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Noncooperative collusion in durable goods oligopoly

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Coase conjectured that a durable-goods monopolist who can make offers to sell arbitrarily frequently will lose the ability to extract positive profits. This result, which has now been proved, can be attributed to the inability of the monopolist to commit to maintaining sufficiently high prices in the near future. For the case of durable-goods oligopoly, we show that letting the firms make offers arbitrarily frequently enhances their ability to commit to high prices and in the limit enables the firms to enjoy total market profits equal to the full commitment (one-shot) monopoly profit.

1. Introduction

■ Coase (1972) conjectured that if a durable-goods monopolist can make offers to sell arbitrarily frequently, then in equilibrium he must always charge the competitive price. Gul, Sonnenschein, and Wilson (1986) prove this conjecture within the framework of a discrete-time, infinite-horizon, extensive-form game with a continuum of buyers. In particular, they show that the difference between the highest price charged by the monopolist and the constant unit cost is small whenever the time between offers is small.

The purpose of this article is to show that the competitive results of durable-goods monopoly are reversed for the case of oligopoly. First, we prove that for any demand f and any number of firms, the total profit associated with any equilibrium sequence of sales and prices is no greater than the one-shot monopoly profit. Next, we show that if the number of firms is at least two, then as the time between offers becomes arbitrarily small, there exist equilibria yielding total (industry) profits arbitrarily close to the one-shot monopoly profit. Finally, we show that any division of an arbitrary level of total profits can be (approximately) realized, provided the total does not exceed the one-shot monopoly profit and the time between offers is sufficiently small.

Intuitively, our results can be attributed to the interplay of two factors: (i) the existence of a price-war equilibrium—an equilibrium that yields zero profit for each firm; and (ii) the ability of consumers to detect deviations of firms from the equilibrium price path and hence to anticipate the ensuing price war.

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This article is a revised version of the second chapter of my Ph.D. dissertation. I am grateful to Hugo Sonnenschein for helpful comments and discussions. After finishing this article, I learned about a working paper version of Ausubel and Deneckere (1987). In that paper they analyzed the same problem for the case of linear demand. Not surprisingly, their conclusions are identical. Ausubel and Deneckere also provide a partial characterization of optimal subgame-perfect equilibria for a fixed δ.

The second of these factors—that is, our modelling of the demand sector as a continuum of rational consumers with intertemporal preferences—distinguishes our analysis from the familiar folk theorems of (nondurable goods) oligopoly supergames (Abreu, 1983). Those results show that for sufficiently high discount factors, any individually rational outcome (in payoff space) of the one-shot game can be supported as a subgame-perfect equilibrium of the repeated game (after the appropriate normalization that renders the outcomes of the one-shot and repeated games comparable). The (credible) price-war threat and the existence of rational consumers who can anticipate the price war (and hence delay buying) yield a surprising result. For the durable goods model, monopoly is more competitive than oligopoly: higher total profits and prices can be sustained under oligopoly than under monopoly.

2. The model

■ The specification used here is a straightforward generalization of the framework developed by Gul, Sonnenschein, and Wilson (1986). There is a unit mass of consumers indexed by [0, 1], each of whom has use for one unit of a good,[1] which is produced by N firms at zero cost.[2]

The function f: $[0, 1] \rightarrow I\!R_+$ and the discount factor δ define the preferences of the consumers. Specifically, if consumer $a \in [0, 1]$ buys the good in period i at price P_i, then he obtains utility $(f(a) - P_i)\delta^i$. The utility of never purchasing the good is zero. The payoff of firm $j = 1, 2, \ldots, N$ is given by

$$\Pi^j = \sum_{i=0}^{\infty} P_i^j \mu(A_i^j)\delta^i,$$

where P_i^j is the price charged by firm j in period i, A_i^j is the set of consumers who purchased the good from firm j in period i, and μ is the Lebesgue measure. At the beginning of each period i, the firms simultaneously announce prices. After observing the prices, the consumers decide whether and from which firm to buy. After each firm serves its customers, no further sales are made until the next period. In general, the strategy of a firm specifies the price it will charge in period i as a function of all $(i - 1)$-period histories. The strategy of a consumer specifies from which firm (if any) he will buy, as a function of all $(i - 1)$-period histories and n-tuples of ith period prices.

The equilibrium concept is subgame-perfect Nash equilibrium. The extension of the notion of subgame perfection to games with a continuum of players necessitates certain regularity assumptions on the set of admissible strategy profiles. In particular, we restrict attention to strategy profiles that specify for every period i, every $(i - 1)$-period history, and every n-tuple of ith period prices, a measurable set of consumers to purchase from each firm j. Since subgames that require simultaneous deviations to reach but never enter agents' computations of optimal strategies, such deviations are not relevant in determining the optimality or perfection of a strategy profile.[3] Hence, in what follows, when verifying subgame perfection, we shall only consider histories that involve no simultaneous deviations.

The following definitions will be used in stating and proving the main results. For all f: $[0, 1] \rightarrow I\!R_+$, nonincreasing and left-continuous, $\delta \in (0, 1)$, and $N \in I\!N$, $E(f, \delta, N)$ denotes the set of equilibria of the above-defined game. Without loss of generality, we also

[1] Hence, we assume that any additional units provide no additional utility to consumers, and therefore each consumer leaves the market after purchasing one unit.

[2] Analyzing the case of zero cost is equivalent to analyzing the case of arbitrary constant cost c.

[3] To see this, take any finite-player version of our game. Assume that a strategy profile specifies optimal behavior at all subgames that can be reached without simultaneous deviations but does not specify optimal behavior at some subgame(s) that can only be reached by simultaneous deviations. Modify this strategy profile by specifying arbitrary subgame-perfect equilibria on all such subgame(s). Note that the new strategy profile is a subgame-perfect equilibrium and has the same outcome path as the original strategy profile.

assume that $f(a) > 0$ for all $a \in [0, 1)$. In addition, π_f is the one-shot monopoly profit; that is, $\pi_f = \max_{p,q} pq$, subject to $q \in [0, 1]$ and $p \leq f(q)$. Let $\pi^j(\sigma)$ denote the present value of the profit stream of firm j when agents follow the equilibrium strategy profile σ. Let $q_i(\sigma)$ represent the total quantity of sales of all firms before period i, which we refer to as the quantity state at i associated with σ. Denote by $P_i^L(\sigma)$ the lowest price charged by any firm in period i in equilibrium σ. Finally, let $A_i^j(\sigma)$ represent the set of consumers who buy from firm j in period i in equilibrium σ. We sometimes suppress σ and use π^j, q_i, etc.

We state the main results below and provide proofs in the Appendix.

Theorem 1. $\sigma \in E(f, \delta, N)$ implies that $\sum_{j=1}^{N} \pi^j(\sigma) \leq \pi_f$.

Theorem 1 establishes that, regardless of the number of firms or the discount factor (hence the time between offers), the total industry profit that can be sustained in equilibrium is no greater than π_f. Theorem 1 follows from consumer optimization.[4]

Gul, Sonnenschein, and Wilson (1986) show that for any $\epsilon > 0$ there exists $\bar{\delta} < 1$ such that the profit of the monopolist, $\pi^1(\delta)$, is less than $f(1) + \epsilon$ for all $\delta \in (\bar{\delta}, 1)$ and $\sigma \in E(f, \delta, 1)$. That is, in the limit, as the time between offers goes to zero, the best that the monopolist can do is to sell immediately to the entire market at the lowest valuation $f(1)$. This implies zero monopoly profits for the typical case $f(1) = 0$. The next theorem establishes that the opposite holds for oligopoly; that is, with two or more firms total oligopoly profit can approach one-shot monopoly profit as δ approaches one.

Theorem 2. For all f, $N \geq 2$ and $\epsilon > 0$, there exists $\bar{\delta} \in (0, 1)$ such that $\delta \in (\bar{\delta}, 1)$ implies that there exists $\sigma \in E(f, \delta, N)$ such that $\sum_{j=1}^{N} \pi^j(\sigma) > \pi_f - \epsilon$.

Theorem 2 and the Coase conjecture show that while a monopolist who can make offers arbitrarily frequently is forced to behave competitively, two or more firms can extract the one-shot monopoly profit.

One can, in fact, show that arbitrary distributions of profit can be approximated as the equilibrium levels of profit, provided the total profit does not exceed π_f; this is Theorem 3.

Theorem 3. (This is the folk theorem for durable goods oligopoly.) For all f, $N \geq 2$, $\epsilon > 0$, and $(V^1, V^2, \ldots, V^N)$ satisfying $V^j \geq 0$ for all j and $\sum_{j=1}^{N} V^j \leq \pi_f$, there exists $\bar{\delta} \in (0, 1)$ such that $\delta \in (\bar{\delta}, 1)$ implies that there exists $\sigma \in E(f, \bar{\delta}, N)$ such that $|\pi^j(\sigma) - V^j| < \epsilon$ for all $j = 1, 2, \ldots, N$.

3. Concluding remarks

■ Kahn (1986) extends the constant-cost, linear-demand, quantity-setting model developed by Bulow (1982) and Stokey (1982) to incorporate quadratic costs. He shows, for a particular sequence of equilibria (indexed by δ), that in the limit as δ approaches one, the monopolist earns positive profits and saturates the market at a rate slower than the welfare-maximizing rate. He also notes that the monopolist earns less than the full commitment monopoly profit. Kahn therefore concludes that while the Coase conjecture fails to hold with increasing costs, the requirement of time consistency[5] still imposes a constraint on the total profit that

[4] Theorem 1 is formally equivalent to an unpublished theorem due to Rubinstein, Wilson, and Wolinsky. This theorem is cited by Paul Milgrom in his Auction Theory survey presented at the 1985 Econometric Society World Congress.

[5] The model of Kahn (1986) is not explicitly game-theoretic. Off-equilibrium path behavior is made relevant, however, by imposing a time-consistency requirement on the monopolist's plan.

can be extracted. For the case of oligopoly, we have shown that time consistency, interpreted as subgame perfection, imposes no restriction on the total amount of surplus that can be extracted by all firms.

We should note that the results of the previous section hold even with an arbitrary number of prospective entrants, provided that a decision to enter on the part of any prospective entrant can be observed one period in advance.[6] In this case, the credible threat to revert to the zero-profit equilibrium could preempt entry. Alternatively, the same threat could be used on incumbents to sustain equilibria with a more complicated structure. Hence, our model provides a very simple framework in which equilibria with no new entry, entry followed by a price war, and entry followed by acquiescence on the part of the incumbent can all occur. Unfortunately, this flexibility comes at a high cost: the model has a very large number of equilibria.

Appendix

■ We use the following lemma, which is essentially a game-theoretic counterpart of the perfect-foresight condition used by Stokey (1982) and Kahn (1986), to prove Theorem 1.

Lemma A1. $\sigma \in E(f, \delta, N)$ and $q_{i+1}(\sigma) - q_i(\sigma) \neq 0$ imply that

$$P_i^L(\sigma) \leq (1-\delta) \sum_{k=i}^{\infty} f(q_{k+1}(\sigma))\delta^{k-i}.$$

Proof. We first show that if in any equilibrium consumer b purchases the good in period i and $f(a) > f(b)$, then in that equilibrium consumer a purchases the good in period $k \leq i$. To see this observe that by consumer optimization, we have

$$(f(a) - P_k^L)\delta^k \geq (f(a) - P_i^L)\delta^i \tag{A1}$$

$$(f(b) - P_k^L)\delta^k \leq (f(b) - P_i^L)\delta^i. \tag{A2}$$

The inequalities in (A1) and (A2) yield $(f(a) - f(b))(1 - \delta^{i-k}) \geq 0$. But since $f(a) - f(b) > 0$, we have $1 - \delta^{i-k} > 0$ and therefore $i \geq k$.

Let $\{i_t\}$ be the subsequence of periods such that $q_{i_{t+1}} - q_{i_t} \neq 0$ for all t. Then, the preceding observation and the left continuity of f establish that for every period i_t there exist consumers with valuation arbitrarily close to $f(q_{i_{t+1}})$ who purchase the good in period i_t. Hence, we have

$$(f(q_{i_{t+1}}) - P_{i_t})\delta^{i_t} \geq (f(q_{i_{t+1}}) - P_{i_{t+1}})\delta^{i_{t+1}}. \tag{A3}$$

By (A3) we have

$$P_{i_t} \leq (1 - \delta^{i_{t+1}-i_t})f(q_{i_{t+1}}) + \delta^{i_{t+1}-i_t}P_{i_{t+1}} = (1-\delta) \sum_{k=i_t}^{i_{t+1}-1} f(q_{i_{t+1}})\delta^{k-i_t} + \delta^{(i_{t+1}-i_t)}P_{i_{t+1}}. \tag{A4}$$

But since $q_k = q_{k+1}$ for all $k = i_t + 1, i_t + 2, \ldots, i_{t+1}$, we have

$$P_{i_t} \leq (1-\delta) \sum_{k=i_t}^{i_{t+1}-1} f(q_{k+1})\delta^{k-i_t} + \delta^{(i_{t+1}-i_t)}P_{i_{t+1}} \tag{A5}$$

for all i_t.

If the sequence $\{i_t\}$ is unbounded, then (A5) yields the desired result. If not, then let $I = \sup \{i_t\}$. Then we have, by the argument used in establishing (A3) above,

$$f(q_{I+1}) - P_I \geq 0 \qquad \text{and} \qquad q_{I+1+t} = q_{I+1} \qquad \text{for all} \qquad t > 0.$$

Hence,

$$P_I \leq (1-\delta) \sum_{k=I}^{\infty} f(q_{k+1})\delta^{k-I} = f(q_{I+1}). \tag{A6}$$

Thus (A5) and (A6) yield the desired conclusion. *Q.E.D.*

[6] This is not a particularly strong assumption, since we are mainly interested in the case where the length of periods becomes arbitrarily small.

Proof of Theorem 1. Let $A_i = \bigcup_{j=1}^{N} A_i^j(\sigma)$. Then

$$\pi^j(\sigma) = \sum_{i=0}^{\infty} P_i^j(\sigma)\mu(A_i^j(\sigma))\delta^i.$$

But $A_i^j(\sigma) = 0$ whenever $p_i^j(\sigma) \neq P_i^L$. Hence,

$$\pi^j(\sigma) = \sum_{i=0}^{\infty} P_i^L \mu(A_i^j)\delta^i \qquad \text{for all} \qquad j = 1, 2, \ldots, N,$$

so that

$$\sum_{j=1}^{N} \pi^j(\sigma) = \sum_{i=0}^{\infty} P_i^L (\sum_{j=1}^{N} \mu(A_i^j))\delta^i = \sum_{i=0}^{\infty} P_i^L \mu(A_i)\delta^i = \sum_{i=0}^{\infty} P_i^L (q_{i+1} - q_i)\delta^i.$$

But by Lemma A1 we have

$$P_i^L \leq (1-\delta)\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i} \qquad \text{or} \qquad q_{k+1} - q_k = 0.$$

Hence,

$$\sum_{j=1}^{N} \pi^j(\sigma) \leq \sum_{i=0}^{\infty} (1-\delta)(\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i})(q_{i+1} - q_i)\delta^i = (1-\delta)\sum_{i=0}^{\infty}(\sum_{k=i}^{\infty} f(q_{k+1})\delta^k))(q_{i+1} - q_i) = (1-\delta)\sum_{i=0}^{\infty} f(q_{i+1})q_{i+1}\delta^i.$$

But $f(q_{i+1})q_{i+1} \leq \pi_f$ for all $q_{i+1} \in [0, 1]$. Hence,

$$\sum_{j=1}^{N} \pi^j(\sigma) \leq (1-\delta)\sum_{i=0}^{\infty} \pi_f \delta^i = \pi_f. \qquad \textit{Q.E.D.}$$

Proof of Theorem 2. Define $Z(Q) = \max f(x)(x - Q)$ for all $Q \in [0, 1]$ and

$$x \in [0,1], t(Q) = \inf\{x \in [Q, 1] | f(x)(x - Q) = Z(Q)\}.$$

We define a sequence $\{Q_i\}_{i=0}^{\infty}$ as follows:

$$Q_0 = 0$$

$$Q_1 = \begin{cases} t(0) & \text{if} \quad t(0) \neq 1 \\ 1 - \bar{\epsilon}/2 & \text{otherwise,} \end{cases}$$

where $\bar{\epsilon}$ is a small number as defined below. For all $i = 1, 2, \ldots, N$,

$$Q_{i+1} = \begin{cases} t(Q_i) & \text{if} \quad t(Q_i) \neq 1 \\ Q_{i+1} = Q_i + (1 - Q_i)/2 & \text{otherwise.} \end{cases}$$

Observe that the left continuity and the nonincreasingness of f guarantee that Z and t are well defined, $Z(Q) = f(t(Q))(t(Q) - Q)$ for all $Q \in [0, 1]$, and, in particular,

$$Z(0) = f(t(0))(t(0) - 0) = \pi_f \qquad \text{provided } t(0) \neq 1.$$

Furthermore, it is easy to verify that Q_i is a strictly increasing sequence and that $\lim_{i\to\infty} Q_i = 1$.

Define

$$\bar{\epsilon} = \min\left\{\frac{1}{2}, \frac{\epsilon}{f(0)}\right\} \qquad \text{and} \qquad \bar{\delta} = \frac{2N - \bar{\epsilon}(1-\bar{\epsilon})}{2N}.$$

Choose m such that $\delta^m < \bar{\epsilon}$ and $\delta^{m-1} \geq \bar{\epsilon}$. Note that $1/2 \leq \bar{\epsilon} < 1$ and $m \geq 1$.

For all $i = ms + r$ for some $s, r \in IN$, and $r \leq m - 1$ define $q_i = Q_{s+1}$ if $i > 0$ and $q_i = 0$ if $i = 0$. Hence, $q_1 = q_2 = \ldots = q_m = Q_1$, $q_{m+1} = q_{m+2} = \ldots = q_{2m} = Q_2$, etc. Finally, we define

$$\bar{P}_i = (1-\delta)\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i}.$$

Now we construct an equilibrium for the game that has $\{q_i\}_{i=0}^{\infty}$ as its sequence of quantity histories. For any consumer a, after any $(i - 1)$-period history in which he has not purchased the good, his strategy in period i is defined below.

If $P_k^j = \bar{P}_k$ for all $j = 1, 2, \ldots, N$ and all $k \leq i$, then $\sigma_i(a) = j$ whenever $a \in (b_{i,j}, B_{i,j})$ for some $j = 1, 2, \ldots, N$, where $b_{i,j} = q_i + (j - 1)(q_{i+1} - q_i)/N$ and $B_{i,j} = q_i + j/N$. $\sigma_i(a) = j$ means that a buys from firm j and $\sigma_i(a) = 0$ means a does not buy in period i. If $P^j_k \neq \bar{P}_k$ for some $k \leq i$ and j, then $\sigma_i(a) = 0$ if $\inf_j \{P^j_i\} \geq (1-\delta)f(a)$ and $\sigma_i(a) = \inf\{j' | P^{j'}_i = \inf\{P^j_i\}\}$ otherwise.

Hence, the strategies of the consumers are such that if every firm j, in every period $k \leq i$, has charged $\bar{P}_k$, then all consumers between q_i and q_{i+1} buy in a way to generate equal revenues for every firm. If some firm has charged a price other than $\bar{P}_k$ for some $k \leq i$, then only those consumers who prefer buying at the lowest available price today (period i) to buying at price zero tomorrow buy in period i (at the lowest price). Furthermore, such consumers buy from the firm with the lowest index j, if there is more than one firm charging the lowest available price.[7]

The strategy of any firm j after any $(i-1)$-period history is defined by

$$\sigma_i(j) = \begin{cases} 0 & \text{if} \quad P^j_k \neq \bar{P}_k \quad \text{for some} \quad k \leq i-1 \\ \bar{P}_i & \text{otherwise.} \end{cases}$$

Next, we verify that these strategies constitute an equilibrium.

Consumer optimality along the equilibrium path. First, we show that every consumer who buys according to his specified strategy obtains a nonnegative utility so that never buying is not a preferred response:

$$\bar{P}_i = (1-\delta)\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i} \leq (1-\delta)\sum_{k=i}^{\infty} f(q_{i+1})\delta^{k-i} = f(q_{i+1}).$$

Hence $f(q_{i+1}) - \bar{P}_i \geq 0$. But if a is designated to buy in period i, then $a \leq q_{i+1}$; hence $(f(a) - \bar{P}_i)\delta^i \geq 0$.

Next, we prove that any consumer who is designated to buy in period i can do no better by buying in any other period. To see this observe that

$$\bar{P}_i = (1-\delta)\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i} \qquad \text{for all } i,$$

which implies that

$$\bar{P}_i = (1-\delta)\sum_{k=i}^{s-1} f(q_{k+1})\delta^{k-i} + \delta^{s-i}\bar{P}_s.$$

Therefore, $\bar{P}_i \leqq (1-\delta)\sum_{k=i}^{s-1} f(q_{i+1})\delta^{k-i} + \delta^{s-i}\bar{P}_s$ and $\bar{P}_i \leqq (1-\delta^{s-i})f(q_{i+1}) + \delta^{s-i}\bar{P}_s$. Hence,

$$(f(q_{i+1}) - \bar{P}_i)\delta^i \geq (f(q_{i+1}) - \bar{P}_s)\delta^s \qquad \text{for all} \qquad s > i.$$

But if a is designated to buy in period i, then $a \leq q_{i+1}$. Hence, $(f(a) - \bar{P}_i)\delta^i \geq (f(a) - P_s)\delta^s$ for all $s > i$. Therefore, a can do no better by purchasing after period i.

Similarly, for $s < i$ we have

$$\bar{P}_s = (1-\delta)\sum_{k=s}^{i-1} f(q_{k+1})\delta^{k-s} + \delta^{i-s}\bar{P}_i \geq (1-\delta)\sum_{k=s}^{i-1} f(q_i)\delta^{k-s} + \delta^{i-s}\bar{P}_i = (1-\delta^{i-s})f(q_i) + \delta^{i-s}\bar{P}_i.$$

Hence, we obtain $(f(q_i) - \bar{P}_s)\delta^s \leq (f(q_i) - \bar{P}_i)\delta^i$. But if a is to buy in period i, $a > q_i$, and hence $f(a) \leq f(q_i)$, which yields

$$(f(a) - \bar{P}_s)\delta^s \leq (f(a) - \bar{P}_i)\delta^i \qquad \text{for all} \qquad s < i.$$

This establishes that a cannot do better by purchasing before i.

Noting that $\lim_{i\to\infty} q_i = 1$, so that consumer $a = 1$ is the only consumer who never purchases the good, and that

$$\bar{P}_i = (1-\delta)\sum_{k=i}^{\infty} f(q_{k+1})\delta^{k-i} \geq (1-\delta)\sum_{k=i}^{\infty} f(1)\delta^{k-i} = f(1),$$

so that $f(1) - \bar{P}_i \leq 0$ for all i, completes the proof of the optimality of consumer behavior along the equilibrium path.

Consumer optimality off the equilibrium path. Note that if a deviation (by some firm) has occurred in period $k \leq i$, then every firm will charge zero in period $i+1$. Hence it is optimal for consumers to buy if and only if they prefer the lowest available price today to buying at price zero tomorrow. Thus, we have shown the optimality of consumer behavior.

Firm optimality. To check the optimality of firms' strategies, note that if a deviation has occurred in any previous period, all other firms will charge zero this period and clear the market; hence, charging zero is optimal for each

[7] Any other tie-breaking rule would do equally well.

firm. Assume that no previous deviation has occurred and consider a firm that is contemplating deviating at period $i = ms + 1$ for some $s \in IN$. By deviating the firm can earn positive revenue only in the current period. Furthermore, the return to such a deviation is at most $D \equiv (1 - \delta)Z(q_i)$ since consumer a is not buying at any price above $(1 - \delta)f(a)$. If a firm does not deviate, then $m - 1$ periods later it will earn

$$\bar{Z} \equiv \frac{1}{N}\bar{P}_{(s+1)m}(q_{(s+1)m+1} - q_{ms+1}),$$

which is worth $V \equiv \delta^{m-1}\bar{Z}$ today.

First, consider the case where $t(q_{ms+1}) \neq 1$. Then $\bar{Z} = Z(q_{ms+1})/N$ so that

$$V = \frac{\delta^{m-1}D}{(1-\delta)N}.$$

But $\delta^{m-1} \geq \bar{\epsilon}$ and $1 - \delta \leq 1 - \bar{\delta} \leq \bar{\epsilon}(1 - \bar{\epsilon})/2N$, so that $(\delta^{m-1})/(1 - \delta)N \geq 2$. This shows that $V > D$, and hence the firm can do no better by deviating. If $t(q_{ms+1}) = 1$, then by not deviating the firm will earn

$$V = \delta^{m-1}\bar{Z} \geq \frac{Z(q_i)}{2N} = \frac{\delta^{m-1}}{(1-\delta)2N}D.$$

Once again, the firm can do no better by deviating since $(\delta^{m-1})/(1 - \delta)2N \geq 1$.

Next, note that if any firm cannot beneficially deviate in any period $ms + 1$, then it cannot beneficially deviate in any period $i = ms + l$ for $l \neq 1$, $l \leq m - 1$, since, in such cases, the gain from deviating D remains the same, whereas the cost of deviating V is greater, since the earliest subsequent period involving a positive mass of sales is less than $m - 1$ periods away.

Hence, the strategies constitute an equilibrium.

Finally, consider the total oligopoly profit under these strategies:

$$\sum_{j=1}^{N} \pi^j(\sigma) = \sum_{j=1}^{N} \frac{1}{N} \sum_{i=0}^{\infty} \bar{P}_i(q_{i+1} - q_i)\delta^i$$

and

$$\bar{P}_0 = (1-\delta)\sum_{i=0}^{\infty} f(q_{i+1})\delta^i > (1-\delta)\sum_{i=0}^{m-1} f(q_{i+1})\delta^i = (1-\delta)\sum_{i=0}^{m-1} f(q_i)\delta^i = (1-\delta^m)f(q_1)f(q_1) - \bar{\epsilon}f(q_1) \geq f(q_1) - \frac{\epsilon}{2}.$$

Hence $\sum_{j=1}^{N} \pi^j(\sigma) \geq \left(f(q_1) - \frac{\epsilon}{2}\right)q_1 \geq f(q_1)q_1 - \frac{\epsilon}{2}$. If $t(0) \neq 1$, then $f(q_1)q_1 = \pi_f$ so that

$$\sum_{j=1}^{N} \pi(\sigma) \geq \pi_f - \frac{\epsilon}{2}.$$

If $t(0) = 1$, then

$$\sum_{j=1}^{N} \pi^j(\sigma) \geq f(q_1)q_1 - \frac{\epsilon}{2} = f(1)\left[1 - \frac{\bar{\epsilon}}{2}\right] - \frac{\epsilon}{2} \geq f(1)(1-\epsilon) = \pi_f - \epsilon. \qquad Q.E.D.$$

Proof of Theorem 3. Let $\bar{V}^j = \sup\{V^j, \epsilon'\}$ for some small $\epsilon' > 0$ and $\alpha^j = \dfrac{\bar{V}^j}{\sum_{k=1}^{N} \bar{V}^k}$. Change the construction of consumer strategies (in the proof of Theorem 2) so that α^j percent of the consumers who purchase the good purchase it from firm j in each period i. Since $\alpha^j > 0$, with some obvious adjustments, we can show that for δ sufficiently close to one, there exist equilibria that yield approximately $\alpha^j\pi_f$ for each firm j. But for δ close to one, the firms can be forced to delay initial sales long enough that the discounted value of the jth firm's revenue stream, that is, the discounted value of $\alpha^j\pi_f$, is approximately equal to $\bar{V}^j$, and this is true for all j. *Q.E.D.*

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