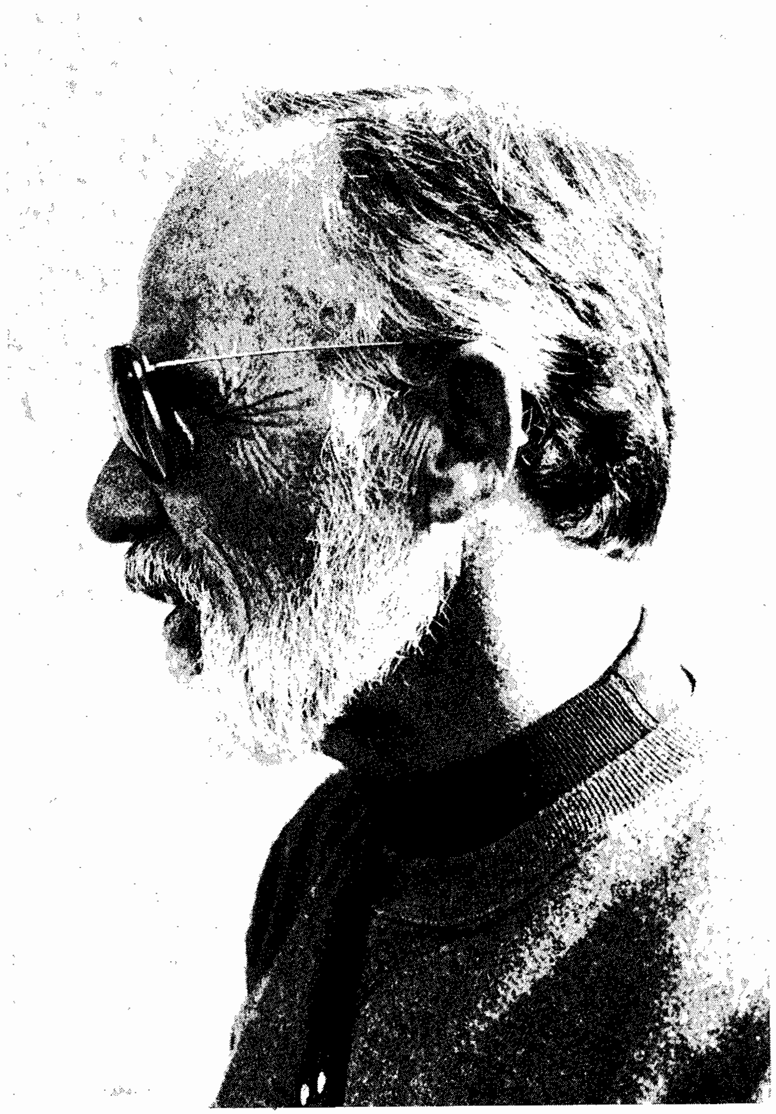

KAREL LAMBERT

# EXISTENCE AND EXPLANATION

*Essays presented in Honor of Karel Lambert*


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namely with colours and orientations and their internal relations, reviewing, comparing, and in fact reviving Meinong's and Wittgenstein's opinions on that subject matter. Ermanno Bencivenga, finally, adds five nice pieces.

The other focal point of the collection is Lambert's philosophy of science. Three essays span historical space. Jules Vuillemin explains some difficulties with kinematics and dynamics and in the development of physics in general by the philosophical analysis of motion given by Plato and Aristotle. Paul Weingartner shows that Aristotle anticipated both Meinong's principle of independence, as Lambert called it, and the modern requirement of finding interpolation sentences for giving scientific explanations. And Erhard Scheibe sharply distinguishes the problems raised in the Einstein-Podolski-Rosen paradox and those raised by Bell's inequality.

Wolfgang Spohn takes up the relation between scientific explanation and understanding in the way proposed by Lambert and arrives at a positive view in which search for explanation is construed as search for coherentistic truth. It is no accident that the notion of stability is also crucial for Brian Skyrms who develops and refines a de Finettian picture of objective probability by generalizing what has been called Miller's principle and by employing ergodic theory. Daniel Hunter, finally, is rather occupied with subjective probability; he defends the method of maximum entropy updating against objections by Brian Skyrms and interprets it as a sound method of belief revision not reducible to any forms of conditionalization.

Thus the collection demonstrates the far-reaching direct and indirect philosophical impact of Karel Lambert's work for which his friends wish to offer warm thanks.

Finally three words of indebtedness: We are very grateful to Ulrike Kleemayer for preparing the index, to Kluwer Academic Publishers, in particular to Mrs. Annie Kuipers, for the effective and friendly co-operation and to Domenico Costantini who invited all three of us to a wonderful conference at the Lago Maggiore in May 1988 where the idea of this Festschrift was born.

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# ON (THE $X$) ($X$ = LAMBERT)

The first few years of my philosophical life were so entangled with learning from Karel Lambert that I can scarcely separate the two. In the fall of 1959 I entered my first philosophy class at the University of Alberta, a class of about seventy students with Lambert as instructor. He told us firmly that he had no intention of discussing ethics or morals, told us to read something about the pre-Socratics for next time, and dismissed the class. It became clear quite soon that he expected us to learn philosophy by doing: despite the size of the class, there was a great deal of discussion and we were constantly challenged. His response was always measured. People who were struggling found sympathy, and he would turn their questions into something significant for discussion. But students who were catching on would immediately find themselves made to face greater difficulties. It was also, I can think of no other way to put it, a lot of fun. Twice, I remember, he put me down, to everyone's amusement. The first time I wouldn't back down from a point when I was clearly losing, and Lambert ended the discussion with "Van Fraassen, you are logical, but you are not reasonable." The second time we had gotten into that subject he did not want to do at all, and I brought up Sartre's famous example of the moral dilemma of the young Frenchman, who had to choose between care for his aging mother and joining the Free French. Lambert listened patiently, then retorted: "Van Fraassen, there comes a time when a boy has to leave his mother."

The course — which like all our courses then, was a year long — ended with Russell, and the theory of definite descriptions. I was working in the university library, part time and then as a summer job, and I immediately started reading as much of Russell as I could. That was also when Lambert called me into his office, to ask me to think about going into philosophy. I said it's all right, I've already decided to do that. So he offered me a Coke, and told me to come see him and talk with him as often as I liked. Looking back on that period, and the courses I took from him, I think one thing stands out most of all. Although he was teaching us logic and related subjects, very near to his

*Wolfgang Spohn et al. (eds.), Existence and Explanation*, 1—18.

heart, he would never especially push or even reveal his own position. We had to come to conclusions via the problems, via questions to be struggled with, not by digesting answers. Even by the end of my fourth year, when I had become quite preoccupied with free logic, there was still the sense that it was entirely more important for me to work on problems that had become real for me, in whatever way and on whatever subject, than that I should get engaged in his.

It took in any case a while before I could even understand those. In the first summer I also started reading Reichenbach, partly because Russell had awakened my interest in the problems of space and time. Early in the second year, I read a new article by Milic Capek on eternal recurrence, in the *Journal of Philosophy.* Nietzsche was of course one of my heroes since high school, and I had thought a lot about that doctrine. Now here was an argument, based on twentieth century ideas about space, time, and relativity (my newest infatuation), and it seemed to make nonsense of this wonderful vision. I wrote a critique, applying what I had learned from Russell and Reichenbach, and showed it to Lambert. Without any meta-comments about the project, he discussed it with me, raised objections, made me clarify and rewrite — 'made me' is right, I think, for I still remember how I felt about it by the seventh and final draft — and told me to submit it. It was my first publication; but more importantly, this was how Lambert taught me in my sophomore year to do research, to criticize myself and to criticize my own criticisms, and to write. And still more importantly, to gain the sense that my ideas could be taken seriously, that ideas had the democratic right to compete, regardless of who voiced them.

Before I discuss the papers Lambert wrote in those years, let me complete this sketch of how I got to the point where I could understand them at all. In my third year, I learned about names, free logic, and the difficulties with Russell's theory of descriptions. We took up Quine's *From a Logical Point of View* and *Word and Object*, believing wholeheartedly that ontological scruples came first, and semantics a very distant second. Lambert mentioned that free logic could be interpreted by postulating some reality for non-existent objects (items treated as designata for names like 'Pegasus') but added that such an interpretation couldn't be of any interest at all. For after all, there are no non-existents, 'existence' is univocal, and the philosophical foundations of logic needed to be part of philosophy generally. I still agree with all of that, and I think he does too, and I am grateful for Quine's insistence on philosophical integrity. But in retrospect, a touch of instrumentalism would have helped. For my fourth year Lambert proposed a directed reading course, on a subject of my choice, with weekly tutorials. I opted for induction and probability, and he chose Kyburg's new book, written entirely in Quine's protosyntax, as one of the main texts. So obviously I had to study Quine's *Mathematical Logic* first, during the preceding summer. That way I also encountered limitative metatheorems in the "'yields a falsehood if appended to its own quotation' yields a falsehood if appended to its own quotation" form. At the very end of the academic year, an article appeared which showed that Kyburg had not been saved from inconsistency by this heroic attempt at formal precision. But in the meantime, I had learned a great deal more logic. I had also had the opportunity to discuss Lambert's own papers with him, with an increasing appreciation of why he wrote them in English rather than in protosyntax.



## HENRY LEONARD AND THE GENESIS OF FREE LOGIC

In 1956, when Lambert was a graduate student, his teacher Henry Leonard published his seminal paper 'The Logic of Existence'. This paper set the problems in philosophical logic that preoccupied Lambert for the next ten years. With hindsight, we can discern two main problems which Leonard posed and for which he proposed solutions.

The paper begins by explaining how logic could have presuppositions, which could be removed so as to make it more widely applicable. The historical example given is the existential import of general terms in the traditional square of opposition. There the I sentence 'Some $S$ is $P$' could be inferred from the corresponding A sentence 'All $S$ is $P$'. Today we reject that inference, allowing only that if the A sentence is true, and there exist some $S$'s, then the corresponding I sentence is true too.

As Leonard saw it, this does not mean that traditional logic was in error. Rather it had a limitation of a sort that perhaps logic will always have, but which could be eliminated once it comes to light. The way he put it was that "traditional logic was a quite correct abstract system of logic; but . . . it was set up and developed with a tacit, or unexpressed presupposition: namely, that its terms $S$, $P$, etc., were terms having existent exemplars." (page 5)

Next Leonard pointed out that contemporary logic also had such a presupposition, with respect to singular terms. This presupposition comes to light in the inference schemes of Existential Generalization and Universal Instantiation:[1]

*Sj*; therefore $(Ex)Sx$
$(x)Sx$; therefore *Sj*

These schemes appear to yield invalid inferences if the term *j* is one which does not refer to anything, such as 'Santa Claus'. Again, that is not an error, but limits applicability. No errors will result if we restrict the allowable substituents for *j* to singular terms that have a referent. But now there is an obvious problem to address: how shall we make the presupposition explicit — i.e. state just what is allowed to replace *j* and what is not — and can we devise an abstract system of logic which is more widely applicable?

Leonard proposes that we characterize the category of singular terms as those terms which purport to refer to some entity, and divide them into those which do refer and those which do not. The term '*E!*' (pronounced '*E*-Shriek') which appears in Russell's theory of descriptions (though not as a primitive) he proposed for the job of marking this division: '*E!j*' is to be read as '*j* exists' and is true if and only if the term *j* refers to something. We are here in an area of philosophical contention, and Leonard discusses for example Quine's treatment of singular terms, and the idea (which he regarded as then prevalent) that ordinary names are all short for definite descriptions. His critique is acute; but I will restrict my exposition here to his own positive proposal.

Leonard does not shrink back from higher order quantification and modal logic. He proposes that we revise the logic of Principia Mathematica, and lays down the following principles:

(L1) $E!x$

(L2) $(Ex)Fx \equiv (Ex)(E!x \,\&\, Fx)$

(L3) $(x)Fx \equiv (x)(E!x \supset Fx)$

(L4) $E!(Ix)Fx \equiv (Ey)(x)(Fx \equiv x = y)$

(L6) $Fy \,\&\, E!y \cdot \supset \cdot (Ex)Fx$

(L7) $(x)Fx \,\&\, E!y \cdot \supset \cdot Fy$

(L8) $(x)Fx \supset (Ex)Fx$



I will leave L4, a principle concerning definite descriptions, for discussion below. Omitted altogether is L5, which pertains to a separate proposal concerning the possibility of defining *E!* by means of higher order quantification into modal contexts. On page 60, before stating the above principles, Leonard proposed to take *E!* as primitive for the time being, and I shall just stick to that here. Finally, Leonard himself points out that L1 and L8 together entail $(Ex)E!x$, i.e. that at least one thing exists. A little later various authors, including Lambert, saw that as a still further presupposition which limits the applicability of logic as well.

What is not clear from Leonard's paper, is just how much the above principles are meant to do. To what are they to be added, so as to yield a satisfactory system of (first order) quantificational logic? That is a question about completeness, and Leonard was not in a position at that point to give that question a precise content. The history of this and related problems is treated adequately in the introduction and selections included in Bencivenga's *Le Logiche Libere*. Roughly speaking, satisfactory systems of that sort came into the literature in 1959, at the hands of Hailperin and Leblanc, and Hintikka. In his abstract of a paper for the International Congress of Logic, Methodology, and Philosophy of Science at Stanford in 1960, Lambert introduced the term 'free logic' to stand generally for systems free of presuppositions of the sort Leonard had discussed.

That was the first problem Leonard set, and his partial solution. Let us take a careful look at its general character. Quite in accordance with his way of introducting the problem, Leonard thought of free logic (as I shall now continue to call it) as a fragment of the standard logic. The theorems of free logic as he sketched it, were part of the theorems of *Principia Mathematica*. But the class of terms that qualified as substituents for free variables in the axioms and rules, was larger, it was not restricted to referring terms. Bound variables were unaffected, for the quantifier retained its standard interpretation. As Leonard codified it:

> We agree with [Quine] that "To exist is to be the value of a variable." But our revised logic is such that we disallow his claim that to name an existent is to be a substituent of a variable. Instead, our logic comports with "To purport to name an existent is to be a substituent of a variable." In other words, not all substituents designate values. (page 60)

When I came to Pittsburgh as a graduate student in the fall of 1963, I took a seminar from Nuel Belnap, with as main topic the logic of questions. It began with an introduction to the semantic analysis of logic, and Belnap presented the short, elegant completeness proof for quantificational logic that he and Alan Anderson had recently published. In a review, Belnap also indicated how this can be amended for free logic, with the singular terms all being given referents in a domain only part of which supplies the range for the quantifier. The logic of 'Santa Claus' is then regarded through the fiction that Santa Claus does exist, but outside the class spanned by our 'All' and 'Some'.

Looking through my correspondence with Lambert during that year, I found a letter from Lambert, responding to what must have been one from me about what I had been learning there.

> Yes, I was quite well aware of the sort of proof of completeness Belknap (*sic*) suggested to you for "free" logic — as much as four years ago. My proof was parallel to his; split the domain into real and imaginary objects, restrict the range of the quantifiers to real objects, replace Specification by Ramified Spec., and proceed a la Henkin. Hintikka suggested this sort of proof to me, though I already knew how to do it. But like you, I have been trying to find a "standard" model, as you call it. (Nov. 26, 1963)

Had the problem — the first problem Leonard set — been solved satisfactorily? In my mind, and as the last sentence of this passage indicates, in Lambert's mind, the answer was *no*. It had been shown that the indicated fragment of standard logic had a certain autonomy: it was exactly the logic obtained from the standard one by restricting the quantifier to the extension of a non-empty but otherwise arbitrarily chosen predicate. Perhaps the right understanding of 'exists' entails that its logic must coincide with that logic — or perhaps there is more to it.

But the problem had been handled well enough to give a satisfactory setting for dealing with the second main problem which Leonard had set — and which preoccupied Lambert a good deal more.

### LEONARD ON DEFINITE DESCRIPTIONS

This second problem was to develop an adequate free logic of descriptions. Leonard rejected Russell's theory, at least partly for the same reason as before: that the logical treatment of terms should be uniform, regardless of matters of fact such as whether this or that thing exists. The above departure with respect to terms generally allows a new treatment of descriptions too. The first 'big change', says Leonard, is that definite descriptions are allowed as substituents for free variables



in the theorems of logic. As first example, Leonard gives

(L11) $(Ix)Fx = (Ix)Fx$

as substitution instance of the law of self-identity. This is not a theorem of *Principia*; but on the other hand, some of those theorems will have to be rejected. Recall that Leonard had already listed for retention

(L4) $E!(Ix)Fx \equiv (Ey)(x)(Fy \equiv x = y)$

which was Principia *14.02. He also retained half of Principia 14.01, namely

(L14) $(Ey)[(x)(Fx \equiv x = y) \,\&\, Gy] \supset G(Ix)Fx$

But the other half, the converse of the L14 would via L4 entail

(a) $G(Ix)Fx \supset E!(Ix)Fx$

Could that be maintained? Not for arbitrary open formulas $G\ldots$ So at best we would land into the mess of scope problems, trying for example to retain (a) for primitive predicates $G$, but distinguishing between $[-G](Ix)Fx$ and $-[G(Ix)Fx]$. After all, the new policy for substitution generates as theorem

(b) $H(Ix)Fx \vee -H(Ix)Fx$

so that $H\ldots$ and $-H\ldots$ could not both replace $G\ldots$ in (a) without yielding the disastrous theorem

(c) $E!(Ix)Fx$

In another respect, however, Leonard wanted to go quite definitely beyond *Principia*, which has

(*14.22) $E!(Ix)Fx \equiv F(Ix)Fx$

Leonard comments "The right-hand member appears to be analytic, and should be assertible without restriction to descriptions that exist." I cannot make out what the last two words are doing here, but it seems that he wanted something stronger than *14.22. Leonard's next statement certainly denies, however, that the attribution "appears to be analytic" can be taken at face value. In the case of a predicate F for which $(Fx \supset E!x)$ is analytic, he says,

(d) $F(Ix)Fx$

cannot be generally true. Specifically, if $F = E!$, then (d) would be

(e) $E!(Ix)E!x$

which says, via L4, that only one thing exists.

Leonard goes on to propose an extension of the little theory of descriptions sketched so far (L4, L11, L14) within modal logic. The problem which came to concern Lambert, however, was the theory of descriptions for first order logic with identity alone. This is what I had in mind as the second main problem Leonard set for his philosophical posterity.

Because the theory of descriptions will be our focus, I will now provide the important formulas with mnemonic manes.

(LEON-EX) $E!(Ix)Fx \equiv (Ey)(x)(Fy \equiv x = y)$

(LEON-ID) $(Ix)Fx = (Ix)Fx$

(LEON-AT) $(Ey)[(x)(Fx \equiv x = y) \,\&\, Gy] \supset G(Ix)Fx$

(MEINONG) $F(Ix)Fx$

Here EX, ID, AT stand for *existence, identity, attribution*, and MEINONG is mean to insinuate something about Meinong — in fact the latter's principle studied by Lambert later on, of the independence of Sein and Sosein.

## LAMBERT'S EARLY ESSAYS ON DESCRIPTION THEORY

In 1959 Jaakko Hintikka had published two papers, one each for the two problems set out above. The second, 'Towards a theory of definite descriptions' appeared to provide just what was called for, a theory of definite descriptions, allowed as substituents for free variables in the theorems of first-order free logic with identity. But Lambert, in his 'Notes on *E!* III', pointed out that (MEINONG) above is provable in Hintikka's system, which has axiom

(HINT) $y = (Ix)Fx \equiv \cdot\, (x)(Fx \supset x = y) \,\&\, Fy$

Via (LEON-ID) this leads to (MEINONG) and also — as Lambert points out further on (page 56) — to

$$(x)(Fx \supset x = (Ix)Fx)$$

which has equally disastrous consequences.



Hintikka replied in 1964, proposing to repair his system by rejecting (LEON-ID). To do this, he had to reject substitution of definite descriptions for the free variable in his general axiom ($y = y$) of self-identity. The paper notes that the restriction must not be easily circumventable, but suggests still that it is a matter only of restricting such substitution. Lambert pointed out in a later rejoinder (1966) that the restriction would have to be extensive — perhaps universal. For suppose we allow the inference from (HINT) to

(g) $y = (Ix)(x = y) \equiv (x)(x = y \supset x = y) \,\&\, y = y$

where $F \ldots$ is replaced by $\ldots = y$. Then we can infer to

(LAMB-ID) $y = (Ix)(x = y)$

If now the restriction does not forbid replacement of $y$ by $(Iz)Fz$, that gives

(h) $(Iz)Fz = (Ix)(x = (Iz)Fz)$

Now, again if the restriction allows it, substitute $(Iz)Fz$ for $y$ in (g) to get, via (h), to

(i) $(x)(x = (Iz)Fz \supset \cdot\, x = (Iz)Fz) \,\&\, (Iz)Fz = (Iz)Fz$

which yields (LEON-ID) after all. At this point it is not clear just how much Hintikka's restriction should forbid; the problem seems to be out of hand.

In 'Notes on *E!* III' Lambert had advanced his own proposal. He proposed (in the context of a general rule that all singular terms — both names and descriptions — are substitutable for free variables in the general theorems of logic) the special principles:

(LAMB-ID) $y = (Ix)(x = y)$

(MFD) $(y)[y = (Ix)Fx \equiv \cdot\, (x)(Fx \supset x = y) \,\&\, Fy]$

The tag MFD was introduced later by Lambert as mnemonic for 'Minimal Free Description theory' (see his 1972). Note that it restricts Hinitikka's basic principle to existents — it says under what conditions an existent is $(Ix)Fx$. For non-existents the information we get is only that from general logic — including therefore (LEON-ID) — plus the special principle (LAMB-ID). But in the meanwhile, existence had been proved to have its natural general explanation in free logic, namely

$$E!y \equiv (Ex)(x = y)$$

and via this biconditional, we get at once from (MFD) to (LEON-EX) and (LEON-AT).

In his 'Notes on '*E!*' IV', Lambert proposed a strengthening of this theory. The new single principle to be added to first-order free logic with identity was

(LAMB-IV) $y = (Ix)Fx \equiv (z)[z = y \equiv \cdot Fz \;\&\; (x)(Fx \supset x = z)]$

The tag IV just refers to the title 'Notes on '*E!*' IV' of course. Note the difference from (HINT): this principle gives the truth conditions for $(y = (Ix)Fx)$, and it agrees with (HINT) when the terms $y$ and $(Ix)Fx$ refer. But if one refers and the other doesn't, it makes the statement false — as Leibniz's law of course requires — while if neither term refers, it makes the statement automatically true.

This is a stronger theory, for it implies both (LAMB-ID) and (MFD). In addition, it implies, as we just saw, in effect:

(LAMB-NONEX) $-E!y \;\&\; -E!(Ix)Fx \cdot \supset y = (Ix)Fx.$

That is all; (LAMB-IV) is in turn entailed by (MFD) and (LAMB-NONEX) together.

## THE SPECTRUM OF FREE DESCRIPTION THEORIES

Leonard had divided the logic of descriptions into two parts for free logic. The first part, encapsulated by Lambert in (MFD) says in effect that Russell was right in his treatment of those definite descriptions which do refer. The second part was the set of principles to be added to free logic, elaborating on the meaning of 'the so and so' generally, and applicable as well to those definite descriptions which do not refer. While the first part is a complete and unique solution to its proper problem, the second part allows for a whole spectrum of options, from complete neutrality (accept (MFD) alone), to a complete and arbitrary fiat for the treatment of all non-referring terms, such as (LAMB-IV).

Should we think that there must be a uniquely right point on this spectrum? Even if the answer were *yes*, we might have to add that the understanding of 'the so and so' cannot be complete within the compass of first-order free logic with identity. Leonard was perhaps right to think that this understanding would also require a study of 'the' in modal discourse — at least. A unique right point on 'our' spectrum (of first order theories) would then be entailed by the right account in that richer logical context.



But I suspect that in the richer context, all we will find is a more richly nuanced spectrum. For there is a general problem that arrives with the modalities: exactly that we can start cataloging alternative possibilities. Let us, like Leonard, ask how (MEINONG) might be restricted so as to yield a valid general principle. Leonard proposed

(LEON-MOD) $-(x)\,\Box\,(Fx \supset E!x) \supset \Box\, F(Ix)Fx$

But now consider

$Fx = [x$ is such that exactly N objects exist$]$.

This can be asserted of $x$ without entailing that $x$ exists. So by (LEON-MOD) we get

$(Ix)(x$ is such that exactly N objects exist)
is such that exactly N objects exist

which is false for most N. Clearly, (LEON-MOD) is not saved by adding to its antecedent

$\ldots \;\&\; \Diamond\, Fy$

for after all, you or I could be such — i.e. inhabit a world such — that there are exactly N things, for many numbers *N*.

The only restriction that would prevent this sort of problem is one that guarantees that $Fy$ does not conflict with any of the facts about existents in our world. But what if $F$ says nothing about the real things? Let us assume the following as given:

$-E!y \;\&\; Fy$

It is clear now that $(Ix)(x = y \;\&\; -Fx)$ does not exist. It also can't be a non-existent, so to say, or at least, we can't assert

$(Ix)(x = y \;\&\; -Fx)$ has the property that it is identical with $y$ and is not $F$

though I don't see what modal fact could give the right restriction for (LEON-MOD) to prevent this as consequence. After all, $(Ix)(x = y \;\&\; -Fx)$ possibly exists, is possibly identical with $y$, and so on. So if we want to restrict (MEINONG), the predicate should not even be such that, if it applies truly anywhere, that could contradict given facts about anything, existents or non-existents. That leaves us only with necessary properties. But they should be very necessary. That is, if $F$ is to qualify for use in (MEINONG) so as to yield a truth, we would need a guarantee that there is no term $t$ at all such that $Ft$ could contradict anything else. And that we can't express unless we add a new sort of quantifier — call it $(/x)$ — such that Universal Instantiation does hold for arbitrary singular terms when it comes to that quantifier.

Perhaps we could, meaningfully, have that. Lambert and I explored this sort of quantifier in a later joint paper. But at this point in our present argument, such principles don't give us any useful information. We are near to the stipulation that $Fy$ itself must be a theorem — in which case the relevant instance of (MEINONG) is a theorem already — and are not genuinely told any more. The answer to the question about what is true about non-existents is still left entirely undetermined.

## QUESTIONS OF COMPLETENESS

Just as Lambert's main problems were initially set by his teacher, so Lambert set my initial problems in philosophical logic. The first derived from his insistence that a philosophically illuminating semantics for free logic would have no recourse to entities outside the range of the quantifiers, as surrogate designata for non-referring terms. In the fall of 1963, shortly after I had come to Pittsburgh as a graduate student, I wrote Lambert about this. Whatever proposal I made, I doubt that it had much to it; he answered

> Later, I shall comment on your proposal for proving the completeness of free logic. You are right. Your proposal differs from Belknap (*sic*) and mine only in a theoretical way. But I have some misgivings, nevertheless, about the way you propose to establish completeness via the technique of models. (Dec. 18, 1963)

In the summer of 1964 I wrote a paper for a directed reading course with Nicholas Rescher on tense logic, 'Tense logic for corruptible entities', in which the quantifiers in present tensed sentences (devoid of tense-modal operators) were interpreted as ranging over presently existing objects only. Since the language contained names that could refer to people already dead or not yet born, that meant that the logic of the present tense fragment had to be free. With that excuse, the paper became the vehicle for work in free logic, and I gave the semantics and completeness proof (which I wrote up properly later on in 'The completeness of free logic'). On August 28, 1964 I sent relevant parts of the essay to Lambert, together with an exposition called 'Two interpretations of free logic' in which I explained why and how free logic should and could be neutral between the various proposals about assigning truth values and leaving truth value gaps when it comes to sentences like 'Pegasus has a white hind leg'. On September 9 already Lambert sent me a letter about it, and three pages of comments, to which I replied two days later — well, when you're hot, you're hot!



Through Lambert's wholehearted and charitable interest in what I was doing, correspondence and conversations almost imperceptibly grew into joint work.[2] In February 1965 we were corresponding intensely about Lambert's 'Notes on E!' (III and IV), and had in effect begun work on our eventual joint paper 'On free description theory'.

Let me explain briefly about how the formal context had changed. Since Lambert had made me learn the system of Quine's *Mathematical Logic*, that is what I began with. Now this is a system designed for a language in which there are no names, and in which formulas containing free variables are not genuine statements at all — just formal conveniences. If we now just add names to this language, and keep everything else the way Quine had it, we have a free logic already. For the formula '$x = a$' takes its place among the formulas containing a free variable, and is treated no differently from '$Fa$', where '$F$' is an arbitrarily chosen monadic predicate. Of course this is uninteresting until we add principles that have to do with names, specifically:

$$\vdash t = t$$
$$\vdash t = t' \supset \cdot A \supset A(t'/t)$$

where Quine's $\vdash$ indicates that the universal closures of what follows are all theorems (and where $t$, $t'$ stand for any singular terms or free variables, and $(t'/t)$ is the substitution of occurrences of $t'$ for all occurrences of $t$).

Understood in this way, free logic is developed not as a fragment of standard logic, but as an extension; yet it is obviously the same thing — in so far as quantificationally closed formulas are concerned — as what we had before. Specifically, we can prove e.g.:

Suppose $Fa$ and $(Ey)(y = a)$. Then $(Ey)(Fa \,\&\, y = a)$.
But $\vdash Fa \,\&\, y = a \cdot \supset \cdot Fy$; therefore, $(Ey)Fy$.

which is the right, free version of Existential Generalization.

Suppose we forget about names, do not add them, but add the description operator $I$ to the syntax, and let the singular terms (substituents for $t$, $t'$) be just exactly the free variables and the expressions $(Ix)A$, where $A$ is any well-formed formula. Then what goes for '$a$' above will obviously go for '$(Ix)A$'. What could be added by way of axioms to yield a free description theory?

The identity axioms above already tell us implicitly that $(Ix)Fx$ can be identical with at most one thing; any additional principle which applies to referring descriptions serves therefore just to narrow down which existent $(Ix)Fx$ is, if it exists.

The question is now what we should add as principles for description theory, to cover also the non-referring cases. If like Leonard we agree that Russell was right in the case of referring descriptions, we still have to worry about his bothersome scope rules. But if Lambert's discussions taught us anything, it is that it will suffice to concentrate on simple assertions of identity. So the agreement with Russell should be expressible as:

(RUSSELL-ID) $\vdash E!(Ix)Fx \cdot \supset \cdot (y)(y = (Ix)Fx \cdot \equiv \cdot Fy \,\&\, (x)[Fx \supset x = y])$

But of course, since the quantifier ranges only over existents, tautologically so, we also have:

$$\vdash -E!(Ix)Fx \cdot \supset \cdot (y)(-[y = (Ix)Fx])$$

But Russell had said, and Leonard agreed to this too, that if there is exactly one $F$, then $(Ix)Fx$ exists:

(RUSSELL-EX) $\vdash (Ey)(Fy \,\&\, (x)[Fx \supset x = y]) \supset E!(Ix)Fx$

It was Lambert's insight that putting all this together amounts exactly to the main principle of his 'Notes on *E!* III', namely (MFD).

Thus we have a rock bottom minimum for free description theory: the one that says when $(Ix)Fx$ exists, and says which existent it is identical with if it does exist. This closes the book, so to say, on the treatment of referring descriptions. There is nothing that can be subtracted without leaving a glaring open question. And there is nothing that can be added which pertains just to the referring ones, for there is nothing — describable in the object language — that is common to just those entities which are designated by definite descriptions. So, any additions will have to be about the non-referring cases.



Here we have just one sort of guidance from the previous literature: the persistent temptation, leading into error on a number of occasions, to subscribe to

(MEINONG) $\vdash F(Ix)Fx$

So one sort of addition will be to assert (MEINONG) for a restricted, 'safe', class of cases. That is just the sort of addition Lambert made in 'Notes on E! III', with

(LAMB-ID) $\vdash (Ix)(x = t) = t$

here written so as to show how it is a special case of (MEINONG). Rich Thomason had another candidate at one point:

(THOMASON) $\vdash Ft \,\&\, -E!t \cdot \supset \cdot F(Ix)Fx$

i.e. $(Ix)Fx$ is $F$ if anything unreal is. My own single contribution to the eventual 'spectrum of free description theories' was of a different sort, having nothing to do with (MEINONG) directly. I proposed a rule which, like Universal Generalization, generates theorems from theorems, rather than conclusions from premises *ueberhaupt*:

($vF$) if $\vdash Ft \equiv Gt$ then $\vdash (Ix)Fx = (Ix)Gx$

The mere fact that this rule is not derivable from the foregoing had disturbed me — but of course, if you are not saying anything about non-existents, and you know that $(Ix)(Fx \,\&\, Gx)$ does not exist, then you do know that $(Ix)(Gx \,\&\, Fx)$ does not exist either, but not whether they are identical or distinct!

A strengthening of ($vF$) can be suggested, but only if we do not care too much about non-existents — if we don't think that there are interesting true stories about Pegasus and the like. In that case, we might be willing to accept:

$$(x)(Fx \equiv Gx) \supset (Ix)Fx = (Ix)Gx$$

with its corollary that the existent golden mountain *is* the golden mountain, and that if in fact no real men eat quiche, then the man = the man who does not eat quiche. I was not willing to go so far. Lambert, in 'Notes on E! IV' had been willing to go further, as I pointed out above. The last two additions I have discussed saw the light later, but in our joint paper, we exhibited the system of 'Notes on E! IV' as the joining of Russellian and Fregean approaches to description theory. For as we showed, it can be produced by adding to the minimal free description theory either, the *E!* IV principle suitably formulated:

$$\vdash t = (Ix)Fx \cdot \equiv \cdot (y)(t = y \equiv \cdot Fy \;\&\; (x)[Fx \supset x = y])$$

(where $x$ and $y$ are distinct variables), or the principle that any two non-existents are identical:

$$\vdash -E!t \;\&\; -E!t' \cdot \supset \cdot t = t'$$

or else the triple of principles, two marking agreement with Russell, and the third the Fregean principle which Kalish and Montague had introduced in their treatment of descriptions:

$$\vdash E!(Ix)Fx \equiv (Ey)(x)(Fx \equiv x = y)$$
$$\vdash (Ey)(x)(Fx \equiv x = y) \supset F(Ix)Fx$$
$$\vdash -(Ey)(x)(Fx \equiv x = y) \supset (Ix)Fx = (Ix)(-[x = x])$$

Short of adding a specific story about the non-existent, this is the strongest free description theory one can have — it reduces the possible stories that can be true about non-existents to those which have only a single character. At the conclusion of the article we suggested that this might be appropriate in special, technical contexts, such as the foundations of mathematics, but not in general.

## CONCLUSION

When I set out to reconstruct what happened at the conception, birth, and maturing of free logic and description theory, I decided to limit the story to what I had seen from close up at the time. It brought back a wealth of memories of how Karel Lambert had allowed me to enter and participate in this intellectual journey — beginning already well before I understood very well what was going on. His example as a teacher, which I have tried to keep as a model, was never to proselytize or dictate, but to provide rich and varied opportunities for learning. His encouragement consisted especially in giving us the feeling that any ideas we came up with would be treated as possible contributions, as quite possibly containing great promise. Then he would point us to new literature — quite often, and without saying so, including views at odds with his own — and suggest new problems, again as opportunities we could take to advance our thinking. Then if we did come to learn his views, they would appear as solutions to problems which we had



struggled with ourselves, and after we had gained some independent ability to evaluate the proffered solutions.

Reading this last paragraph over again, I realize that in trying for a summary of my debt, I have produced something rather abstract, and perhaps a little stilted. I remember quite well Lambert's ironic smile when I'd go off into something pretentious, pedantic, or overly abstract. I have tried to keep that in mind as well, even if not always equally successfully. But I wanted to explain just how much I learned, much of it not at all theoretical, in learning how to answer the question *who* = $(Ix)$ ($x$ = Lambert)?

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## NOTES

[1] The use/mention distinction will not be strictly observed, but in general my symbols are not object language expressions but names thereof, or placeholders for such names.

[2] Lambert had moved from Alberta in the summer of 1963 to become chairman of the Philosophy Department of West Virginia University, in Morgantown, W. Va. Since that is not very far from Pittsburgh, it was possible to meet from time to time. His letters make frequent mention about his trips to Pittsburgh to take part in golf tournaments, and our meetings on those occasions. Eventually Lambert hired me as a full time instructor in his department for the spring term of 1966, while I was finishing my dissertation at Pittsburgh.

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ERMANNO BENCIVENGA

# FIVE EASY PIECES

Lambert has shown in a number of occasions how to make significant philosophical points while using space very economically. The following pieces are conceived as homage to his example in this regard.

## 1. FREGE'S OTHER DUALISM

One way of understanding Frege's distinction between concept and object is as an approach to the problem of the third man argument: if concepts are irreducible to objects, to the point of not even being possible objects of discourse, then that argument cannot get started. Here I will show that another Fregean distinction — that between grasping and asserting a thought — can be seen as an approach to another Platonic puzzle: the reduction of knowledge to a form of recollection. I leave it to others to decide whether this analogy points to a more general, deeper opposition between the conceptual structure of Frege's realism and that of other, more traditional variants of the same general position.

According to Frege, there would be no objectivity to human knowledge if it weren't for thoughts. It is by mobilizing thoughts that one can account for the fact that what I know when I know the Pythagorean theorem is the same as what you know when you know that theorem. The problem is: how exactly is this 'mobilization' supposed to work?

Thoughts are expressed by sentences, they are the senses of sentences, and understanding a sentence means grasping the thought expressed by it. So suppose that grasping a thought be the only relation one can have with it. Suppose also that I am sitting in a windowless, soundproof room, and you come in and tell me:

(1) It is raining outside.

If (1) is true and I have reasons to believe you, we will consider this a case in which knowledge of (1) is transmitted from you to me. But, even before your utterance, I was in a position to grasp the sense of (1), that is, more specifically, I grasped the senses of the component expressions

*Wolfgang Spohn et al. (eds.), Existence and Explanation*, 19—24.