

Homework 10

1. Let $A(c)$ be a predicate logic sentence containing the constant “ c ”, and suppose that $\vdash A(c) \leftrightarrow B$, where B is in prenex normal form. Show that: there is a sentence B' in prenex normal form such that $\vdash \forall v A(v) \leftrightarrow B'$.
2. Let A be a prenex sentence with the following form:

$$A = \exists x_1 \dots \exists x_n \forall y_1 \dots \forall y_m R(x_1, \dots, x_n, y_1, \dots, y_m).$$

- That is, A begins with $n > 0$ existential quantifiers, followed by $m \geq 0$ universal quantifiers. Show that if A is consistent, then there is an interpretation \mathcal{I} such that the domain of \mathcal{I} has n elements, and A is true in \mathcal{I} .
3. Find a sentence A that is consistent, but which is true only in interpretations whose domain is larger than the number of quantifiers in A .
 4. Let \mathcal{A} be the set of propositional logic sentences built with just the two atomic sentences P and Q . Show that there is a finite subset \mathcal{B} of \mathcal{A} such that for every sentence A in \mathcal{A} , there is a sentence B in \mathcal{B} such that $\vdash A \leftrightarrow B$. How small can \mathcal{B} be?