

Homework 3

1. Write valid proofs of the following arguments. You are permitted to use any of the rules of inference that we have introduced, i.e., any of the rules in Chapter 1 of Lemmon's book. (Note: you should *not* use truth tables for these problems.)

(a) $\vdash ((Q \rightarrow P) \rightarrow (Q \rightarrow \neg P)) \rightarrow (Q \rightarrow \neg P)$

(b) $P \rightarrow (R \vee S) \vdash (P \rightarrow R) \vee (P \rightarrow S)$

2. Write out full truth tables for the following sentences. Highlight in some way (e.g. draw a circle around) the column under the main connective of the sentence, and say whether the sentence is a contingency, a tautology, or an inconsistency.

(a) $(P \rightarrow Q) \vee (Q \rightarrow P)$

(b) $\neg(P \vee R) \wedge (\neg Q \rightarrow (P \wedge R))$

3. Determine if the following arguments are valid. You do not need to show all of your work. But if an argument is *invalid*, then give a counterexample (i.e. a truth-assignment relative to which the premises are true and the conclusion is false).

(a) $(P \rightarrow Q) \vee (Q \rightarrow R), \neg R \rightarrow \neg(P \wedge Q) \vdash Q \rightarrow \neg P$

(b) $(P \vee Q) \rightarrow (R \vee S), P \leftrightarrow \neg(R \wedge S), Q \leftrightarrow \neg(P \wedge R) \vdash (S \wedge P) \rightarrow \neg(P \vee R)$

(c) $\vdash ((P \leftrightarrow Q) \vee (P \leftrightarrow R)) \vee (Q \leftrightarrow R)$

4. Which of the following sentences is a *substitution instance* of the sentence $P \rightarrow \neg Q$?

(a) $\neg Q \rightarrow \neg P$

(b) $P \rightarrow R$

(c) $(P \rightarrow \neg Q) \rightarrow \neg(P \rightarrow \neg Q)$.