

Homework 9

1. We write $A \equiv B$ when there is a proof $\vdash (A \leftrightarrow B)$. Prove that: For any sentences A, A', B, B' , if $A \equiv A'$ and $B \equiv B'$ then $(A \rightarrow B) \equiv (A' \rightarrow B')$
2. Let Σ be the set of sentences generated from a single atomic sentence P . That is, Σ is defined inductively as follows:
 - Base case: the atomic sentence P is in Σ ;
 - Generating clauses: if $A \in \Sigma$ and $B \in \Sigma$ then $\neg A \in \Sigma$, $A \wedge B \in \Sigma$, $A \vee B \in \Sigma$ and $A \rightarrow B \in \Sigma$.

Show that for any $A \in \Sigma$, either $P \vdash A$ or $P \vdash \neg A$. (Do *not* cite completeness in your proof!)