A Systematic Approach to Modeling Impedances and Current Distribution in Planar Magnetics

Minjie Chen[1], Mohammad Araghchini[1], Khurram K. Afridi[2], Jeffrey H. Lang[1]
Charles R. Sullivan[3] and David J. Perreault[1]

1. Massachusetts Institute of Technology
2. University of Colorado Boulder
3. Dartmouth College

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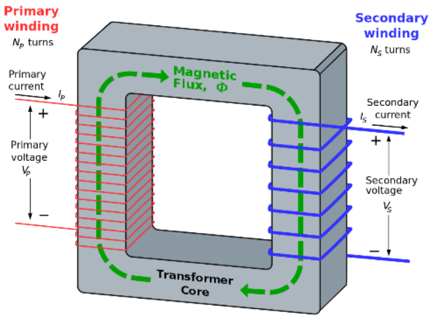
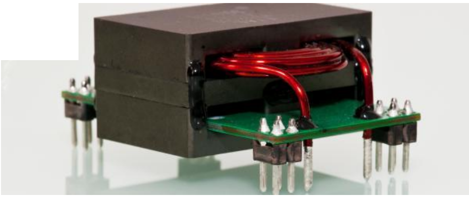
Planar Magnetics in Power Electronics

Magnetics with wire windings



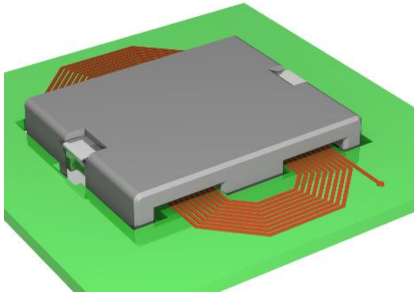


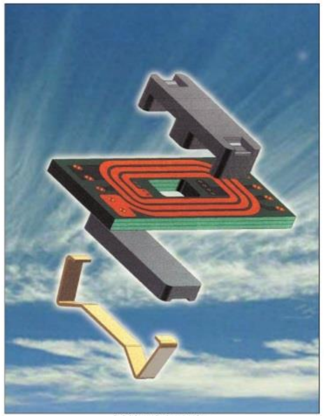
Advantages of Planar Magnetics:

1. High repeatability
2. Suitable for high frequency
3. Good thermal performance
4. High power density

Magnetics with planar windings

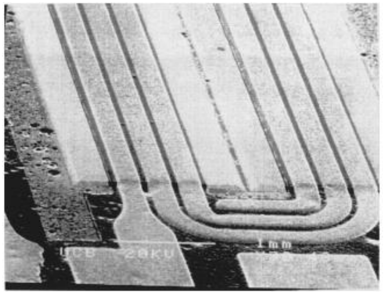
on PCB

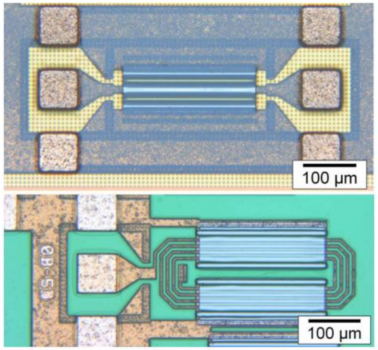




Exploded view of a planar transformer

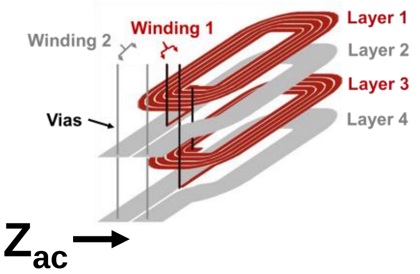
on Chip

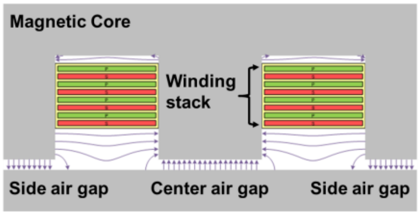


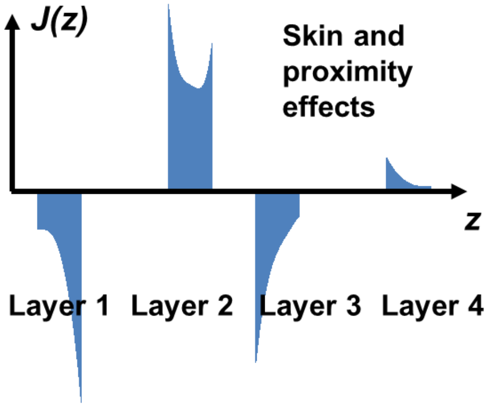


- L. Daniel, “Design of microfabricated inductors”, *IEEE Trans. Power Electron., 1999*
- D.S. Gardner, “Review of on-chip inductor structures with magnetic films”, *IEEE Trans. Magn., 2009*

Motivation







Current distribution in four layers

1. Skin- and proximity- effects makes the modeling challenging.

2. Solving Maxwell's equations for all design options is not practical.

3. Existing analytical models usually have specific assumptions and are not easy to use.

4. Finite element modeling are:

- Time consuming
- Not analytical

An analytical approach that is:

Accurate Fast Easy to Use Widely Applicable

Example Applications

1. What is the most appropriate way to interleave many layers?

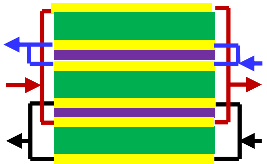
Alternating interleaved

1 & 3 in parallel
2 & 4 in parallel

Symmetric interleaved

1 & 4 in parallel
2 & 3 in parallel

More complicated?



…?

2. What is the most appropriate PCB spacing?

Thin Middle Spacing

Thick Middle Spacing

3. Other Design Options?

1) Leakage & Shielding Layers?
2) Hybrid Materials (Ni/Cu/FR4)?
3) Multi-Resonant Devices?
4) Etc…?

Two Commonly Shared Assumptions

Every model starts from assumptions …

(1) MQS assumption

- Assume $\frac{\partial E}{\partial t} = 0.$
- Applicable when the wavelength is much longer than the device size (usually lower than ~100MHz).

Magneto-Quasi-Static Maxwell's equations

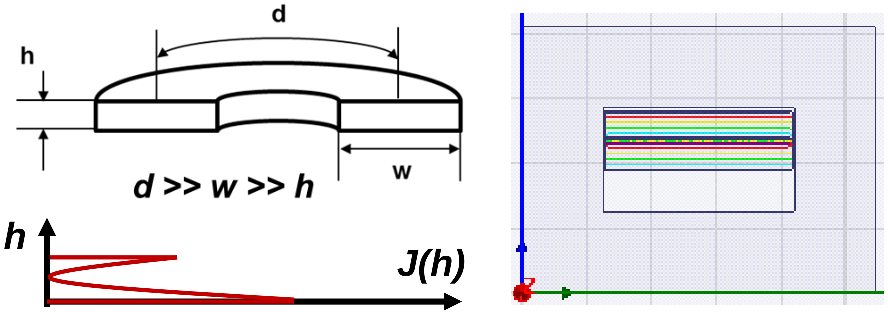
$$\begin{cases} \nabla E = \frac{\rho}{\varepsilon_0} \\ \nabla B = 0 \\ \nabla \times E = -\frac{\partial B}{\partial t} \\ \nabla \times B = \mu_0(J + \cancel{\varepsilon_0 \frac{\partial E}{\partial t}}) \end{cases}$$

Ignore the time evolution of the electric field

(2) 1-D assumption

- Fields vary only along the thickness direction.
- Applicable when the flux is guided by the magnetic core.

Magnetic core guides the flux



Skin and proximity effects change current distribution

Modeling a Single Conductor Layer

Field diffusion equations:

$$H_X(z) = \frac{H_T\, sinh(\Psi z) + H_B\, sinh(\Psi(h-z))}{sinh(\Psi h)}$$

Ampere's law:

$$\nabla \times H \;= J \;= \sigma E \qquad \Psi = \frac{1+j}{\delta} \qquad \delta = \sqrt{\frac{2}{\mu\omega\sigma}}$$

E field as a function of H and K:

$$\begin{cases} E_T \;= E_Y(h) = \dfrac{\Psi}{\sigma}\left(\dfrac{H_T e^{\Psi h} - H_B}{e^{\Psi h} - e^{-\Psi h}} - \dfrac{H_B - H_T e^{-\Psi h}}{e^{\Psi h} - e^{-\Psi h}}\right) \\ E_B \;= E_Y(0) = \dfrac{\Psi}{\sigma}\left(\dfrac{H_T - H_B e^{-\Psi h}}{e^{\Psi h} - e^{-\Psi h}} - \dfrac{H_B e^{\Psi h} - H_T}{e^{\Psi h} - e^{-\Psi h}}\right) \end{cases}$$

$$\boldsymbol{Z_a = \frac{\Psi(1 - e^{-\Psi h})}{\sigma(1 + e^{-\Psi h})}}$$

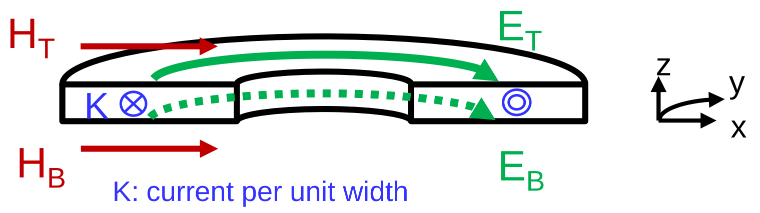
$$\boldsymbol{Z_b = \frac{2\Psi e^{-\Psi h}}{\sigma(1 - e^{-2\Psi h})}}$$

KVL/KLC relationships:

$$V/m \qquad \Omega \qquad A/m$$

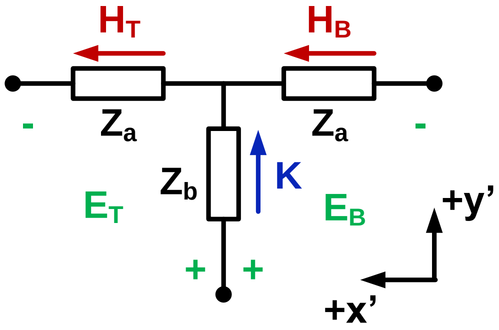
$$\begin{cases} E_T \;= Z_a H_T + Z_b K & \text{KVL} \\ E_B \;= Z_b K - Z_a H_B & \text{KVL} \\ \quad K = H_T - H_B & \text{KCL} \end{cases}$$

Electromagnetic Fields



K: current per unit width

Modular Layer Model



***H & K*:** through variables ~ unit (**A/m**)

***E*:** across variable ~ unit (**V/m**)

***Z_a, Z_b*:** impedances ~ unit (**Ω**)

Modeling Two Adjacent Layers

Intuition:

- *Two three-terminal networks*
- *Connected by the H field between them*

Faraday's Law and Field Continuity

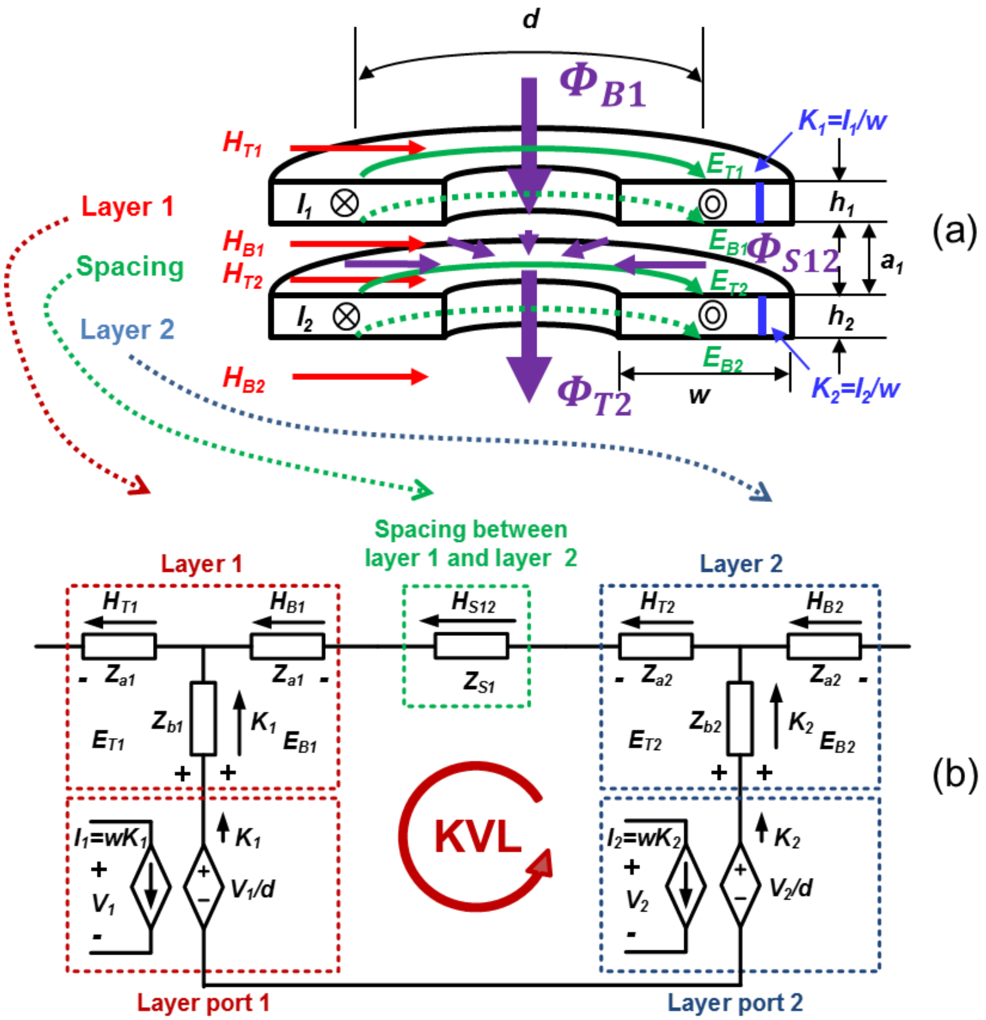
$$E_{B1}d - V_1 = -\frac{d\Phi_{B1}}{dt} \qquad E_{T2}d - V_2 = -\frac{d\Phi_{T2}}{dt}$$

$$\frac{d\Phi_{T2}}{dt} = \frac{d\Phi_{B1}}{dt} + \frac{d\Phi_A}{dt}$$

Flux Linking Two Layers:

An additional KVL equation

$$\underbrace{j\omega\mu_0 a_1}_{\Omega}\underbrace{H_{S12}}_{A/m} = \underbrace{\frac{V_2}{d} - E_{T2} - \frac{V_1}{d} + E_{B1}}_{V/m}$$

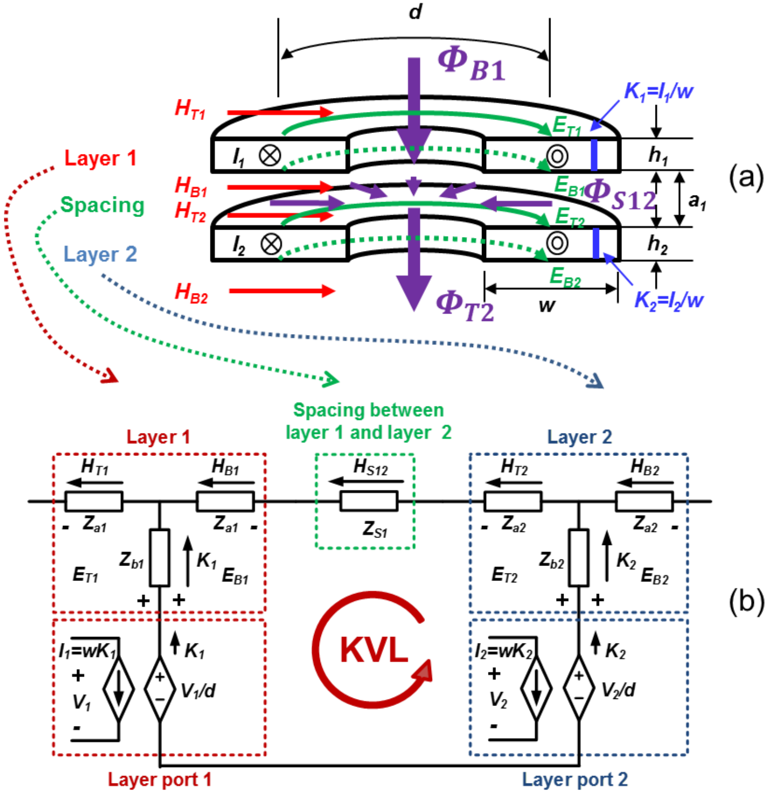


Circuit Domain ⟷ *Electromagnetic Domain*

Modeling Layers with Multiple Turns

Fields distributions in multiple-turns layers are linearly related to those in single-turn layers

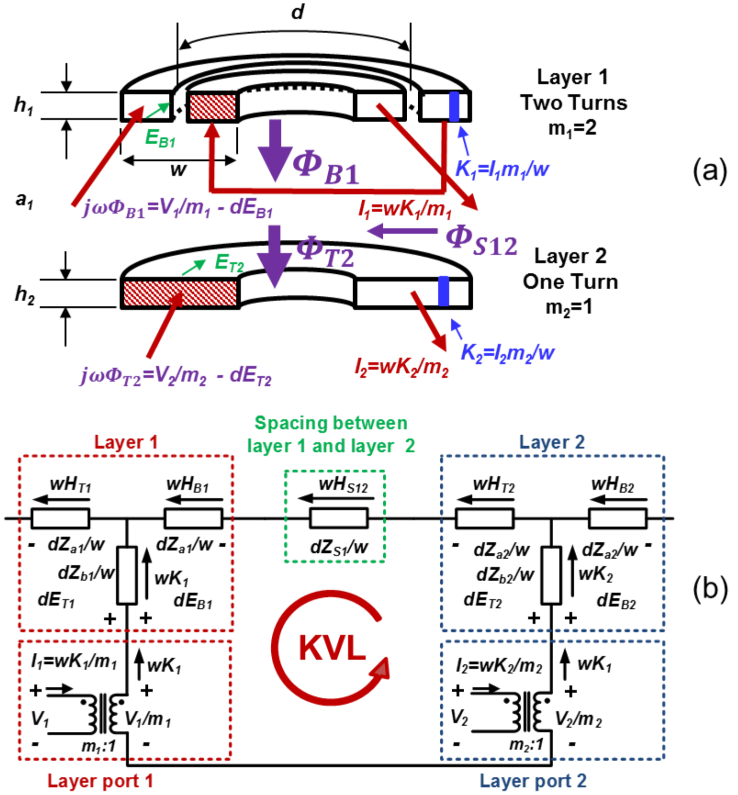
Multiple turns → Additional Linear Conversions



E&M Domain

Circuit Domain

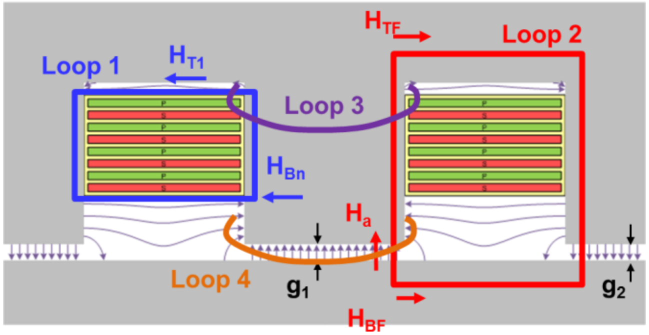
Intuitive Ideal Transformers



Modeling n Layers, the Core and the Air Gap

Additional Impedances Representing the Cores and Air Gaps

TOP Side



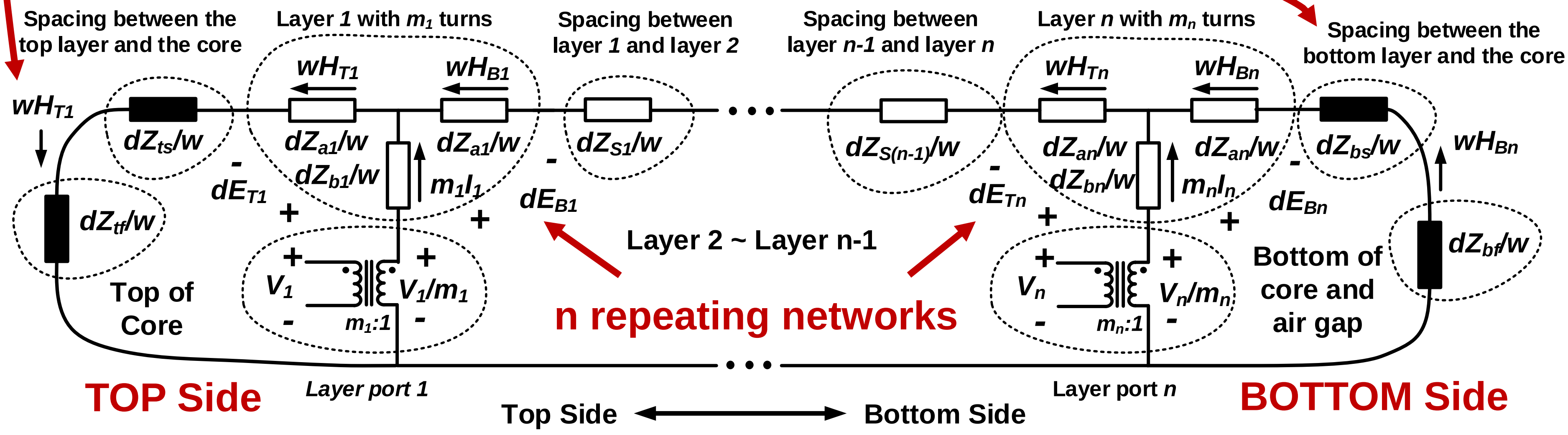
BOTTOM Side

1. Top Side

$$E_{T1} - \frac{V_1}{d} = -j\omega\mu_r c H_{T1} - j\omega\mu_0 b H_{T1}$$

2. Bottom Side

$$E_{Bn} - \frac{V_n}{d} = j\omega \frac{\mu_0 A_c w}{\left(g_1 + g_2 + \frac{\mu_0 A_c w}{\mu_r c d}\right) d} H_{Bn} + j\omega\mu_0 b H_{Bn}$$

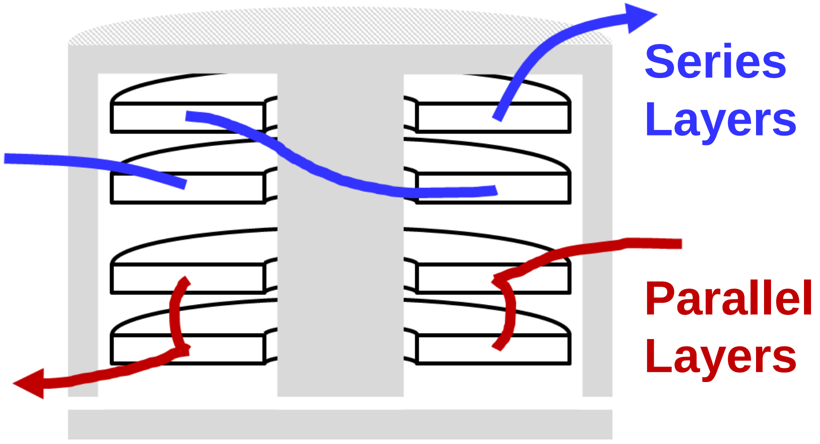


Modeling Electrical Interconnects (Vias)

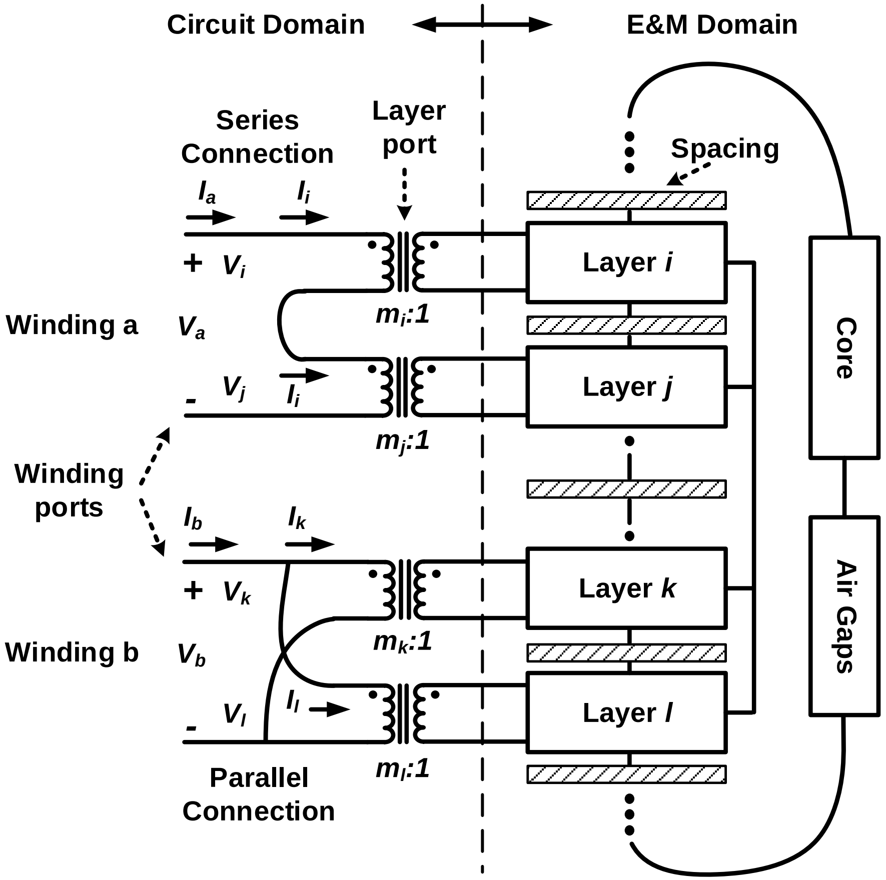
Modeling vias is equivalent to adding KVL, KCL constraints:

Layer i and Layer j in series
Layer k and Layer l in parallel

$$\begin{cases} V_i + V_j = V_a \\ V_k = V_l = V_b \end{cases} \qquad \begin{cases} I_i = I_j = I_a \\ I_k + I_l = I_b \end{cases}$$

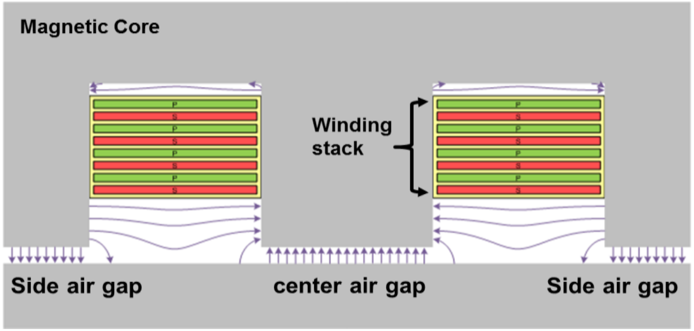


Connect the layer ports in the same pattern as they are in the real circuit

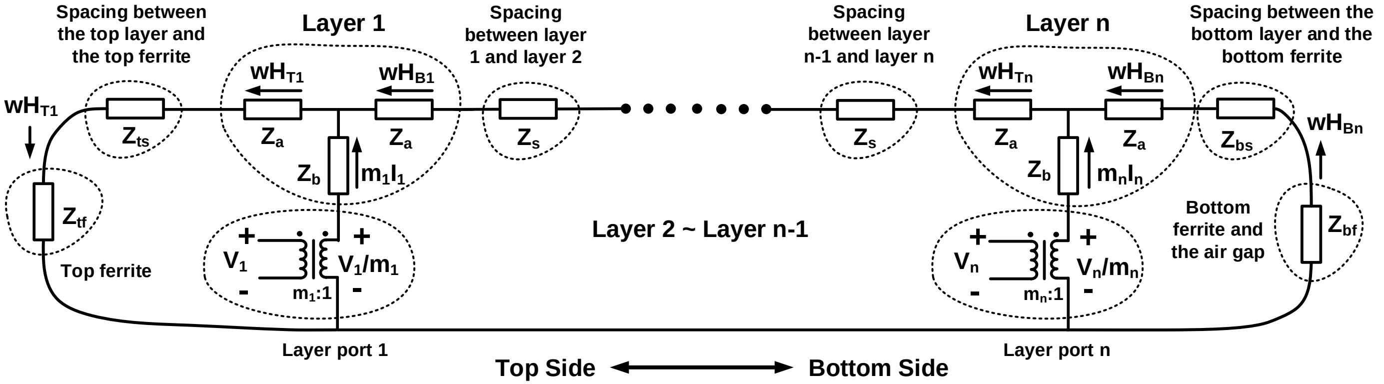


Summary of the Model

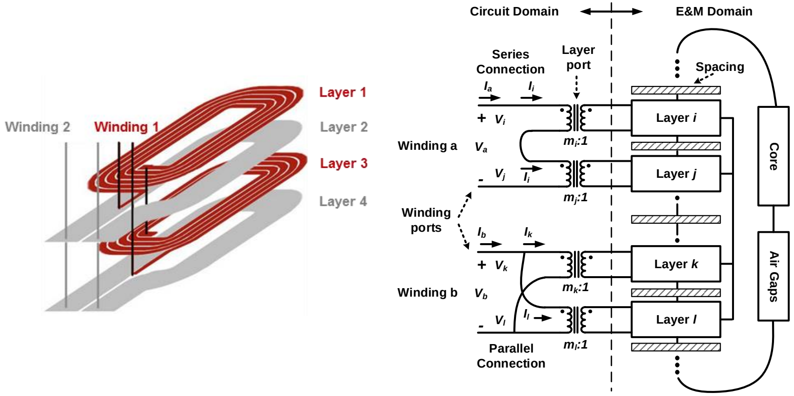
1 Geometry Information



2 Modular Layer Model



3 Cross Layer Connections

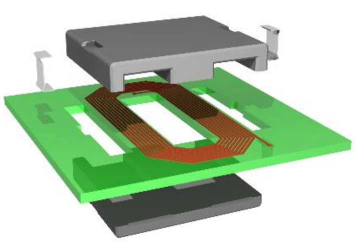


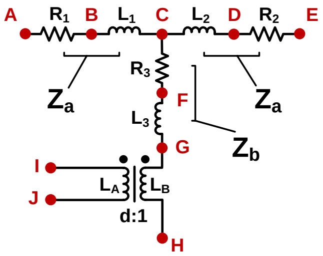
Use the model!

Use the Model Numerically

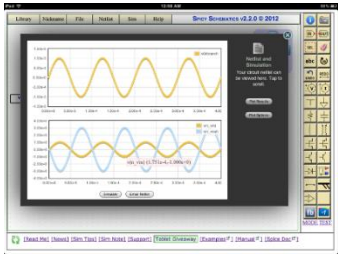
Netlist generation and full circuit simulation

Geometry → **Impedances** → **Netlists** → **Simulations (SPICE)**

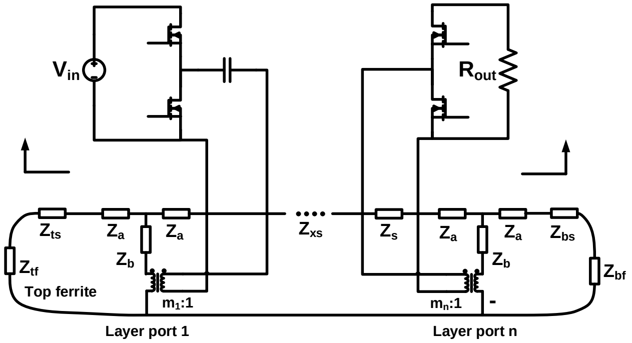


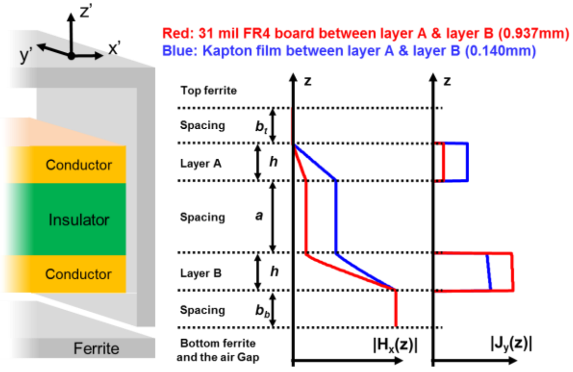


```
*----------Netlist for a single layer-------------------
*-----------1. Describing the Impedances-----------
*--Name---Node 1------Node 2-----------Value------
   R1        A         B          real(Z_a)
   L1        B         C          imag(Z_a)/ω
   L2        C         D          real(Z_a)
   R2        D         E          imag(Z_a)/ω
   R3        C         F          real(Z_a)
   L3        F         G          imag(Z_a)/ω
*----------2. Describing the ideal transformer-----
   LA        I         J          d^2
   LB        G         H          1
   K         LA        LB         1
```



→ **"Magnetics-in-the-Loop" Simulations** → **Visualizing H_X, E_Y, J_Y, just as in FEM**





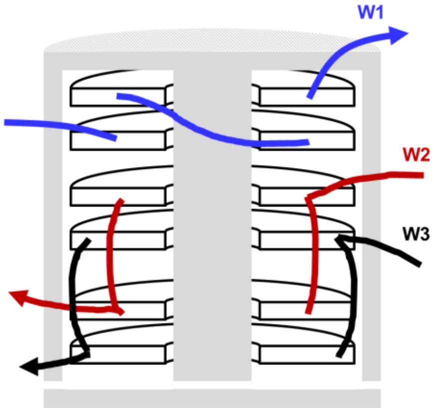
Use Python/Matlab scripts to rapidly generate the netlist ~~

Use SPICE to rapidly solve the netlist ~~

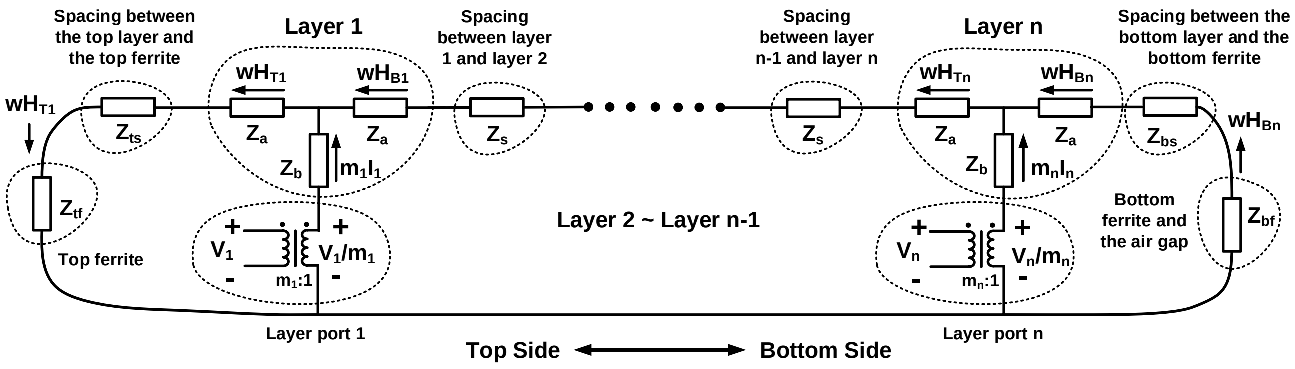
A GUI is under development ~~

Use the Model Analytically

Physical Structure



Modular Layer Model (SPICE netlist)



Simplify

Parameter Extractions Using:

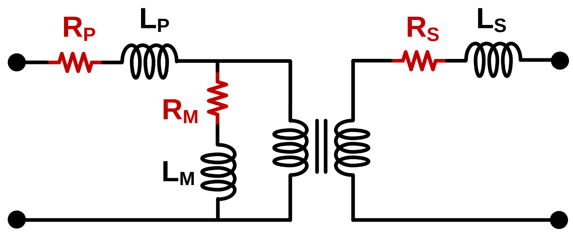
Open and Short Circuit *Simulations.*

- Impedance Matrix

$$V_{N\times1} = Z_{N\times N} \times I_{N\times1}$$

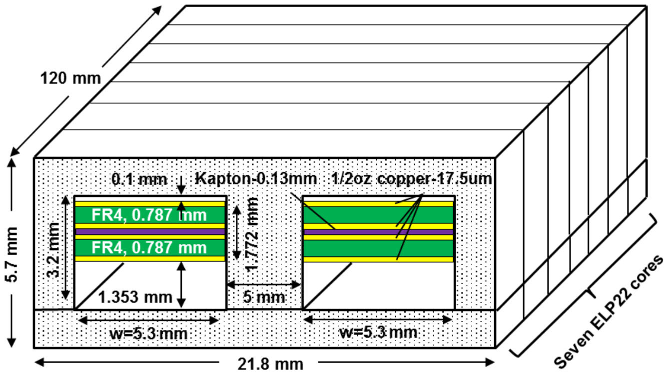
- Extract Parameters for Other Circuit Models.

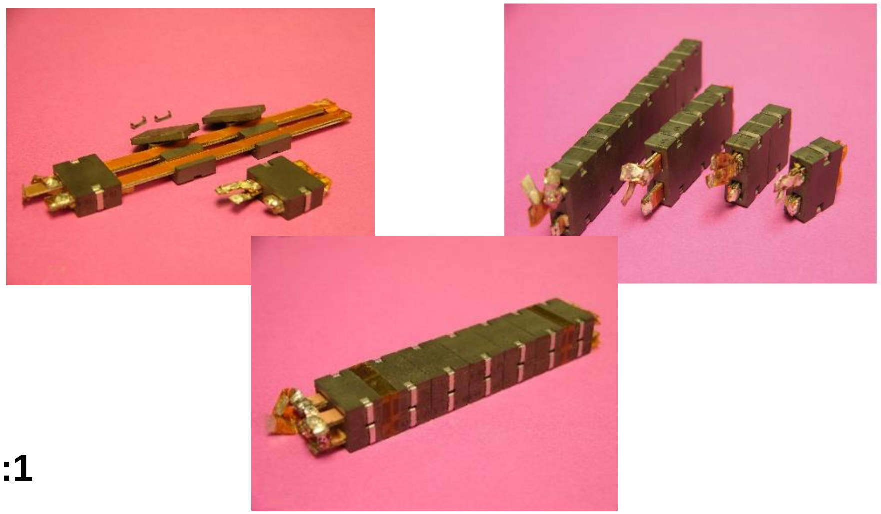
Conventional Transformer T Model:



Verifications and Application Examples

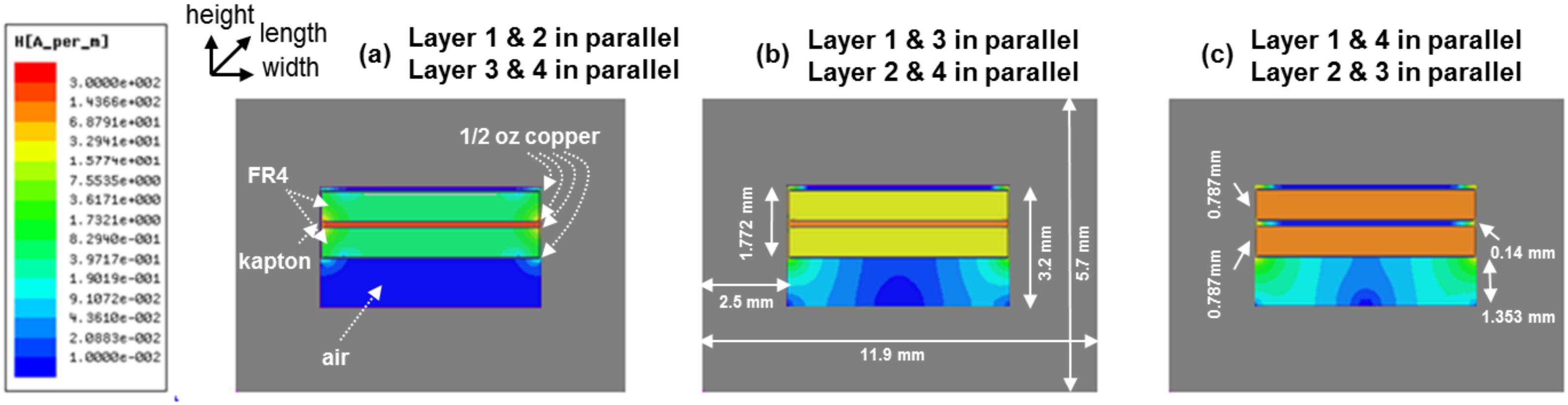
Experimental Measurements





ELP22 Cores: window width/height ratio ~ 3:1

ANSYS Maxwell FEM Simulations



(a) Layer 1 & 2 in parallel
Layer 3 & 4 in parallel

(b) Layer 1 & 3 in parallel
Layer 2 & 4 in parallel

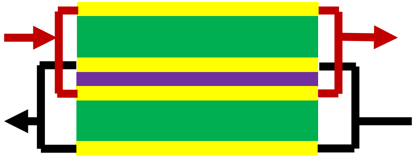
(c) Layer 1 & 4 in parallel
Layer 2 & 3 in parallel

Impacts of Interleaving Patterns

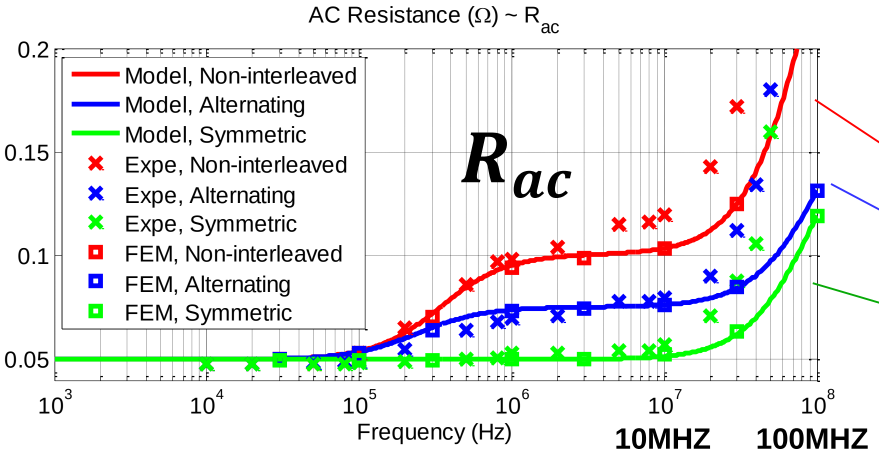
Comparing the P_{ac} and E_{ac} of three 1: 1 transformers with three different interleaving patterns

Non interleaved

Alternating



Symmetric

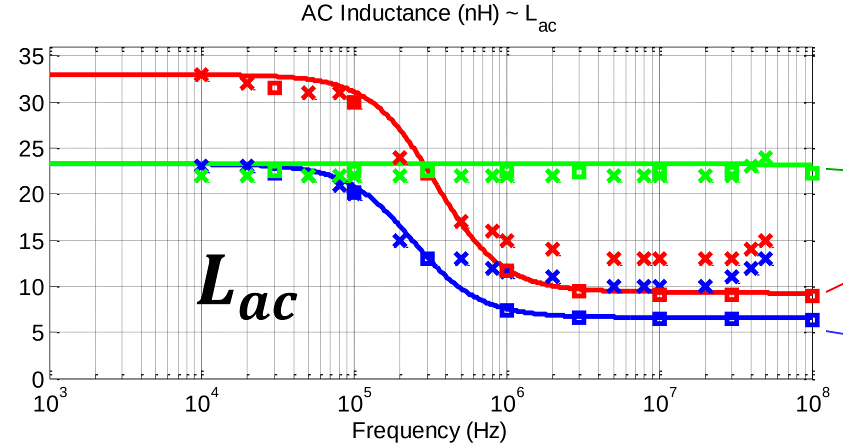


$$P_{ac} = \sum I^2 R_{ac}$$

Non Interleaved

Alternating

Symmetric



$$E_{ac} = \frac{1}{2}\sum I^2 L_{ac}$$

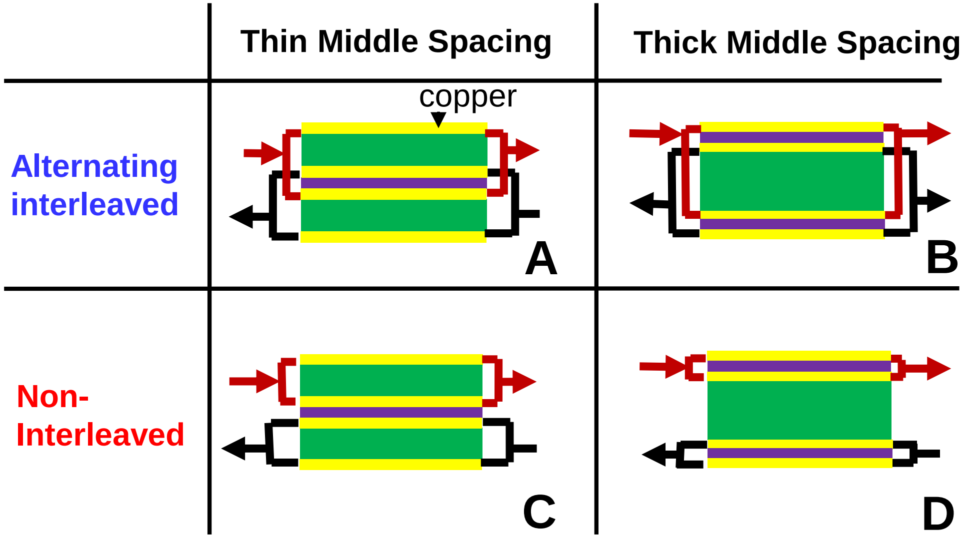
Symmetric

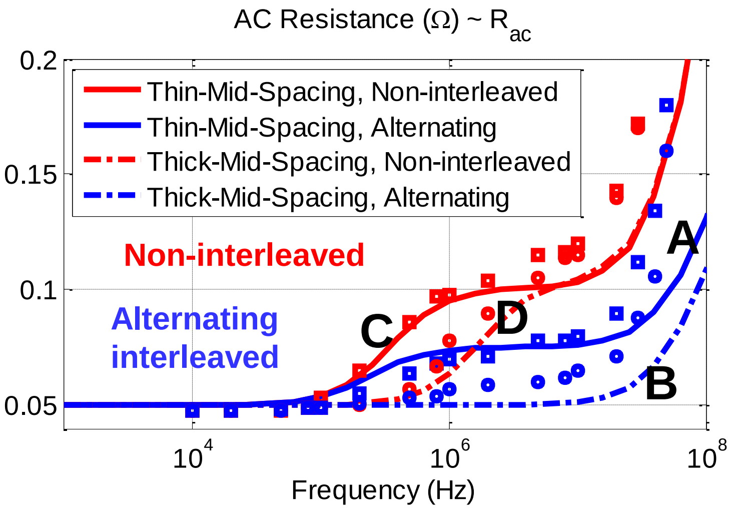
Non interleaved

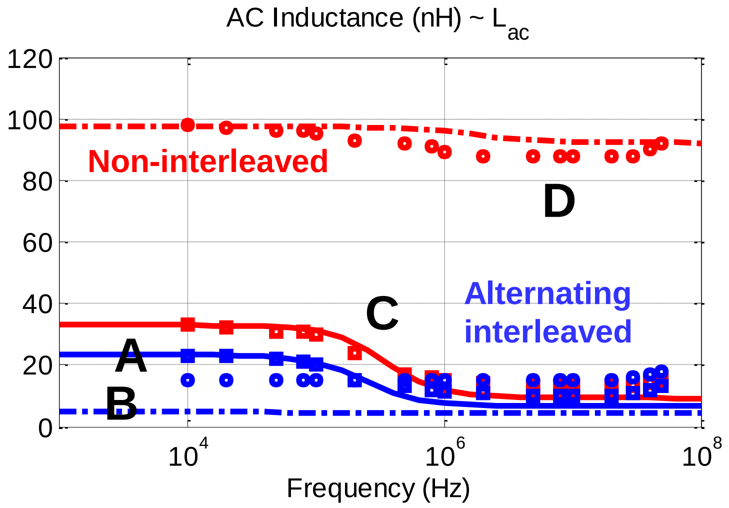
Alternating

Interleaving has to be done in the right way !!!

Impacts of PCB Layer Stacks



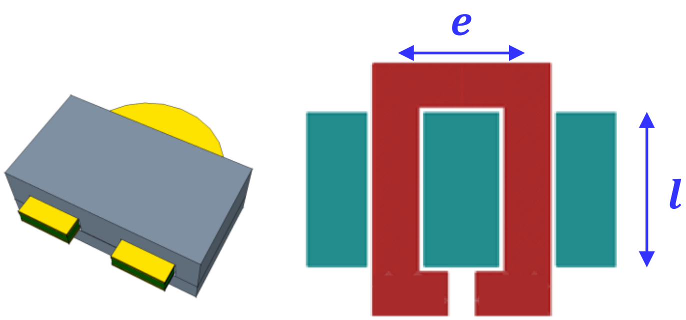




1. ***Quantifying the impacts of PCB stacks on impedances.***

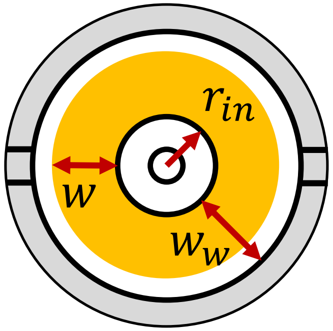
2. ***Choosing the optimal combination of interleaving strategies and PCB layer stacks/materials.***

Practical Considerations



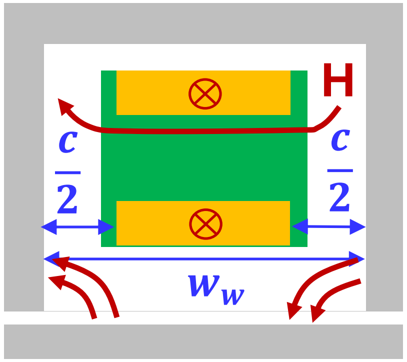
(e/(2l+e)<25%, err<15%)

(a) End effects



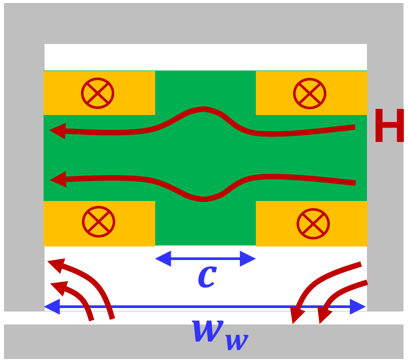
$$\boxed{w_e = r_{in}\ln\frac{r_{in}+w}{r_{in}}}$$

(b) Radius effects for pot cores



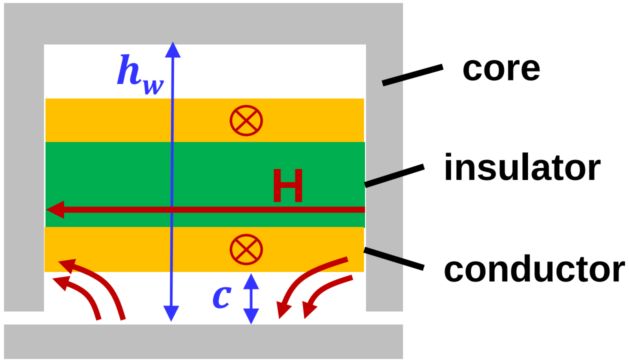
(c/h_w<40%, err<10%)

(c) Conductor to core clearances (side spacing)



(c/w_w<40%, err<10%)

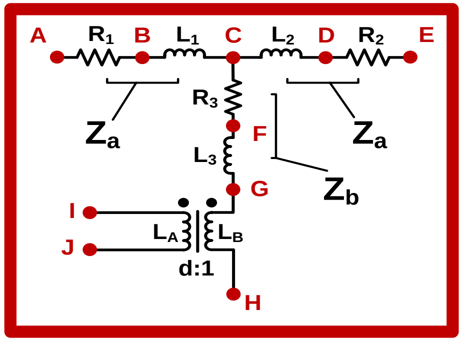
(d) Conductor to Conductor clearances (middle spacing)



(c/h_w>40%, err<10%)

(e) Fringing effects

Summary



Geometry + Operating Frequency

MQS Maxwell's Equations & 1-D Assumption

Lumped Circuit Model

Simulations (SPICE)

Circuit Theory

Numerical

1. Circuit Simulation
2. Field Visualization

Analytical

1. Impedance matrix
2. Impedance-based cantilever model

Applications

1. Multiple primary/secondary windings
2. Interleaving
3. PCB Spacing
4. Board Materials
5. Other design options

Please check the reference for more details include:

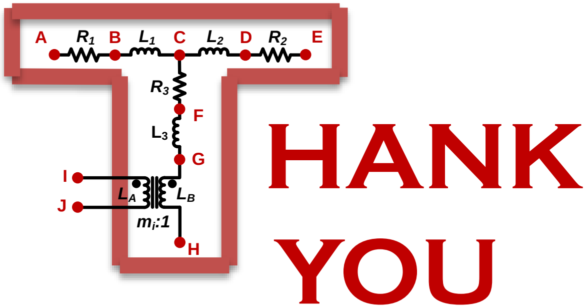
Derivations & Verifications

Empirical Design Rules

END

Acknowledgement:

Prof. David J. Perreault
Prof. Jeffrey H. Lang
Prof. Khurram K. Afridi
Prof. Charles R. Sullivan
Mohammad Araghchini



Sponsorship: MIT CICS, Texas Instruments

Reference:

- **M. Chen, M. Araghchini, K.K. Afridi, J.H. Lang, C.R. Sullivan, and D.J. Perreault, "A Systematic Approach to Modeling Impedances and Current Distribution in Planar Magnetics," *Proc. of the IEEE Workshop on Control and Modeling for Power Electronics (COMPEL)*, June 2014.**