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FACTS AND TAUTOLOGICAL ENTAILMENTS*

NOT very long ago, facts (and their various relatives in the philosophical entourage) played a central role in the explication of logical relationships. But today the prevalent¹ opinion seems to be that facts belong solely to the prehistory of semantics and either have no important use or are irredeemably metaphysical or both. In this paper we shall explore first what kinds of facts must be countenanced if we are to take them seriously at all and, secondly, what we can do with them once we have them. We argue that there are several tenable positions concerning what kinds of facts there are, but reach two main conclusions which are independent of these positions. The first is that facts can be represented within the framework of standard metalogic; the second is that facts provide us with a semantic explication of tautological entailment.² These seems to us to be sufficient reasons to take facts seriously, and we shall argue in section III that doing so involves no objectionable metaphysical commitment.

I

"The first truism to which I wish to draw your attention" Bertrand Russell said in his 1918 lectures on Logical Atomism, ". . . is that the world contains facts. . . ." And he added, "When I speak of a

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¹ An apparent exception is Roger Wertheimer, "Conditions," this JOURNAL, LXV, 12 (June 13, 1968): 355-364.

² This notion was introduced by A. R. Anderson and Nuel D. Belnap, Jr., in "Tautological Entailments," *Philosophical Studies*, XIII, 1-2 (January/February 1962): 9-24; it is a refinement of similar notions of entailment discussed by Geach, Smiley, Strawson, von Wright, et al.

fact . . . I mean the kind of thing that makes a proposition true or false."³ But what facts are there? A most generous answer would consist in allowing that, if *A* is any sentence, then (*the fact*) that *A* names a fact. As is well known, Russell himself argued against this answer; but, to begin, let us consider the consequences of generosity.

The first consequence is clearly that some facts obtain (are the case), some facts do not obtain, some must obtain, and some cannot obtain. This notion of *obtaining*, or being the case, is somewhat like that of existence. Indeed, it may plausibly be held that "*X* is the case" means simply "*X* exists and *X* is a fact." Thus, Whitehead and Russell (using 'complex' where Russell would later use 'fact') say that an elementary judgment (i.e., an atomic statement) "is true when there is a corresponding complex, and false when there is no corresponding complex."⁴ We face here a difficulty (avoided by Russell) since in the terminology of many philosophers it makes no sense to say that some things do not exist and, so, presumably, no sense to say that some facts do not, or cannot, obtain. But we have argued elsewhere⁵ that sense can be made of it, and we shall use these expressions without further comment.

In what follows we shall everywhere accept that some facts obtain and others do not. But we shall not begin by committing ourselves to what we have called the "generous answer." Instead we wish to follow first Russell's procedure of admitting only such facts as we find ourselves forced to admit, given that we wish to have a viable theory of facts.

II

As the weakest possible principle of any theory of facts, we offer the following minimal explication of Russell's "first truism":

1. The truth value of a sentence is determined by the facts that are the case.

The question is now what kinds of facts there must be for this principle to hold. The minimal commitment would appear to be to "atomic facts," the *complexes* of *Principia Mathematica*: "We will give the name of 'a complex' to any such object as '*a* in the relation *R* to *b*' or '*a* having the quality *q*' . . ." (*loc. cit.*). The atomic fact *a*'s bearing relation *R* to *b* is the case if and only if the atomic sentence *aRb* is true. It is important to see, however, that 1 does not require us to say that there is any other kind of fact. For with respect to more

³ *Logic and Knowledge* (London: Allen & Unwin, 1956), p. 182.

⁴ *Principia Mathematica* (Cambridge: University Press, 2d ed., 1963), vol. I, p. 44.

⁵ "Meaning Relations among Predicates," *Nous*, 1, 2 (May 1967): 161-179.

complex sentences, we can now give the usual truth definitions in terms of their components:

Not-A is true if *A* is not true, *A and B* is true if *A* and *B* are both true, $(x)A$ is true if *A* is true for all values of *x*, and so on.

But this makes nonsense of any theory of facts that refuses to go beyond 1, or, correlatively, that seeks only to define truth conditions. For, of course, we do not need facts to define truth conditions for atomic sentences either: *bRc* is true if and only if *b* bears *R* to *c*. So we must look for a relation between sentences and facts more intimate than the relation defined by "*A* is true if and only if fact *X* obtains." And our first clue here is the remark added by Russell to his "first truism": a fact is the kind of thing that *makes* a sentence true.

As a less trivial explication of Russell's truism we therefore propose

2. A sentence *A* is true [false] if and only if some fact that makes *A* true [false] is the case.

If *A* is an atomic sentence, say *bRc*, and *A* is true, this still leads us only to the conclusion that the atomic fact of *b*'s bearing *R* to *c* is the case. But what if *bRc* is false? Russell reports here on his (moderately famous) debate with Demos.⁶ The latter argued that one need not postulate "negative facts" for if *bRc* is false, this is because there is some (positive) fact that rules out that *b* bears *R* to *c*. Russell objected to this for various reasons.

Let us rephrase the question this way: suppose *A* is atomic, and *not-A* is true, made true by fact *e*. Is there then some other atomic sentence *B* that is made true by *e*? It appears that Demos answered "Always," and Russell "Never." But surely the answer depends on the structure of the language, specifically on the set of predicates. If some of these predicates have disjoint extensions or, better yet, have necessarily disjoint extensions, then the answer may sometimes be "Yes" and sometimes "No." Russell held, of course, that there is a unique "ideal language," of which the predicates express logically independent properties, from which point of view his answer is correct. But this atomism does not seem essential to the theory of facts, and it will suffice for us to say that the answer to this question depends on the structure of the language.

Russell argues secondly that there are no disjunctive facts; that is, that we need not postulate special facts whose function it is to make

⁶ *Logic and Knowledge; Essays, 1901-1950* (New York: Macmillan, 1956), Robert Charles Marsh, ed., pp. 211-214.

disjunctions true. For A or B will be made true by any fact that makes A true as well as by any fact that makes B true; and if no such facts obtain, then it is not true. It is not clear whether Russell sees any new problem arising from a false disjunction; he says that the truth value of A or B depends on two facts, one of which corresponds to A and one to B . We may take this to mean either that Russell vacillated between 1 and 2 or that he would have rejected 2. For if we accept 2, this does lead to a problem: if there are only atomic facts, there is no fact that makes A or B false.

Since A or B is false if and only if *not- A and not- B* is true, we may rephrase this as: if there are only atomic facts, then there is no fact that makes any conjunction true. Acceptance of 2, therefore, implies the acceptance of *conjunctive facts*. For every two facts e and e' there is a conjunctive fact $e \cdot e'$ that is the case if and only if both e and e' obtain.

And we face the same problem, essentially, for the quantifiers.⁷ There is no reason (as yet) to admit existential facts, for if some fact makes A true (for some value of x), then it also makes $(\exists x)A$ true. But we must say that there are universal facts, if we accept 2, these universal facts being somewhat like infinite conjunctions.

III

Now it appears that we have reached (with Russell) a position where we can say, defensibly, that we have admitted into our ontology only such facts as we found it necessary to admit. But there are, of course, various possible ways to challenge that. It must be noted especially that Russell was not the only philosopher to attempt a semantic explication of logical and metalogical relationships through a theory of facts. For example, C. I. Lewis developed such a theory—for what Lewis calls “states of affairs” seems quite definitely to be the kind of thing that Russell called “facts.” And Lewis’s basic principles seem not to have been 1 and 2; indeed, he does not appear to address himself at all to the relation of *making true*.

Lewis says that a sentence *signifies* a state of affairs.⁸ He explains this to mean that the use of a sentence to make an assertion “*attributes the state of affairs signified to the actual world*” (*op. cit.*, 242). Besides the signification of a sentence, Lewis also discusses its “denotation,” “comprehension,” and “intension.” From his discussion

⁷ The suspicion that Russell vacillated on his basic principles concerning facts is reinforced by his immediate postulation of both existential and universal facts (*op. cit.*, 236–237) after having denied molecular facts; he admits that this may not be consistent (237).

⁸ “The Modes of Meaning,” *Philosophy and Phenomenological Research*, iv, 2 (December 1943): 236–249.

it appears that in standard logic we deal with denotations, and in modal logic with comprehensions and intensions. But Lewis did not develop the account of signification very far; he does not consider the question whether there are not some logical relationships for the explication of which signification is needed. It seems clear, however, that instead of the Russellian principle, 2, Lewis accepted

3. A sentence *A* is true if and only if every fact that *A* describes as being the case (or *signifies*) is the case, and false if and only if every fact that *A* describes as not being the case, is the case.

The question is now whether acceptance of 3 rather than 2 leads to an *essentially* different theory of facts.

For an atomic sentence *A*, the fact signified by *A* would seem to be exactly that fact which makes *A* true (if it obtains). But for molecular sentences one sees an obvious difference. *A* and *B* presumably describes as being the case whatever is so described by *A* and whatever is so described by *B*. There is, therefore, no need now to postulate conjunctive facts. On the other hand, *A* or *B* cannot describe as being the case any fact so described by *A* or by *B* (in general); so now we must postulate disjunctive facts.

This is not surprising: signification is a relation "dual" to *making true*, and principle 3 is dual to 2; so the consequences are also dual to each other. But this is a bothersome problem for anyone who is seriously considering the question of which kinds of facts to admit into his ontology. For Russell's argument that we can do without disjunctive facts is good reason not to admit those, and the argument on the basis of Lewis's theory that we can do without conjunctive facts, is good reason not to admit those. But if we admit neither, our theory of facts is codified in principle 1 alone, and hence trivial; if we admit either to the exclusion of the other, we are arbitrary; and if we admit both, we are generous but not parsimonious.

To cut this Gordian knot, I propose that we retain our ontological neutrality, and treat facts as we do possibles: that is, explicate "fact" discourse in such a way that engaging in such discourse does not involve ontological commitment.⁹ This means that we must represent facts, relations among facts, and relations between facts and sentences; this representation can serve to explicate fact discourse without requiring the claim that it also represents a reality. (Indeed, such a claim would, if unqualified, be necessarily false; for we wish to explicate discourse about nonexistents and impossibles as well as about existents.) The nature of the representation is of course dictated by methodological considerations; unlike the ontologist, we

⁹ Cf. the reference in fn. 5.

cannot be embarrassed by achieving parsimony at the cost of being arbitrary. Purely for convenience our representation will be Russellian; and because we know that Lewis's approach is the dual of this, and the generous policy admits simply the sum of what is admitted by both Russell and Lewis, we can be assured that our results will be independent of this arbitrary choice.

IV

We have so far talked about relations between facts and sentences, and before we go on we need to take a look at relations among facts. Suppose we say, with Whitehead and Russell, that to the atomic sentence aRb there corresponds the complex *that- aRb* . The first question is whether this determines entirely the class of complexes. To put it more clearly: are we to conceive of complexes as language-dependent entities, so that every complex corresponds to an atomic sentence? Or are we to say that there is a complex *that- aRb* whether or not the relation R and the individuals a , b are named by expressions of the language? Russell's debate with Demos suggests that he accepts the former (and we argued that, even granting that, he did not win the argument). For if we accept the latter, then for every complex *that- aRb* there is also a complex *that- $a\bar{R}b$* , which makes aRb false; and there will be no need to postulate any negative facts.

From here on we shall accept the latter course, partly for the convenience of not having to admit special negative facts, and partly because facts have traditionally been held to be independent of what anyone may think, or say, or be able to say, about them. The representation of the complex *that- aRb* may now conveniently be achieved by identifying it with the triple $\langle R, a, b \rangle$:

4. A *complex* is an $(n + 1)$ -tuple, of which the first member is a relation of degree n .

Various relations among complexes are now easy to define; for example, complexes e and e' are incompatible (in the sense of Demos) if they differ only in that their first members are disjoint relations. For the explication of logical relationships, however, it is much more interesting to look at relations among molecular facts.

To represent the conjunctive fact whose components are complexes d and d' , I propose we use simply the set $\{d, d'\}$. This is convenient, for a universal fact can then be represented in the same way by just allowing an infinite set of complexes to be a fact also. (This would not be so easy if we had not decided that a complex need not correspond to a sentence; for, after all, all individuals might not be named in the language.) We shall make this precise in the next section, but in the meantime we adopt:

5. A *fact* is any nonempty set of complexes, and the fact $e \cdot e'$ is the union of the two facts e, e' .

Clearly, the fact $\{d\}$ plays the same role as the complex d , and so we have a certain amount of redundancy in our representation; from here on we shall find it convenient to say that the fact $\{\langle R, a, b \rangle\}$, rather than the complex $\langle R, a, b \rangle$ is what makes aRb true.

There is clearly an intimate relation between $e \cdot e'$ and its components e, e' both in the theory of facts and in our representation. We shall say that $e \cdot e'$ *forces* e and e' , and this relationship is then represented by the relation of inclusion as subsets.¹⁰ Thus e forces e' if and only if e' is (represented by) a subset of e . Clearly e forces e , and we also have that if e forces both e' and e'' , then e forces $e' \cdot e''$. We shall say that a sentence is made true [false] *in the wider sense* by any fact that forces some fact that makes it true [false]. After these remarks, the formal representation, to which we now turn, is straightforward.¹¹

v

In the standard interpretation of a first-order predicate language, truth is defined for sentences relative to a model and to an assignment of values to the variables. A model M comprises a domain D and relations R_1, R_2, \dots on that domain. Intuitively speaking, D is the set of existents involved in some (possible) situation, and R_i is the extension of the i th atomic predicate P_i in that situation. The values $d(x)$ of the variables x are chosen from that domain, and the atomic sentence $P_i x_1 \dots x_n$ is true (in M , relative to d) if $\langle d(x_1), \dots, d(x_n) \rangle$ belongs to R_i , and false otherwise. The truth values of complex sentences are defined in the familiar way which we need not recount here. The question now is how we can represent in M the facts that constitute the situation which, intuitively speaking, M is meant to represent.

Following the intuitive remarks in the preceding section, we call a *complex in M* any $(n + 1)$ -tuple, whose first member is an n -ary relation on D and whose other members are members of D . We call a *fact in M* any nonempty set of complexes in M . We designate the union of facts e_1, \dots, e_n as $e_1 \cdot \dots \cdot e_n$, and call this a conjunctive fact with components e_1, \dots, e_n . We say that e *forces* e' in M if both e and e' are facts in M , and e' is a subset of e .

¹⁰ The term 'forces' has already a use in metamathematics, but confusion seems hardly likely; and there are some analogies that make this adaption not to inappropriate.

¹¹ In this I am much indebted to J. C. C. McKinsey, "A New Definition of Truth," *Synthèse*, VII (1948/49): 428-433, and R. Schock, "A Definition of Event and Some of Its Applications," *Theoria*, XXVII, 3 (1962): 250-268.

6. A complex $\langle R, b_1, \dots, b_n \rangle$ in M is the case in M if and only if $\langle b_1, \dots, b_n \rangle \in R$, and a fact e in M is the case in M if and only if all its members are the case in M .

Turning now to the subject of truth and falsity, we shall define for every sentence A the set $T(A)$ of facts that make A true in M and the set $F(A)$ of facts that make A false in M .

First, there is exactly one fact that makes $P_{ix_1 \dots x_n}$ true in M (relative to d)—namely, $\langle R_i, d(x_1), \dots, d(x_n) \rangle$ —and exactly one fact that makes it false in M —namely, $\langle \bar{R}_i, d(x_1), \dots, d(x_n) \rangle$, where we designate the complement of R in M as \bar{R} . To define the sets $T(A)$ and $F(A)$ for a complex sentence A , some new notation may be helpful. When X and Y are two sets of facts, we shall call the *product* $X \cdot Y$ of X and Y the set of facts $e \cdot e'$ such that e is in X and e' is in Y . The product of an infinite family of sets of facts is defined analogously. Note that such a product is still a set of facts. Secondly, we shall symbolize *not- A* as $\sim A$, A and B as $A \& B$, and A or B as $A \vee B$. The latter will be thought of as defined in terms of \sim and $\&$, and it will be seen that this yields the correct result for the sets $T(A \vee B)$ and $F(A \vee B)$.

7. $T(\sim A) = F(A)$; $F(\sim A) = T(A)$
 $T(A \& B) = T(A) \cdot T(B)$
 $F(A \& B) = F(A) \cup F(B)$
 $T_d((x)A) =$ the product of the sets $T_{d'}(A)$ such that d' is like d except perhaps at x
 $F_d((x)A) =$ the union of the sets $F_{d'}(A)$ with d' as above

We shall not have much occasion to use the definitions for the case of quantified sentences, and from here on we shall not treat this case in detail. The sets $T(A)$ and $F(A)$ are related to the sentence A as follows:

8. A is *true* (in model M) if and only if some fact in $T(A)$ obtains, and *false* (in model M) if and only if some fact in $F(A)$ obtains.

Recalling our notion of forcing, let us designate as $T^*(A)$ —respectively, $F^*(A)$ —the set of all facts that force some fact in $T(A)$ —respectively, in $F(A)$. The members of $T^*(A)$ make A true in the wider sense. It is easy to see that 8 still holds if we replace $T(A)$ and $F(A)$, respectively, by $T^*(A)$ and $F^*(A)$.

VI

Facts will hardly be of interest if they serve only to redefine truth in a model; if facts are to have a use, they must serve to define interesting new semantic relations. And here the most promising avenue of approach would seem to be the replacement of the notion of "being

true" by that of "being made true." Specifically, let us consider the relation of *semantic entailment* which is defined in terms of "being true." We say that A semantically entails B ($A \Vdash B$) if, whenever A is true, so is B . (More precisely: if A is true in model M , then B is true in M .) To this corresponds then the tighter relationship:

9. Whatever makes A true, also makes B true.

We shall abbreviate this as $A \Vdash\!\!\Vdash B$. When we take 9 to mean that $T(A)$ is included in $T(B)$, this is a very tight relationship which even $A \& B$ does not bear to A , although A bears it to $A \vee B$. But if we explicate 9 as

10. $A \Vdash\!\!\Vdash B$ if and only if $T^*(A)$ is included in $T^*(B)$ in any model.

then this relationship turns out already to have a name: it is *tautological entailment*. This is a very encouraging result, for this relation did not previously have a semantic explication within the confines of standard model theory.¹² Before showing that 10 really does define tautological entailment, we shall make some intuitive remarks about this.

C. I. Lewis introduced strict implication to eliminate the paradoxes of material implication. For example, if A is true, then B materially implies A , but this does not hold for strict implication. However, if A is a tautology, then B strictly implies A . This Anderson and Belnap argued to be a fallacy of relevance: the premise of an inference should be relevant to its conclusion, and the conclusion's being tautological does not make it so. So according to them, A entails $A \vee \sim A$, and so does $\sim A$; but not just any sentence whatsoever. With this, our principle 10 agrees: what makes $A \vee \sim A$ true is exactly what makes A true if A is true, or what makes $\sim A$ true if $\sim A$ is true. But what makes B true has, in general, nothing to do with what makes A or $\sim A$ true.

We shall now prove that we have here an explication of tautological entailment, in two steps. In this proof we shall feel free to use the terminology explained by Anderson and Belnap in the paper to which we have referred. Also, like them, we confine ourselves to propositional logic. It may be helpful, before we proceed with the proof, to point out that $T^*(A) \subseteq T^*(B)$ if and only if each member

¹² See fn. 2. Anderson and Belnap defined the relation syntactically, and later Belnap provided a matrix-theoretic semantics: "Intensional Models for First Degree Formulas," *Journal of Symbolic Logic*, xxxii, 1 (March 1967): 1-22. Using his elegant representations of these matrices, J. Michael Dunn showed in his doctoral dissertation (Pittsburgh, 1966) that tautological entailment could be explicated in terms of the topics that sentences are "about," but this relation of *being about* was not further explicated.

in $T(A)$ forces some member in $T(B)$. For if $T^*(A) \subseteq T^*(B)$ then $T(A) \subseteq T^*(B)$, so each member of $T(A)$ forces some member of $T(B)$; conversely, if the latter is the case, then, by the transitivity of forcing, the former follows. This will make the proofs shorter.

THEOREM I. If A tautologically entails B , then $A \text{ |||} - B$.

PROOF: (a) If this is a primitive entailment with A being $a_1 \& \dots \& a_n$ and B being $b_1 \vee \dots \vee b_m$, then let $T(A)$ be $\{e_1 \dots e_n\}$ and let $T(B)$ be $\{e'_1, \dots, e'_m\}$. Since $a_i = b_j$ for some i and j , $e_i = e'_j$, so whatever forces $e_1 \dots e_n$ also forces some member of $T(B)$, namely e'_j . From this it follows at once that $T^*(A)$ is included in $T^*(B)$.

(b) If the entailment is in normal form, with A being $A_1 \vee \dots \vee A_n$ and B being $B_1 \& \dots \& B_m$, then each A_i tautologically entails each B_j . So we must validate the rules:

11. (i) If $A \text{ |||} - B$ and $A \text{ |||} - C$, then $A \text{ |||} - B \& C$
- (ii) If $A \text{ |||} - C$ and $B \text{ |||} - C$, then $A \vee B \text{ |||} - C$

Well, if $T^*(A)$ is included in both $T^*(B)$ and $T^*(C)$, then each member e of $T(A)$ forces some member e_1 of $T(B)$ and also some member e_2 of $T(C)$. But then e forces $e_1 \cdot e_2$, which belongs to $T(B \& C)$. Secondly, let $T^*(A)$ and $T^*(B)$ both be included in $T^*(C)$. Then if e belongs to $T(A \vee B)$, it must belong to either $T(A)$ or $T(B)$, hence it forces some member of $T(C)$. Thus both rules are valid.

(c) To allow every tautological entailment to be brought into normal form, we must validate the replacement rules: Commutation, Association, Distribution, Double Negation, and DeMorgan. Of these all but Distribution are immediate; we shall prove one part of Distribution as a representative example: $T(A \& B \vee C) = T(A) \cdot T(B \vee C) = T(A) \cdot (T(B) \vee T(C)) \equiv (T(A) \cdot T(B)) \vee (T(A) \cdot T(C)) \equiv T(A \& B \vee A \& C)$. The third step is the important one; it follows from our definition of the product of two sets of facts.

THEOREM II. If A does not tautologically entail B , then $A \text{ |||} - B$ is not valid.

PROOF: Because of the replacement rules we need consider only entailments in normal form. If such an entailment is not a tautological entailment, its antecedent has a disjunct A and its consequent a conjunct B such that A is a primitive conjunction $a_1 \& \dots \& a_n$ and B a primitive disjunction $b_1 \vee \dots \vee b_m$, where $a_i \neq b_j$ for any i and j . It is easy to check that we can then choose facts in a model such that the single member of $T(a_i)$ does not force the single member of $T(b_j)$, for any i and j .

This finishes the proof.

VII

In conclusion, I should like to make two comments. The first is that our notion of fact can be extended to cover situations dealt with in

nonclassical logics. For example, in modal logic we would say that a complex is an $(n + 2)$ -tuple, where the first member is an n -ary relation, and the last member a possible world. (In many-valued logic, this last member could be an element of the logical matrix.) The facts that make *Necessarily A* true would then be the conjunctions of facts that make *A* true in the various possible worlds; for *Necessarily A* is true if and only if *A* is true in world α_1 , and in world α_2 , and so forth. Thus the relation of tautological entailment can be semantically extended to the case of a modal language; and it may be interesting to investigate its properties there.¹⁸

My second comment is that the question whether the theory of facts is philosophically interesting seems to me to hinge on the further question of what this theory can do regardless of which of the possible approaches to it we follow. For example, if we could explicate tautological entailment only by following Russell's rather than Lewis's approach, then this would not be interesting at all. But as we have seen, we can represent facts within the standard logical framework, and use the relations among them to explicate tautological entailment, regardless of the approach adopted.

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ESSENTIAL PROPERTIES

SOCRATES was essentially a man, but he was not essentially wise or ugly or a philosopher, nor was he essentially an entity, or of flesh and blood, or not a dog.

He was only accidentally wise and ugly and a philosopher. But essential properties are in some fashion necessary, or "natural" properties.

Being an entity is a necessary property of everything, i.e., a transcendental property. But "nothing that is common is substance" (*Metaphysica*, 1040b, 23). Essential properties sort the entities of which they are true in some fashion.

Not in every fashion. Being a snub-nosed, hen-pecked, hemlock-drinking philosopher sorts Socrates from everything in the universe (probably). But not all sorting is individuating, and essential properties don't have to individuate.

Being of flesh and blood and being not a dog are like being a man in that they are necessary, nontranscendental, nonindividuating properties of Socrates. But being of flesh and blood is a material prop-

¹⁸ For motivation, see the last section of my "Logical Structure in Plato's *Sophist*," *Review of Metaphysics*, xxii, 3 (March 1969).