of how it works. That proposal can be precisely that nature privileges certain observables under certain circumstances, so that the measurements in which we have an interest have definite outcomes. Bub has therefore presented us with no mean achievement: an explanation of how a certain range of interpretations have the conceptual resources to change the measurement problem from unsolvable riddle to solved conundrum.

We have seen therefore that we have here a major contribution to the debates concerning the interpretation of quantum mechanics. In addition, the wide scope of these results is illuminating. They give us a new insight into a number of earlier interpretations which Bub is able to locate with respect to his own. But I do want to enter a demurral here to the claims of universality made in the book's early pages (e.g., 1, 4, 5). The uniqueness interpretation does not characterize the entire range of "no collapse" interpretations, nor that of all modal interpretations, and certainly not of all tenable interpretations.

Of course, Bub is quite clear on this. His claims of universality should be taken as relative to the underlying program. That point is made quite clearly in the Coda, particularly pages 240 and 241 around the quote from von Neumann. It is especially clear if those passages are read with the parable about the 19th century math student (pp. 115—117) in mind.

But it may still be illuminating for us if we take stock of how other interpretations can and do differ from the range that Bub characterizes here. To begin, the representation of "Observable A has value k" by the k-eigenspace of the operator that represents A rules out quite a bit. First of all it rules out that an observable can have values other than its eigenvalues (as happens in the Bohm–Hiley way of dealing with observables other than position). Second, it rules out "de-occamized" and "contextual" interpretations in which a single observable ambiguously represents several distinct though statistically equivalent observables. Third, by taking all the subspaces as propositions and respecting their lattice structure, Bub rejects doubts about the meaningfulness of conjoining value attributions to incompatible observables. (For example, in Healey's 1988 interpretation the set of truth-valued propositions is not closed under logical conjunction.)

Some of these limitations of the uniqueness theorem are pointed out in a paper by J. L. Bell and Clifton (*Int. J. Theor. Phys.* **34**, 2409–2421 (1995)). As they also point out, the interpretation this reviewer proposed ("Copenhagen variant of the modal interpretation") is not covered. There the EE-link is violated only when the state is not pure. The divergence from orthodoxy will in general appear for the components of many-body systems. For those components will typically be in mixed states even when the total systems are themselves in pure states. Measurement and Schrödinger's cat are typically represented as dynamic evolution of composite systems (possibly with the environment as one component). The desire for values concerns observables that pertain to individual components (such as the cat, or the pointer on the measuring apparatus). Thus the shared goal to do justice to our intuitions in those cases can perhaps be met without abandoning the EE-link for pure states. In other respects there are significant similarities between this and Bub's interpretations, such as adherence to the *Identity of Observables* principle and hence the representability of propositions by subspaces.

We may note that one intuitive reading of the original Bohm interpretation also cannot belong to the range characterized by Bub. I mean this: that position is the only observable that ever has a value. A quick look at the lattice $D(e, R)$ shows that there will in general be many observables other than the privileged observable itself which receive true and false value attributions. The reason is that the propositions concerning R's value do not exhaust the lattice of truth-valued propositions.

These points are not drawbacks to Bub's interpretation, nor do they diminish its virtues or the importance of his general results. They only mean that still more general results are in the offing (and indeed, the results of the Bell and Clifton paper are more general). The virtues Bub can claim are indisputable, and his achievement gives us a truly successful and illuminating way to understand quantum theory. The book is a must for everyone in the field.

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