PHI 340 Handout

October 18, 2005

Lemmon Style Proofs for S5

The elimination rule for $\Box$, and the introduction rule for $\Diamond$ are obvious ("Stage 1 Rules").

Definition. The *scope* of an instance of a connective is the smallest WFF in which that instance of the connective occurs.

Definition. Let A be a sentence of modal propositional logic. We say that A is *fully modalized* just in case each atomic sentence in A is within the scope of some modal connective.

A line of a Lemmon style proof looks like this:

Δ_n (n) A citation

This is equivalent to the metalanguage sentence "$\Delta_n \vdash A$". The citation is just for the sake of the reader of the proof, so s/he understands why you believe that $\Delta_n \vdash A$.

$\Box$ Introduction

Suppose that A occurs on a line n with dependency set Δ_n. If each sentence in Δ_n is fully modalized, then we can write $\Box A$ on a subsequent line with dependency set Δ_n. We cite "n $\Box$I".

$\Diamond$ Elimination

Suppose that:

1. $\Diamond A$ occurs on line i with dependency set Δ_i;
2. A occurs on line j with dependency set $\Delta_j = \{j\}$, i.e. A is an assumption on line j;
3. B occurs on line k with dependency set Δ_k, and
    (a) B *is fully modalized*;
    (b) each sentence in $\Delta_k - \{j\}$ is fully modalized.

Then we can write B on a subsequent line with dependency set $\Delta_i \cup (\Delta_k - \{j\})$. We cite "$i, j, k$ $\Diamond$E".