Homework 8

1. For each of the following sentences, give an interpretation with non-empty extension of "Fxy" that makes the sentence true, and another such interpretation that makes the sentence false. Arrow diagrams are sufficient — you don't have to put your answer in set-theoretic notation.
   (a) $\exists x \exists y(Fxy \wedge Fyx) \wedge \forall x \forall y[\exists z(Fxz \wedge Fzy) \rightarrow Fxy]$
   (b) $\exists x \exists y(Fxy \leftrightarrow \neg Fyy)$
2. For each of the following pairs of sentences, give an interpretation that shows that the first sentence does not imply the second. Arrow diagrams are sufficient — you don't have to put your answer in set-theoretic notation.
   (a) $\exists x \forall y \neg Fxy \wedge \exists x \forall y Fxy$, $\forall x(\exists y Fxy \rightarrow \forall y Fxy)$
   (b) $\forall y(\exists z Fyz \rightarrow \exists z Fzy)$, $\forall y(\forall z Fyz \rightarrow \forall z Fzy)$
   (c) $\forall x \exists y(Fxy \wedge \neg Fyx)$, $\forall x(\exists y Fxy \rightarrow \exists y Fyx)$
3. Translate the following sentences into predicate logic notation, using the "=" relation if appropriate.
   (a) Maren is the only student who hasn't made a single mistake on any homework or exam. ($Px \equiv x$ has made some mistake on a homework or on an exam, $m \equiv$ Maren)
   (b) There is no greatest prime number. ($Fxy \equiv y$ is greater than x; $Px \equiv x$ is a prime number)
   (c) The smallest prime number is even. ($Fxy \equiv y$ is greater than x; $Px \equiv x$ is a prime number; $Ex \equiv x$ is even.)