Leakage through filtercake into a fluid sampling probe

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Pore fluid can be withdrawn from reservoir rock by means of a probe lowered down a well and clamped against the rock surface. The rest of the rock surface is covered by a drilling fluid filtercake which impedes, but does not totally prevent, flow of filtrate from the wellbore into the rock and thence into the probe. The magnitude of this filtrate flow is investigated in an idealized geometry in which the porous rock, with permeability k, occupies the half-space $z>0$. The probe covers the circular region $r<a$ of the plane $z=0$, and the rest of the plane is covered by a thin filtercake of permeability k_c and thickness h. The fluid is assumed incompressible and obeys Darcy's law, so that the fluid pressure p in the porous rock satisfies the Laplace equation. The pressure in the probe is $p_0<0$, and $p=0$ in the wellbore and in the pore fluid at infinity. This mixed boundary value problem depends only on $K=k_c a/kh$. If $K=0$ the problem is equivalent to that of an electrified disc at constant potential p_0 in unbounded space, and pore fluid is drawn from the rock at infinity. If $K>0$, fluid leaks from the wellbore into the reservoir, and the volume of fluid withdrawn by the probe is equal to the volume of fluid which passes from the wellbore into the rock. When $0<K \ll 1$ fluid streamlines within the rock are similar to those for $K=0$ close to the probe, but emanate from the filtercake on $z=0$ on a length scale $r \sim a/K$. Estimates of the hydraulic resistance of filtercakes usually encountered when drilling for petroleum indicate that this leakage flux is sufficiently small to be neglected over typical time scales for fluid sampling. [DOI: 10.1063/1.1360712]

I. INTRODUCTION

Once an oil well has been drilled, a probe can be lowered down the well in order to extract fluid samples from various layers of reservoir rock.[1–3] The cylindrical surface of exposed rock is by this stage usually coated by a low permeability filtercake[4] which impedes (but does not totally prevent) the flow of fluid from the wellbore into the surrounding rock. Once the probe has been clamped against the rock surface which surrounds the wellbore (Fig. 1), the filtercake that lies between the probe and the rock is removed. Fluid from the pores of the rock (pore fluid) is pumped from the rock into the probe, and thence into a fluid sampling device.

There are many fluid flow problems associated with such sampling techniques; it is likely that certain fluid sampling procedures in other porous media, such as tissue, pose similar questions. Here we shall concentrate on the possibility that it may be easier to suck fluid along a short path through the low-permeability filtercake than to suck fluid from the rock pores far from the probe. This would lead to a sample of wellbore fluid, rather than to a sample of pore fluid, and might, for example, hide the presence of petroleum reserves within the rock. In order to investigate this possibility, we consider an idealized problem in which the curvature of the wellbore is ignored (Fig. 2). The rock occupies the half-space $z>0$, and we adopt cylindrical coordinates (r,θ,z). The rock surface $z=0$ is divided into two regions. The circular probe contacts the rock in the region $r<a$. The rest of the rock surface ($z=0$, $r>a$) is covered by a uniform filtercake of thickness $h \ll a$ and permeability k_c.

We assume that at time $t=0$ the rock is saturated with a single pore fluid, i.e., we neglect any previous invasion by filtrate from the wellbore such as would have occurred during drilling. Invasion by filtrate starts at $t=0$, and although we shall distinguish filtrate from the original fluid in place, we shall assume that the fluids have the same viscosity and are miscible, i.e., there are no effects of interfacial tension/capillarity.

We measure all pressures relative to that within the wellbore, so that from now on the wellbore pressure is $p_w=0$. The fluid pressure within the probe is assumed uniform and equal to p_0, and we take $p_0<0$ so that flow is from the rock into the probe. We assume that the difference between the wellbore pressure and the pore pressure in the rock far from the probe is negligibly small compared to p_0, so that $p \to 0$ within the rock at infinity. Thus we neglect the slow filtration from the wellbore to the rock which occurs even in the absence of suction by the probe. We assume that the flow is incompressible, and look for steady Darcy flow within the rock and filtercake. This will be incorrect at very early times, but will be a good approximation close to the probe once the pressure perturbation caused by fluid extraction by the probe has diffused a distance large compared to the probe radius a.

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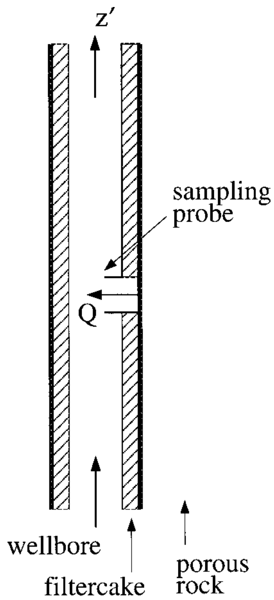

FIG. 1. Schematic of a wellbore, with axis z'. The probe is clamped against the cylindrical wellbore, and withdraws fluid from the rock pores at a volumetric flow rate Q.

In Sec. II we set down the dual integral equations and boundary conditions which describe the problem. The simplest case of an impermeable filtercake is solved in Sec. III A, and the solution for the more general case of a permeable filtercake is then given in terms of an infinite series in Sec. III B. The limit in which the filtercake permeability decreases to zero is noteworthy and is discussed in Sec. III C. Numerical results are presented in Sec. IV, and we close in Sec. V with a discussion of the implications of the analysis for a real wellbore.

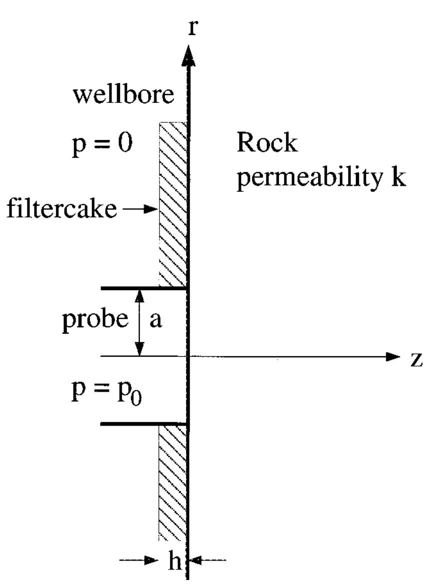

FIG. 2. Schematic of the simplified geometry considered in Secs. II–IV. The probe is clamped against a plane rock surface $z=0$, and the problem is axisymmetric about the z axis.

II. THE LAPLACE EQUATION AND BOUNDARY CONDITIONS

The rock has permeability k, so that the fluid velocity within the rock is given by Darcy's law

$$\mathbf{u}=-\frac{k}{\mu}\nabla p, \tag{1}$$

where μ is the fluid viscosity. Hence, by incompressibility

$$\nabla^2 p=0. \tag{2}$$

On $z=0$ the pore pressure $p=p_0$ over the surface of the probe $r<a$, but we must determine the boundary condition to be applied for $r>a$ where the rock surface is covered by a filtercake. The filtercake thickness is h, with $h\ll a$ so that within the filtercake we need consider only one-dimensional filtrate flow normal to the filtercake surface. The filtrate velocity u_z in the z direction is

$$u_z=\frac{k_c\Delta p}{\mu h}, \tag{3}$$

where

$$\Delta p=p_w-p|_{z=0}=-p|_{z=0} \tag{4}$$

is the jump in pressure across the filtercake and varies with r. But by continuity this flow through the filtercake must equal the Darcy flow within the rock normal to the surface $z=0$, and so

$$u_z=-\frac{k_c p}{\mu h}=-\frac{k}{\mu}\frac{\partial p}{\partial z},\quad z=0,\quad r>a. \tag{5}$$

We scale all lengths with respect to the probe radius a. Thus we wish to find a solution of the Laplace equation (2) which decays at infinity and which satisfies the boundary conditions

$$p=p_0,\quad z=0,\quad r<1, \tag{6a}$$

$$\frac{\partial p}{\partial z}=Kp,\quad z=0,\quad r>1, \tag{6b}$$

$$p\rightarrow 0,\quad z\rightarrow\infty, \tag{6c}$$

where

$$K=\frac{k_c a}{kh}. \tag{7}$$

The problem is axisymmetric, so that we seek a solution independent of θ of the form

$$p=p_0\int_0^\infty B(s)J_0(sr)e^{-sz}\,ds, \tag{8}$$

where J_0 is a Bessel function and $B(s)$, which is unknown, must be chosen such that

$$\int_0^\infty B(s)J_0(sr)ds=1,\quad r<1, \tag{9a}$$

$$\int_0^\infty (s+K)B(s)J_0(sr)ds=0,\quad r>1. \tag{9b}$$

III. SOLUTION OF THE LAPLACE EQUATION

A. Impermeable filtercake: $K=0$

The case $K=0$ corresponds to the problem of an electrified disk at uniform potential in free space, with the potential vanishing at infinity. This is a classical problem[5] with solution

$$B(s)=\frac{2\sin s}{\pi s}. \tag{10}$$

Thus when $K=0$, if we insert (10) into expression (8) for the pore pressure p, we find that on the plane $z=0$,

$$p=\frac{2p_0}{\pi}\int_0^\infty s^{-1}J_0(sr)\sin s\,ds, \quad z=0. \tag{11}$$

The integral (11) is a special case of the Weber–Schafheitlin discontinuous integral, with

$$p=\frac{2p_0}{\pi}\sin^{-1}(r^{-1}), \quad r>1, \quad z=0 \tag{12a}$$

$$=p_0, \quad r<1, \quad z=0. \tag{12b}$$

In the far field $p\sim 2p_0/\pi r$, corresponding to (spherically symmetric) radial flow toward a point sink.

The total volumetric flow rate of fluid through the probe when the filtercake is impermeable is

$$Q_p^{(0)}=2\pi a^2\int_{r=0}^1 ru_z\,dr$$
$$=\frac{4ap_0k}{\mu}\int_{s=0}^\infty s^{-1}J_1(s)\sin s\,ds=\frac{4ap_0k}{\mu}, \tag{13}$$

where in (13) we have used

$$\int_0^z tJ_0(t)dt=zJ_1(z), \tag{14}$$

and the final integral of (13) is again a special case of the Weber–Schafheitlin discontinuous integral (e.g., Eq. 11.4.35 of Ref. 9). The definition (13) gives the flow through the probe in the direction z increasing, and is thus in the opposite direction to Q depicted in Fig. 1. As indicated previously, $p_0<0$ and hence $Q_p^{(0)}<0$.

B. Series solution for arbitrary K

To solve the dual integral equations (9) we follow the standard procedure set down by Tranter,[5] and seek an expansion of the unknown $B(s)$ in the form

$$(s+K)B(s)=s^{1-\beta}\sum_{m=0}^\infty a_mJ_{2m+\beta}(s), \tag{15}$$

where β is an arbitrary parameter which later will be required to satisfy $\beta>0$. The coefficients $\{a_m\}$ satisfy an infinite linear system of equations

$$\sum_{m=0}^\infty a_m\int_0^\infty \frac{s^{1-2\beta}}{s+K}J_{2m+\beta}(s)J_{2n+\beta}(s)ds$$
$$=\frac{\delta_{0n}}{2^\beta\Gamma(\beta+1)}, \quad n=0,1,2,\ldots. \tag{16}$$

The integrals on the left-hand side of (16) may be evaluated numerically using routines developed by Lucas.[6,7] If the infinite set of equations (16) is truncated, the resulting finite set of linear equations for the $\{a_m\}$ may be solved numerically. In the case $K=0$ we know from (10) that

$$(s+K)B(s)=\frac{2\sin s}{\pi}=\left(\frac{2s}{\pi}\right)^{1/2}J_{1/2}(s). \tag{17}$$

In this case if we choose $\beta=\frac{1}{2}$ only the term $m=0$ in the expansion (15) is nonzero, and all the computations presented here were performed with this choice for β.

Once we have solved (16) to determine the $\{a_m\}$, we know the expansion (15) for $B(s)$. The pressure p is, by (8),

$$p=p_0\int_{s=0}^\infty \frac{s^{1-\beta}e^{-sz}}{s+K}\sum_{m=0}^\infty a_mJ_{2m+\beta}(s)J_0(sr)ds. \tag{18}$$

The Darcy velocity (1) within the rock is

$$u_r=\frac{p_0k}{a\mu}\int_{s=0}^\infty \frac{s^{2-\beta}e^{-sz}}{s+K}\sum_{m=0}^\infty a_mJ_{2m+\beta}(s)J_1(sr)ds, \tag{19a}$$

$$u_z=\frac{p_0k}{a\mu}\int_{s=0}^\infty \frac{s^{2-\beta}e^{-sz}}{s+K}\sum_{m=0}^\infty a_mJ_{2m+\beta}(s)J_0(sr)ds, \tag{19b}$$

and the total volumetric flow rate of fluid through the probe is

$$Q_p=2\pi a^2\int_{r=0}^1 ru\,dr, \quad z=0$$
$$=2\pi a^2\left(\frac{p_0k}{\mu a}\right)\int_{s=0}^\infty \frac{s^{1-\beta}}{s+K}\sum_{m=0}^\infty a_mJ_{2m+\beta}(s)J_1(s)ds, \tag{20}$$

where we have again used (14). The total volumetric flow from the filtercake into the rock at the plane $z=0$ is, by (19b),

$$Q_c=Q_c^{(1)}=2\pi a^2\left(\frac{p_0k}{\mu a}\right)\lim_{R\to\infty}\int_{r=1}^R r\,dr\int_{s=0}^\infty \frac{s^{2-\beta}}{s+K}$$
$$\times\sum_{m=0}^\infty a_mJ_{2m+\beta}(s)J_0(sr)ds, \tag{21}$$

where the upper limit of integration with respect to r has been taken to be R, which we shall ultimately allow to go to infinity. An alternative expression for the flux through the filtercake itself may be derived from (3), and leads to

$$Q_c^{(2)} = -2\pi a^2 K\left(\frac{p_0 k}{\mu a}\right)\lim_{R\to\infty}\int_{r=1}^{R} r\,dr \times\int_{s=0}^{\infty}\frac{s^{1-\beta}}{s+K}\sum_{m=0}^{\infty} a_m J_{2m+\beta}(s)J_0(sr)\,ds. \quad (22)$$

Using (9b) we can show that $Q_c^{(1)} - Q_c^{(2)} = 0$, as expected, so that we may define $Q_c \equiv Q_c^{(1)} = Q_c^{(2)}$. Performing the integral with respect to r in (22), we obtain

$$Q_c^{(2)} = -2\pi a^2 K\left(\frac{p_0 k}{\mu a}\right)\lim_{R\to\infty}\int_{s=0}^{\infty}[RJ_1(sR) - J_1(s)]\frac{s^{-\beta}}{s+K}\sum_{m=0}^{\infty} a_m J_{2m+\beta}(s)\,ds. \quad (23)$$

For suitable functions $g(s)$ which decay to zero as $s\to\infty$, integration by parts gives (Ref. 8, p. 344)

$$\int_0^\infty J_1(sR)g(s)\,ds = -\left[\frac{J_0(sR)}{R}g(s)\right]_{s=0}^{\infty} + \int_0^\infty \frac{J_0(sR)}{R}g'(s)\,ds \quad (24a)$$

$$= \frac{g(0)}{R} + O(R^{-3/2}), \quad R \gg 1, \quad (24b)$$

where we assume that the derivative g' is well-behaved and decays as $s\to\infty$ sufficiently rapidly that the integral in (24a) exists. Hence, by (23) and (24)

$$Q_c^{(2)} = 2\pi a^2 K\left(\frac{p_0 k}{\mu a}\right)\left[\int_{s=0}^{\infty}\frac{s^{-\beta}}{s+K}\sum_{m=0}^{\infty} a_m J_{2m+\beta}(s)J_1(s)\,ds - \frac{a_0}{2^\beta K\Gamma(1+\beta)}\right]. \quad (25)$$

Alternatively, the same steps applied to (21) lead to

$$Q_c^{(1)} = -2\pi a^2\left(\frac{p_0 k}{\mu a}\right)\int_{s=0}^{\infty}\frac{s^{1-\beta}}{s+K}\sum_{m=0}^{\infty} a_m J_{2m+\beta}(s)J_1(s)\,ds. \quad (26)$$

Comparing (26) and (20) we see that $Q_p = -Q_c$. Thus all of the fluid withdrawn from the rock by the probe is replaced by filtrate from the wellbore, rather than by pore fluid drawn from the reservoir at infinity. If this filtrate reaches the probe, the fluid sample will be contaminated,

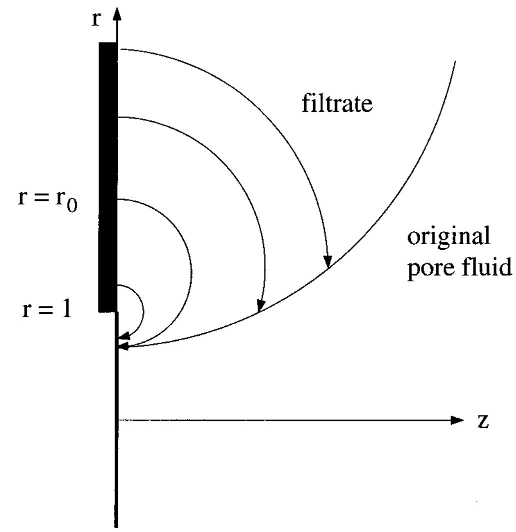

FIG. 3. Sketch showing the front which divides invading filtrate from original pore fluid at time t_0. Fluid from the filtercake at $r=r_0$ has just arrived at the probe, and there is steady flow of filtrate to the probe from filtercake covering the region $1<r\le r_0$.

which is not desired. We shall therefore be interested later in the amount of filtrate which has arrived at the probe having traveled all the way from the filtercake covering the region $1<r<r_0$, and we define

$$Q_c^f(r_0) = 2\pi a^2\int_{r=1}^{r_0} u_z r\,dr, \quad z=0$$
$$= 2\pi a^2 K\int_0^\infty [J_1(s) - r_0 J_1(sr_0)]\frac{s^{-\beta}}{s+K} \times\sum_m a_m J_{2m+\beta}(s)\,ds. \quad (27)$$

The fraction of flow reaching the probe that comes from the region $1<r<r_0$ is therefore

$$F_f(r_0) = Q_c^f/Q_c. \quad (28)$$

We may integrate the fluid velocity (19) to compute trajectories of filtrate fluid particles as they travel from the filtercake at $z=0$ toward the probe, thereby determining the time a filtrate fluid particle takes to reach the probe. At any given time there will be a front separating invading filtrate from the original pore fluid, as sketched in Fig. 3. In Fig. 3 fluid from the cake at $(r,z)=(r_0,0)$ has just arrived at the probe at time t_0. There is by this time a steady flow of filtrate into the probe from the region $1<r\le r_0$ of the filtercake, so that at time t_0 the fraction of the flow into the probe that is filtrate from the wellbore is $F_f(r_0)$ (28).

C. Green's function for $K \ll 1$

The flux through an impermeable filtercake is zero, but we saw in Sec. III B that as soon as $K>0$ the total volumetric flow rate Q_c through the filtercake is equal to the flow rate $-Q_p$ through the probe, which for $K\ll 1$ may be approximated by (13). This discontinuity in Q_c merits further investigation.

We now rescale all lengths by K^{-1}, so that lengths have now been scaled by aK^{-1}. The pressure field still satisfies the Laplace equation expressed in terms of the new scaled variables $(\hat{r},\hat{z})$. In the limit $K\to 0$ the probe, which occupies the disk $\hat{r}<K$, will appear as a point sink, and we may neglect the fine details of the pressure distribution over the probe surface. This allows us to modify the boundary condition over the probe.

We now assume that the entire surface $z=0$ is covered by a filtercake, with $p_w=0$ in $\hat{r}>K$ and $p_w=p_1$ in $\hat{r}<K$. We shall discuss later the relation between p_1 and p_0. The boundary condition over the plane $z=0$ is

$$\frac{\partial p}{\partial \hat{z}}=p-p_w=p, \quad \hat{r}>K \tag{29a}$$

$$=p-p_1, \quad \hat{r}<K. \tag{29b}$$

We look for a solution of the Laplace equation of the form

$$p=p_1\int_0^\infty C(s)J_0(s\hat{r})\exp(-s\hat{z})\,ds \tag{30}$$

and the boundary condition (29) becomes

$$\int_0^\infty (s+1)C(s)J_0(s\hat{r})\,ds=0, \quad \hat{r}>K \tag{31a}$$

$$=1, \quad \hat{r}<K. \tag{31b}$$

The left-hand sides of (31) are identical, unlike the mixed boundary conditions (9) of Sec. II. One can therefore solve for $C(s)$ by means of the Hankel inversion theorem, but it is easier to note, by comparison with the Weber–Schaftheitlin discontinuous integral (e.g., Eq. 11.4.42 of Ref. 9) that if we take

$$(1+s)C(s)=KJ_1(Ks), \tag{32}$$

then

$$p=Kp_1\int_0^\infty \frac{J_1(Ks)}{1+s}J_0(s\hat{r})\exp(-s\hat{z})\,ds \tag{33}$$

satisfies the boundary conditions (31).

On $z=0$ the Darcy velocity through the probe is

$$u_z=-\frac{kK}{a\mu}\frac{\partial p}{\partial \hat{z}}=\frac{kK^2p_1}{a\mu}\int_0^\infty \frac{sJ_1(Ks)}{1+s}J_0(s\hat{r})\,ds \tag{34}$$

so that the total volumetric flow rate through the probe is

$$Q_p=\frac{2\pi a^2}{K^2}\int_0^K \hat{r}u_z\,d\hat{r}$$

$$=\frac{2\pi akp_1K}{\mu}\int_0^\infty \frac{J_1(sK)J_1(sK)}{1+s}\,ds. \tag{35}$$

But if $K\ll 1$ the dominant contribution to the final expression comes from $s\gg 1$, so that we may replace the final integral in (35) by

$$Q_p\approx\frac{2\pi akp_1K}{\mu}\int_0^\infty \frac{J_1(sK)J_1(sK)}{s}\,ds=\frac{\pi Kp_1ka}{\mu}. \tag{36}$$

If we wish this to equal $Q_p^{(0)}$, given by (13), we require

$$p_1=\frac{4p_0}{\pi K}. \tag{37}$$

The pressure p_1 outside the filtercake in the region $0\leqslant\hat{r}<K$ in this modified problem is therefore $O(p_0/K)$ and much larger than the pressure p_0 in the original problem in which there is no filtercake in $0\leqslant\hat{r}<K$. Since the filtercake permeability is small compared to that of the rock, the flow through the filtercake over the region $0\leqslant\hat{r}<K$ will be approximately uniform, with Darcy velocity $Q_p/\pi a^2$. The pressure drop across the filtercake is therefore p_1, to leading order in *K*, and the pressure at the rock surface will be $O(p_0)$ but will vary with *r*.

Expression (33) for *p* is easily evaluated in the limit $\hat{z}\gg 1$ when $K\ll 1$. We expand J_1 as a power series in its argument *sK* to obtain

$$p=\frac{K^2p_1}{2}\int_0^\infty \frac{s}{1+s}J_0(s\hat{r})\exp(-s\hat{z})\,ds+O(K^3). \tag{38}$$

As $z\to\infty$ the dominant part of the integral comes from $s=O(z^{-1})$, so that

$$p\sim\frac{K^2p_1}{2}\int_0^\infty sJ_0(s\hat{r})\exp(-s\hat{z})\,ds=\frac{K^2p_1\hat{z}}{2(\hat{z}^2+\hat{r}^2)^{3/2}} \tag{39}$$

(Ref. 10, p. 1261, Eq. 10.3.20). This represents a dipole pressure field, which should be contrasted with the monopole-like pressure field when $K=0$. For fixed *z*, *p* decays as r^{-3} as $r\to\infty$. More care is required to investigate the limit $\hat{r}\to\infty$ when $z=0$. Integrating first $sJ_0(s\hat{r})$ and then $J_1(s\hat{r})$ in (33) by parts, and then expanding in powers of *K*, we obtain

$$\frac{p}{p_1}=-\frac{K}{\hat{r}}\int_0^\infty J_1(s\hat{r})e^{-s\hat{z}}\left\{\frac{KJ_0(Ks)}{1+s}-\frac{J_1(Ks)(2+3s)}{s(1+s)^2}-\frac{J_1(Ks)\hat{z}}{1+s}\right\}ds$$

$$=-\frac{K^2}{\hat{r}^2}\int_0^\infty J_0(s\hat{r})\left\{\frac{s-1}{(1+s)^2}+\frac{\hat{z}(s-1)}{(1+s)^3}\right\}ds+O(K^3)+O(z^2)$$

$$=\frac{K^2(1+\hat{z})}{2\hat{r}^3}+O(K^3)+O(z^2), \quad \hat{r}\to\infty. \tag{40}$$

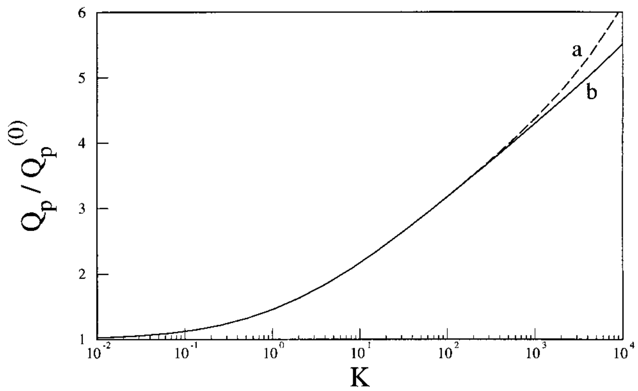

FIG. 4. Total volumetric flow rate Q_p into the probe, scaled by the flow rate $Q_p^{(0)}$ when $K=0$, as a function of the filtercake permeability parameter K (7). Number of terms in the truncated expansion (15): (a) $M=10$; (b) $M=30$.

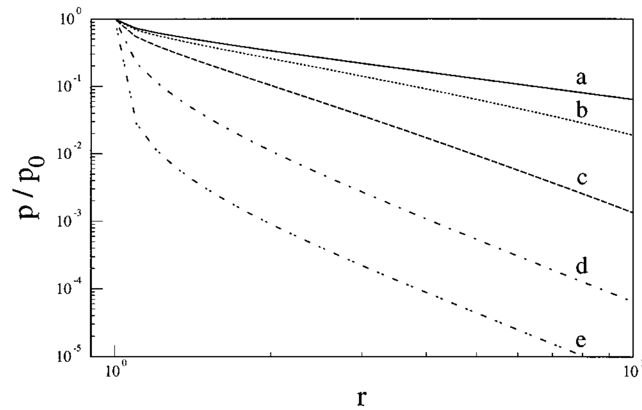

FIG. 5. Pressure p on the surface $z=0$, as a function of radial position r, after scaling by the probe pressure p_0. Filtercake permeability parameter (7): (a) $K=0$, (b) $K=0.1$, (c) $K=1$, (d) $K=10$, (e) $K=100$.

We see that although (39) was incorrect on $z=0$, it appears to have correctly captured $\partial p/\partial \hat{z}$ on the boundary.

Thus as $K\rightarrow 0$ there is an inner region, around the probe, for which the pressure field is similar to that for the case $K=0$. However, on a length scale $r\sim O(K^{-1})$ all streamlines start from the filtercake $z=0$. This breakdown of the solution for $K=0$ at distances $O(K^{-1})$ from the probe can be predicted by simple arguments. If the pressure p on $z=0$ could be approximated for $0<K\ll 1$ by the result (12) for $K=0$, the total volumetric flow rate Q_c through the filtercake over the annulus $1<r<R$ would be

$$Q_c=2\pi a^2\int_{r=1}^{R} r u_z\, dr=-\frac{2\pi a^2 k_c}{\mu h}\int_{r=1}^{R} rp\, dr \sim -\frac{4p_0 RakK}{\mu}, \quad R\gg 1. \qquad (41)$$

This does not converge as $R\rightarrow\infty$, and is comparable in magnitude to the flow $Q_p^{(0)}$, given by (13), when

$$R\sim K^{-1}. \qquad (42)$$

We conclude that at sufficiently large distances $r\sim O(K^{-1})$ from the probe the presence of a filtercake that is slightly permeable (rather than completely impermeable) has a considerable effect upon the pressure distribution within the porous rock, and the solution for $K=0$ breaks down.

IV. NUMERICAL RESULTS

In order to determine the solution, the expansion (15) for $B(s)$ was truncated at $m=M-1$. The M linear equations (16) were then solved by the NAG routine F04ATF. The total volumetric flow rate through the filtercake was evaluated using both (23) for $Q_c^{(2)}$ and (26) for $Q_c^{(1)}$. The results agreed to seven significant figures. However, the integral (23) could be evaluated much more rapidly, presumably because of improved convergence as $s\rightarrow\infty$.

We first consider how the volumetric flow rate Q_p into the probe varies (for fixed suction pressure p_0) as a function of K. Figure 4 shows Q_p, scaled by $Q_p^{(0)}$, as a function of K. We see that ten terms in the series (15) (i.e., $M=10$) suffice to give good results up to $K=200$, since there is negligible change in the computed value for Q_p when M is increased to $M=30$.

We next consider the pressure p on the surface $z=0$, as given by (18). This is shown, scaled by p_0, in Fig. 5, as a function of r, for various values of K.

One measure of the accuracy of the solution of the dual integral equations is to consider how well the boundary condition (9a) is satisfied over $0\leqslant r<1$. When $K=0.1$, $M=10$ gives $p=1.0000$ over this interval. If $K=1$, then $M=10$ gives errors as large as 2×10^{-5}, which are eliminated when $M=30$. If $K=100$, then $M=20$ gives errors in p of order 3×10^{-4}, which are reduced to 1.2×10^{-4} when $M=30$. For $r>1$ these two values of M give results which are identical to five significant figures.

As K increases, so flow becomes localized near the probe. Figure 6 shows the fraction $F_f(r_0)$ of the total flow through the filtercake which passes through the region $1\leqslant r\leqslant r_0$. Figure 7 shows streamlines corresponding to three cases $K=0.1$, 1.0, and 10.0. Note that in the limit $K\rightarrow 0$ the pressure gradient $\partial p/\partial z\rightarrow 0$ on the plane $z=0$. Flow within

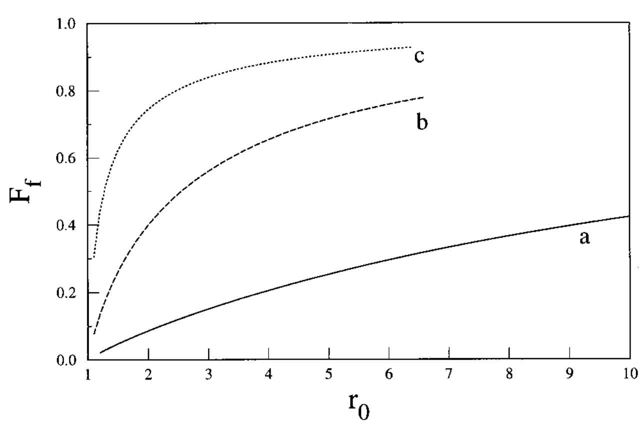

FIG. 6. The fraction F_f of the total flow through the filtercake which has come from the region $1<r<r_0$, as a function of r_0. (a) $K=0.1$, (b) $K=1.0$, (c) $K=10.0$.

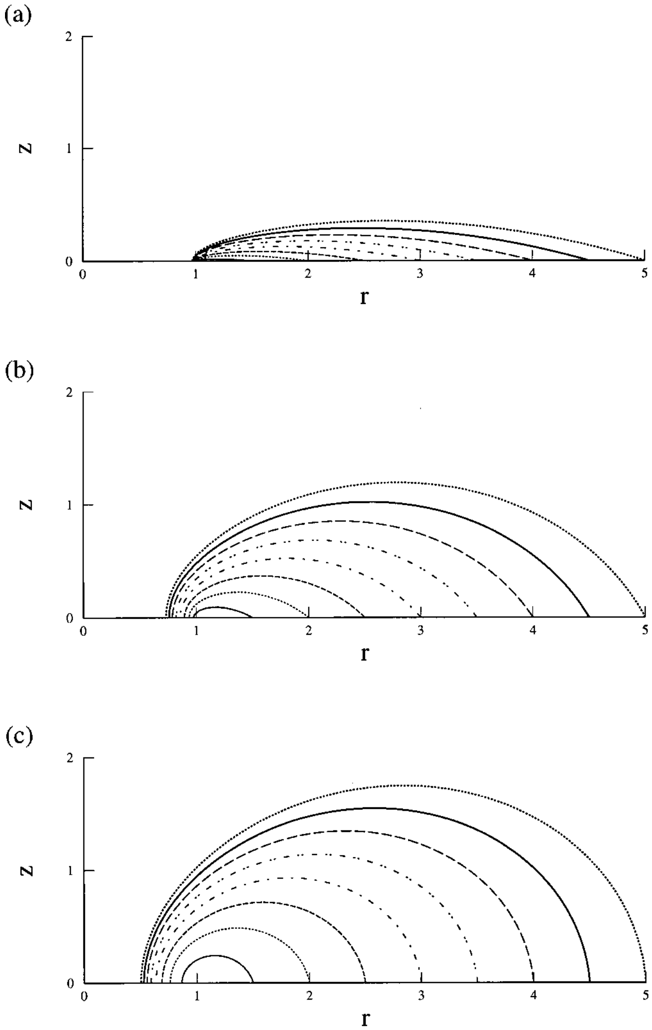

FIG. 7. Streamlines for (a) $K=0.1$, (b) $K=1.0$, (c) $K=10.0$.

the rock originating from the filtercake at any fixed r_0 is increasingly confined to a narrow region adjacent to the surface and enters the probe closer to its outer edge $r=1$ as $K\to 0$. When $K\ll 1$ there will be a region $1<r\ll K^{-1}$ over which the pressure on the plane $z=0$ may be approximated by (12), so that the time taken by a fluid particle to travel from $r=r_0$ to $r=1$ is approximately

$$t_0=\frac{2\pi a^3\phi}{3Q_p}(r_0^2-1)^{3/2},\quad 1<r_0\ll K^{-1}, \tag{43}$$

where the rock porosity ϕ is introduced because the velocity of a fluid particle is higher than the Darcy velocity (19) by a factor ϕ^{-1}.

We have already scaled lengths by the probe radius a, and we now scale time by $a^3\phi/Q_p$. Thus we are effectively comparing results for different values of K at times corresponding to an equal amount of fluid withdrawn through the probe. Figure 8 shows the time $t_0(r_0)$ at which a fluid particle starting at $(r,z)=(r_0,0)$ reaches the probe, for three different values of K. Also shown is the asymptote (43) for the limit $K\to 0$.

Figure 9 reinterprets the results of Figs. 6 and 8. At time t_0, filtrate from the filtercake covering $1<r<r_0(t_0)$ has

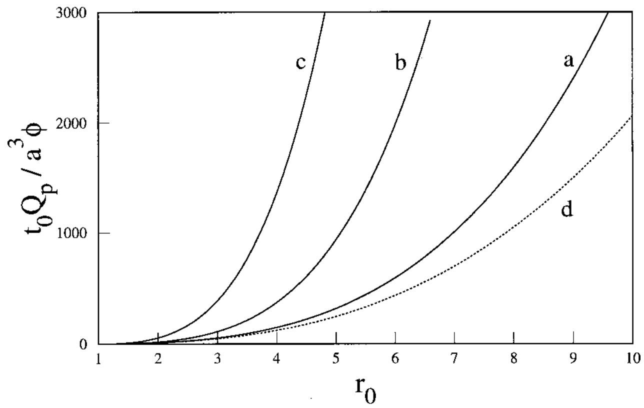

FIG. 8. The nondimensional time $t_0Q_p/a^3\phi$ that a fluid particle takes to travel from $(r,z)=(r_0,0)$ to the probe. (a) $K=0.1$, (b) $K=1.0$, (c) $K=10.0$. (d) The asymptotic prediction (43) in the limit $K\to 0$.

reached the probe, but filtrate from $r>r_0(t_0)$ has yet to do so. The fraction of fluid passing into the probe which is filtrate, rather than original pore fluid, is therefore $F_f(r_0(t_0))$ (28). The nondimensional time $t_0Q_p/a^3\phi$ required for F_f to become large increases as $K\to 0$, and from the analysis of Sec. III C we expect this time to increase as K^{-3}.

V. APPLICATION TO A WELLBORE

A. Typical filtercake properties

Drilling fluid filtercakes are formed by cross-flow filtration within the wellbore. Such filtercakes grow until the filtration rate is sufficiently low that further filtercake growth is prevented by the tangential flow of the drilling fluid adjacent to the filtercake.[11,12] Thus the filtercake thickness depends upon the shear rates within the fluid. Fordham and Ladva[13,14] found that the final steady flow rate through a filtercake formed from a water-based bentonite drilling fluid was typically in the range 0.2–1.0 μm s^{-1} when the filtration pressure was 1.93 MPa. The higher flow rate corresponds to steady Darcy flow through a uniform incompressible filtercake for which

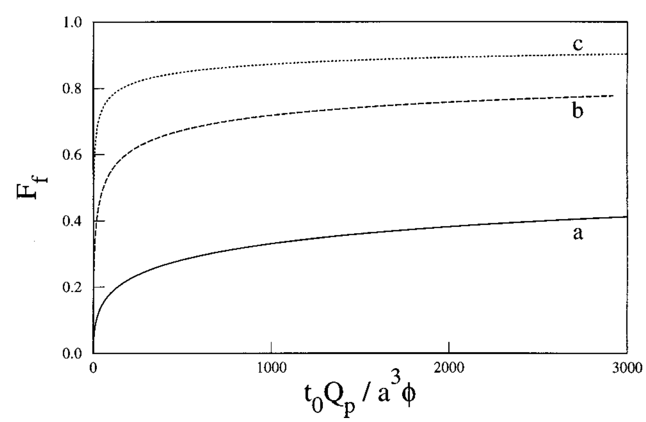

FIG. 9. The fraction F_f of liquid entering the probe at nondimensional time $t_0Q_p/a^3\phi$ which has come through the filtercake [in the region $1<r<r_0(t_0)$], rather than being part of the original pore fluid within the rock at $t=0$. (a) $K=0.1$, (b) $K=1.0$, (c) $K=10.0$.

$$\frac{k_c}{\mu h} \approx 5\times 10^{-13}\,\mathrm{m\,Pa^{-1}\,s^{-1}}. \tag{44}$$

We assume that the filtrate viscosity is $\mu=0.001$ Pa s, corresponding to water at 20 °C, so that

$$\frac{k_c}{h} \approx 5\times 10^{-16}\,\mathrm{m}. \tag{45}$$

Bentonite filtercakes are compressible, and so any estimate of the filtercake properties ought to vary with the differential pressure. In particular, if the probe lowers the pore pressure markedly within the rock, the differential pressure across the filtercake in the vicinity of the probe will increase and the filtercake will be compacted. Pore fluid will be squeezed from the filtercake,[4] and the hydraulic conductivity k_c/h of the filtercake will be reduced close to the probe. However, the analysis presented in this report is valid only for a filtercake with properties independent of radial position r. Although filtercakes formed from oil-based drilling fluids have lower permeabilities than those of water-based fluids, the filtercake is thought to be weaker and therefore thinner when formed under cross flow. The hydraulic resistance of such a filtercake is therefore typically only slightly more than that of a water-based filtercake.[15,16]

We assume a probe diameter $2a=1$ in., so that $a=0.0127$ m. If we assume the rock permeability to be $k=100$ mDarcy$=10^{-13}\,\mathrm{m}^2$, we conclude that

$$K=\frac{k_c a}{kh}\approx 6\times 10^{-5}. \tag{46}$$

Thus if there is a good filtercake, and if the seal between the probe and the filtercake is sound, any filtrate flow through the filtercake should be negligibly small in the vicinity of the probe.

A typical flow rate through a probe in the field would be of order 30 l/h.[17] We take $Q_p=10^{-5}\,\mathrm{m^3\,s^{-1}}$, and assume a rock porosity $\phi=0.1$, so that the time scale $t_1=a^3\phi/Q_p$, which represents the typical time for pore fluid to move a distance comparable to the probe radius, is of order $t_1\approx 0.02$ s. This time scale t_1 is much shorter than a typical fluid sampling time t_s, which might be of the order to 2000 s,[17,18] so that a nondimensional sampling time is of order $\hat{t}_s=t_s/t_1=10^5$.

When K is very small, the arguments of Sec. III C leading to (42) imply that when fluid from the filtercake at $r_0=K^{-1}$ starts to reach the probe, a considerable proportion of the fluid flowing into the probe will be filtrate. We find from (43) that this occurs after a nondimensional time

$$\hat{t}_K=t_K/t_1\approx\frac{2}{3K^3}. \tag{47}$$

Taking $K=6\times 10^{-5}$ we find that this nondimensional time is $\hat{t}_K\approx 3\times 10^{12}$. Thus $\hat{t}_s\ll\hat{t}_K$, and very little filtrate will reach the probe in the time scale over which fluid is sucked from the rock.

B. A cylindrical wellbore

The analysis has so far assumed that the rock surface covered by filtercake is an unbounded plane, as depicted in Fig. 2. A real wellbore is a long cylinder (Fig. 1), with radius r_w (typically of order 0.1 m), and the above-presented results are inappropriate at distances away from the probe greater than or comparable to r_w.

We now consider a cylindrical wellbore. We follow arguments similar to those used to derive for the plane rock surface of Sec. III C the distance R (42) at which filtrate flux through a low-permeability filtercake affects the flow toward the probe.

If the filtercake is impermeable, we expect the probe to appear to be a point sink when observed from sufficiently far away. In the far field, to a first approximation, this point sink may be taken to be on the wellbore axis, rather than on the cylindrical wellbore/rock interface. In terms of a spherical radial coordinate r_s the pore pressure and Darcy velocity are

$$p=\frac{\mu Q_p}{4\pi k r_s},\quad u_r=\frac{Q_p}{4\pi r_s^2},\quad r_s\gg 1, \tag{48}$$

where $Q_p<0$. We assume (as in Sec. III C) that this pressure field is unaltered when the filtercake becomes slightly permeable.

We take a new set of cylindrical polar coordinates, with the z' axis along the axis of the wellbore and with the probe at $z'=0$. The filtercake area per unit length of the wellbore is $2\pi r_w$, and the pore pressure in the rock immediately adjacent to the cake may be approximated by (48) with $r_s\approx|z'|$ for $|z'|\gg r_w$. The total volumetric flow rate from the wellbore through the filtercake over the interval $-Z<z'<Z$ might be expected to be

$$Q_c=-\frac{2\pi r_w k_c}{\mu h}\int_{-Z}^{Z} p\,dz'\sim-\frac{r_w k_c Q_p}{kh}\log(Z/a), \tag{49}$$

where a cutoff has been introduced at $|z'|=a$ in order to avoid the singularity near the origin, where (48) breaks down. The magnitude of this flow rate Q_c will be comparable to that of the volumetric flow rate Q_p through the probe at a distance

$$Z\sim a\exp\left(\frac{kh}{r_w k_c}\right)=a\exp\left(\frac{a}{r_w K}\right). \tag{50}$$

This distance is much larger than the equivalent result (42) for a plane surface. The surface area of the wellbore grows only linearly with distance from the probe, unlike the quadratically growing surface area of the plane rock surface considered in Secs. II–IV. In consequence, the filtrate that flows from the wellbore into the rock has less effect upon the unperturbed flow (48) than in the plane case. For typical sampling times and filtercake hydraulic resistances, flow of filtrate through the plane filtercake of Sec. III is negligibly small, as discussed in Sec. V A. We conclude that filtrate flow is even smaller in the cylindrical geometry of a real oil well.

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