Online Appendix

to

“What to do when you can’t use ‘1.96’ Confidence Intervals for *IV*"

For supplementary material, please visit: http://www.princeton.edu/∼davidlee/wp/SupplementVtF.html

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Appendix

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A Notation Reference

This appendix lays out the notation we use throughout the paper. In general, we use a circumflex ("hat") to indicate sample rather than population quantities. As two leading examples of this principle, we write $\widehat{F}$ for the estimated F-statistic from the first stage regression, and we write $\widehat{t}$ for the estimated t-statistic for a null hypothesis β_0. When we speak of the limiting distribution for an estimated quantity, we drop the circumflex. So for example, under weak instrument asymptotics (WIA), $\widehat{F}$ converges to F, which follows a non-central χ^2 with one degree of freedom and non-centrality parameter f_0^2; $\widehat{t}$ converges to a non-normal distribution that we write simply as t.

Appendix Table A1

Notation	Description
N	sample size
Y, X, and Z	representative observations: outcome, endogenous regressor, and instrument
$\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{Z}$	N-vectors: data on the outcome, endogenous regressor, and instrument
β	population coefficient of interest, i.e., the structural parameter
π or π_N	population coefficient on Z in the first-stage regression of X on Z; under WIA π_N shrinks to zero at rate $1/\sqrt{N}$
$\pi\beta$	population coefficient on Z in the reduced-form regression of Y on Z; also equal to $\pi_N\beta$
$\widehat{\beta}$	IV estimate: $\widehat{\beta} = \mathbf{Z}'\mathbf{Y}/\mathbf{Z}'\mathbf{X}$
$\widehat{\pi}$	sample first-stage coefficient: $\widehat{\pi} = \mathbf{Z}'\mathbf{X}/\mathbf{Z}'\mathbf{Z}$
$\widehat{\pi\beta}$	sample reduced-form coefficient: $\widehat{\pi\beta} = \mathbf{Z}'\mathbf{Y}/\mathbf{Z}'\mathbf{Z} = \widehat{\pi}\cdot\widehat{\beta}$
$\widehat{se}(\widehat{\beta})$, $\widehat{se}(\widehat{\pi})$, and $\widehat{se}(\widehat{\pi\beta})$	robust (HAC) standard errors for $\widehat{\beta}$, $\widehat{\pi}$, and $\widehat{\pi\beta}$, respectively
u	population structural error ($u = Y - X\beta$)
v	population first stage error ($v = X - Z\pi$)
$\widehat{u}$ and $\widehat{v}$	sample residuals corresponding to u and v
ρ	population correlation between Zu and Zv; not consistently estimable under WIA
$\rho(\beta_0)$	population correlation between Zu and Zv implied by β_0; see equation (6)
$\hat{\rho}(\beta_0)$	sample correlation between Zu and Zv implied by β_0; consistently estimable under WIA and $H_0 : \beta = \beta_0$
σ_{11}	asymptotic variance of the reduced-form coefficient $\widehat{\pi\beta}$
σ_{22}	asymptotic variance of the first-stage coefficient $\hat{\pi}$
ρ_{RF}	asymptotic correlation of the reduced-form coefficient $\widehat{\pi\beta}$ and the first-stage coefficient $\hat{\pi}$
$\hat{\sigma}_{11}$	consistent estimator of σ_{11}
$\hat{\sigma}_{22}$	consistent estimator of σ_{22}
$\hat{\rho}_{RF}$	consistent estimator of ρ_{RF}
t, t_{AR}, F, and r	limiting distributions of $\widehat{t}$, $\widehat{t}_{AR}$, $\widehat{F}$, and $\widehat{r}$, respectively, under WIA
$\widehat{t}$	sample t-statistic corresponding to $H_0 : \beta = \beta_0$
$\widehat{t}_{AR}(\beta_0)$ or $\widehat{t}_{AR}$	sample *signed* AR-statistic corresponding to $H_0 : \beta = \beta_0$
$\widehat{AR}$	sample *unsigned* AR-statistic: $\widehat{AR} = \widehat{t}_{AR}^2$

$\widehat{F}$	sample F-statistic; equal to $\frac{N \cdot \widehat{\pi}^2}{\widehat{\sigma}_{22}}$
$\widehat{r}$	sample correlation between $Z\widehat{u}$ and $Z\widehat{v}$: $\widehat{r} = \widehat{\rho}(\widehat{\beta})$; see footnote 8
$c_{tF}(\widehat{F};\alpha)$ or $c_{tF}(\widehat{F})$	tF critical value function of size α; depends on $\widehat{F}$
$c(\widehat{\rho}(\beta_0),\widehat{F};\alpha)$ or $c(\widehat{\rho}(\beta_0),\widehat{F})$	VtF critical value function of size α; depends on both $\widehat{F}$ and on $\widehat{\rho}(\beta_0)$
$k^-_{AR}(\widehat{r},\widehat{F};\alpha)$ or $k^-_{AR}(\widehat{r},\widehat{F})$	left-hand adjustment factor for AR of confidence $1-\alpha$
$k^+_{AR}(\widehat{r},\widehat{F};\alpha)$ or $k^+_{AR}(\widehat{r},\widehat{F})$	right-hand adjustment factor for AR of confidence $1-\alpha$
$k^-(\widehat{r},\widehat{F};\alpha)$ or $k^-(\widehat{r},\widehat{F})$	left-hand adjustment factor for VtF of confidence $1-\alpha$
$k^+(\widehat{r},\widehat{F};\alpha)$ or $k^+(\widehat{r},\widehat{F})$	right-hand adjustment factor for VtF of confidence $1-\alpha$

Notes:

1. As in LMMP (2022), Y, X, and Z (and their N-vector analogues $\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{Z}$) are all sample residuals from a regression of the outcome, endogenous regressor, and instrument, respectively, on a constant and any covariates. This obviates the need for additional notation regarding covariates and means.

2. Our notation for the coefficient on Z in the reduced-form regression of Y on Z is motivated by the fact that $\widehat{\pi\beta} = \widehat{\pi} \cdot \widehat{\beta}$.

3. While the t-statistic depends on the hypothesized β_0, we suppress that dependence and simply write $\widehat{t}$.

B VtF Critical Values and Confidence Interval Adjustment Factors

B.1 VtF Critical Values

Tables A3 and A4 display the critical values for the 5% and 1% tests respectively. For hypothesis testing with a 2SLS t-statistic, calculate the first-stage $\hat{F}$ and the $\hat{\rho}(\beta_0)$ for the hypothesized β_0, then reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0),\hat{F};\alpha)}$. The tables display the critical values $\sqrt{c(\hat{\rho}(\beta_0),\hat{F};\alpha)}$ that should be used in the VtF procedure. Every subtable is associated with a given $\hat{F}$ value, and each cell displays $\sqrt{c(\hat{\rho}(\beta_0),\hat{F};\alpha)}$ for a given $\hat{\rho}(\beta_0)$.

Table A3: *VtF* Critical Values: 5 percent level.

$\hat{F} = 4$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.400	1.400	1.452	1.513	1.567	1.616	1.660	1.702	1.742	1.780
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.817	1.852	1.887	1.920	1.953	1.986	2.018	2.050	2.082	2.113
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.144	2.176	2.207	2.238	2.269	2.300	2.332	2.363	2.395	2.427
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.459	2.492	2.525	2.558	2.592	2.625	2.660	2.695	2.730	2.766
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.802	2.839	2.877	2.915	2.954	2.994	3.035	3.076	3.118	3.162
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.206	3.251	3.297	3.345	3.394	3.444	3.495	3.548	3.603	3.659
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.717	3.776	3.838	3.902	3.968	4.037	4.108	4.182	4.259	4.339
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
4.423	4.510	4.602	4.698	4.799	4.906	5.018	5.137	5.263	5.397
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
5.540	5.694	5.858	6.036	6.228	6.437	6.667	6.919	7.199	7.513
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
7.866	8.271	8.739	9.290	9.952	10.770	11.817	13.226	15.272	18.654

$\hat{F} = 4.101$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.408	1.410	1.458	1.520	1.574	1.623	1.667	1.709	1.749	1.787
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.823	1.859	1.893	1.927	1.960	1.992	2.024	2.056	2.088	2.119
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.150	2.181	2.212	2.243	2.274	2.305	2.336	2.368	2.399	2.431
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.463	2.495	2.527	2.560	2.593	2.627	2.661	2.695	2.730	2.765
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.801	2.837	2.874	2.912	2.950	2.989	3.029	3.070	3.111	3.153
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.196	3.241	3.286	3.332	3.379	3.428	3.478	3.529	3.582	3.636
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.692	3.750	3.810	3.871	3.935	4.000	4.069	4.139	4.213	4.289
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
4.369	4.452	4.539	4.630	4.725	4.825	4.931	5.042	5.159	5.284
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
5.416	5.557	5.707	5.869	6.043	6.231	6.435	6.658	6.904	7.175
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
7.477	7.817	8.204	8.649	9.169	9.789	10.545	11.498	12.751	14.507

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0),\hat{F})}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0),\hat{F})}$.

Table A3 (continued): VtF Critical Values: 5 percent level.

$\hat{F} = 4.265$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.422	1.424	1.469	1.531	1.584	1.633	1.678	1.719	1.759	1.797
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.833	1.869	1.903	1.937	1.969	2.002	2.034	2.065	2.096	2.127
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.158	2.189	2.220	2.250	2.281	2.311	2.342	2.373	2.404	2.435
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.467	2.498	2.530	2.562	2.595	2.627	2.660	2.694	2.728	2.762
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.797	2.833	2.869	2.905	2.942	2.980	3.019	3.058	3.098	3.138
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.180	3.222	3.266	3.310	3.355	3.402	3.450	3.498	3.549	3.600
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.653	3.708	3.764	3.822	3.881	3.943	4.007	4.073	4.141	4.212
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
4.285	4.362	4.442	4.525	4.611	4.702	4.797	4.897	5.002	5.113
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
5.229	5.353	5.484	5.624	5.773	5.933	6.104	6.290	6.491	6.710
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
6.950	7.214	7.509	7.838	8.211	8.638	9.134	9.719	10.426	11.303

$\hat{F} = 4.538$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.442	1.448	1.484	1.546	1.600	1.649	1.693	1.735	1.774	1.812
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.848	1.883	1.917	1.950	1.983	2.015	2.046	2.078	2.108	2.139
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.169	2.199	2.229	2.259	2.289	2.319	2.349	2.379	2.409	2.440
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.470	2.501	2.532	2.563	2.594	2.626	2.658	2.690	2.722	2.755
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.789	2.822	2.857	2.891	2.927	2.963	2.999	3.036	3.073	3.112
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.151	3.190	3.231	3.272	3.314	3.358	3.402	3.447	3.493	3.540
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.589	3.638	3.689	3.742	3.795	3.851	3.908	3.967	4.028	4.090
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
4.155	4.222	4.292	4.364	4.438	4.516	4.597	4.681	4.769	4.862
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
4.958	5.059	5.165	5.277	5.396	5.521	5.653	5.794	5.945	6.105
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
6.278	6.464	6.666	6.885	7.125	7.389	7.682	8.009	8.377	8.797

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$.

Table A3 (continued): VtF Critical Values: 5 percent level.

$\hat{F} = 5.002$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.474	1.485	1.508	1.570	1.623	1.671	1.715	1.757	1.796	1.833
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.868	1.903	1.936	1.969	2.001	2.032	2.063	2.093	2.123	2.152
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.182	2.211	2.240	2.269	2.298	2.326	2.355	2.384	2.413	2.441
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.470	2.499	2.528	2.558	2.587	2.617	2.647	2.677	2.707	2.738
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.769	2.800	2.831	2.863	2.896	2.928	2.961	2.995	3.029	3.064
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.099	3.134	3.170	3.207	3.245	3.283	3.321	3.361	3.401	3.442
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.484	3.527	3.571	3.615	3.661	3.708	3.756	3.805	3.855	3.907
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
3.960	4.014	4.071	4.128	4.188	4.249	4.312	4.378	4.445	4.515
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
4.588	4.663	4.741	4.822	4.907	4.995	5.087	5.183	5.283	5.389
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
5.500	5.617	5.740	5.870	6.008	6.155	6.312	6.479	6.659	6.852

$\hat{F} = 5.838$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.522	1.538	1.541	1.602	1.655	1.702	1.745	1.786	1.823	1.859
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.894	1.927	1.959	1.990	2.020	2.050	2.079	2.108	2.136	2.164
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.192	2.219	2.246	2.273	2.299	2.326	2.352	2.379	2.405	2.431
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.458	2.484	2.510	2.537	2.563	2.589	2.616	2.643	2.669	2.696
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.723	2.751	2.778	2.806	2.833	2.861	2.890	2.918	2.947	2.976
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.005	3.035	3.065	3.095	3.126	3.157	3.188	3.220	3.252	3.285
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.318	3.352	3.386	3.421	3.456	3.492	3.528	3.565	3.603	3.641
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
3.680	3.720	3.760	3.802	3.844	3.887	3.931	3.975	4.021	4.068
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
4.116	4.165	4.216	4.267	4.320	4.375	4.430	4.488	4.547	4.607
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
4.670	4.734	4.801	4.869	4.940	5.014	5.090	5.168	5.250	5.335

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$.

Table A3 (continued): VtF Critical Values: 5 percent level.

$\hat{F} = 7.482$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.593	1.600	1.590	1.645	1.695	1.740	1.781	1.818	1.853	1.886
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.918	1.948	1.978	2.006	2.033	2.060	2.086	2.112	2.137	2.161
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.185	2.209	2.233	2.256	2.279	2.302	2.324	2.347	2.369	2.391
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.413	2.435	2.456	2.478	2.500	2.521	2.543	2.564	2.585	2.607
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.628	2.650	2.671	2.693	2.714	2.736	2.757	2.779	2.800	2.822
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
2.844	2.866	2.888	2.910	2.932	2.954	2.977	2.999	3.022	3.045
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.068	3.091	3.114	3.137	3.161	3.185	3.209	3.233	3.257	3.281
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
3.306	3.331	3.356	3.381	3.407	3.433	3.459	3.485	3.512	3.539
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
3.566	3.594	3.622	3.650	3.678	3.707	3.736	3.766	3.796	3.826
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
3.857	3.888	3.919	3.951	3.984	4.017	4.050	4.084	4.118	4.153

$\hat{F} = 11.214$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.692	1.695	1.699	1.692	1.736	1.775	1.810	1.842	1.871	1.899
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.925	1.950	1.974	1.997	2.019	2.040	2.061	2.081	2.101	2.120
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.138	2.156	2.174	2.192	2.209	2.226	2.242	2.258	2.275	2.290
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.306	2.322	2.337	2.352	2.367	2.382	2.397	2.411	2.426	2.440
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.455	2.469	2.483	2.497	2.511	2.524	2.538	2.552	2.566	2.579
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
2.593	2.606	2.619	2.633	2.646	2.659	2.673	2.686	2.699	2.712
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
2.725	2.738	2.751	2.764	2.777	2.790	2.803	2.816	2.829	2.842
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
2.855	2.868	2.881	2.894	2.907	2.920	2.933	2.946	2.958	2.971
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
2.984	2.997	3.010	3.023	3.036	3.049	3.062	3.075	3.088	3.101
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
3.115	3.128	3.141	3.154	3.167	3.180	3.194	3.207	3.220	3.234

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$.

Table A3 (continued): VtF Critical Values: 5 percent level.

$\hat{F} = 22.516$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.812	1.818	1.808	1.838	1.816	1.805	1.808	1.831	1.851	1.870
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.888	1.904	1.920	1.935	1.949	1.962	1.975	1.987	1.999	2.011
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.022	2.033	2.043	2.053	2.063	2.073	2.082	2.091	2.100	2.109
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.118	2.126	2.135	2.143	2.151	2.159	2.166	2.174	2.181	2.189
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.196	2.203	2.210	2.217	2.224	2.231	2.237	2.244	2.250	2.257
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
2.263	2.269	2.276	2.282	2.288	2.294	2.300	2.306	2.312	2.317
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
2.323	2.329	2.335	2.340	2.346	2.351	2.357	2.362	2.367	2.373
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
2.378	2.383	2.388	2.394	2.399	2.404	2.409	2.414	2.419	2.424
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
2.429	2.434	2.439	2.444	2.448	2.453	2.458	2.463	2.467	2.472
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
2.477	2.481	2.486	2.491	2.495	2.500	2.504	2.509	2.513	2.518

$\hat{F} = 104.67$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.925	1.925	1.925	1.929	1.925	1.930	1.939	1.933	1.925	1.922
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
1.922	1.928	1.935	1.941	1.946	1.950	1.953	1.955	1.957	1.957
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
1.957	1.956	1.952	1.949	1.946	1.943	1.940	1.938	1.936	1.934
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
1.932	1.930	1.929	1.927	1.926	1.924	1.923	1.922	1.921	1.921
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
1.920	1.919	1.919	1.918	1.918	1.917	1.917	1.917	1.917	1.916
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
1.916	1.916	1.917	1.917	1.917	1.917	1.917	1.918	1.918	1.918
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
1.919	1.919	1.920	1.921	1.921	1.922	1.922	1.924	1.925	1.926
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
1.928	1.929	1.930	1.931	1.933	1.934	1.935	1.936	1.937	1.939
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
1.940	1.941	1.942	1.943	1.944	1.945	1.947	1.948	1.949	1.950
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
1.951	1.952	1.953	1.954	1.955	1.956	1.957	1.958	1.959	1.960

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .05)}$.

Table A4: VtF Critical Values: 1 percent level.

$\hat{F} = 6.67$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.824	1.875	1.979	2.065	2.140	2.208	2.270	2.329	2.384	2.438
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.489	2.539	2.588	2.636	2.683	2.729	2.775	2.820	2.865	2.909
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.954	2.998	3.042	3.087	3.131	3.175	3.220	3.265	3.310	3.355
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.401	3.447	3.494	3.541	3.589	3.637	3.686	3.736	3.786	3.837
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.889	3.941	3.995	4.049	4.105	4.162	4.220	4.279	4.339	4.401
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.464	4.529	4.595	4.663	4.733	4.805	4.879	4.955	5.034	5.115
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.198	5.285	5.375	5.467	5.564	5.664	5.768	5.876	5.989	6.108
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
6.231	6.361	6.498	6.641	6.793	6.953	7.123	7.303	7.496	7.702
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.924	8.162	8.421	8.702	9.009	9.348	9.723	10.143	10.616	11.156
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
11.781	12.515	13.396	14.480	15.862	17.711	20.372	24.702	33.787	90.937

$\hat{F} = 6.711$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.827	1.877	1.982	2.068	2.143	2.210	2.273	2.332	2.387	2.441
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.492	2.542	2.591	2.639	2.686	2.732	2.778	2.823	2.868	2.912
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.957	3.001	3.045	3.090	3.134	3.178	3.223	3.268	3.313	3.359
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.404	3.450	3.497	3.544	3.592	3.640	3.689	3.738	3.789	3.839
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.891	3.944	3.997	4.052	4.107	4.164	4.222	4.281	4.341	4.403
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.466	4.530	4.596	4.664	4.734	4.806	4.879	4.955	5.034	5.114
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.198	5.284	5.373	5.465	5.561	5.661	5.764	5.872	5.984	6.102
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
6.225	6.354	6.489	6.632	6.782	6.941	7.109	7.288	7.479	7.683
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.902	8.137	8.392	8.669	8.972	9.305	9.674	10.085	10.549	11.076
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
11.684	12.397	13.248	14.289	15.607	17.352	19.822	23.723	31.367	61.582

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$.

Table A4 (continued): VtF Critical Values: 1 percent level.

$\hat{F} = 6.799$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.832	1.883	1.987	2.073	2.149	2.216	2.279	2.338	2.394	2.447
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.499	2.549	2.597	2.645	2.692	2.738	2.784	2.829	2.874	2.919
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.963	3.008	3.052	3.096	3.140	3.185	3.230	3.274	3.319	3.365
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.411	3.457	3.503	3.550	3.598	3.646	3.695	3.744	3.794	3.845
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.897	3.949	4.002	4.057	4.112	4.168	4.226	4.284	4.344	4.406
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.468	4.533	4.598	4.666	4.735	4.806	4.880	4.955	5.033	5.113
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.195	5.280	5.369	5.460	5.555	5.653	5.756	5.862	5.973	6.089
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
6.210	6.337	6.471	6.611	6.758	6.914	7.079	7.255	7.441	7.640
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.854	8.083	8.331	8.600	8.893	9.214	9.569	9.963	10.406	10.907
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
11.481	12.150	12.941	13.899	15.093	16.641	18.763	21.939	27.500	41.716

$\hat{F} = 6.986$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.845	1.894	1.999	2.085	2.160	2.228	2.291	2.350	2.406	2.460
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.511	2.561	2.610	2.658	2.705	2.751	2.797	2.842	2.887	2.932
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.976	3.021	3.065	3.109	3.153	3.198	3.242	3.287	3.332	3.377
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.423	3.469	3.515	3.562	3.609	3.657	3.706	3.755	3.805	3.855
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.906	3.958	4.011	4.065	4.120	4.176	4.233	4.291	4.350	4.411
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.473	4.536	4.601	4.668	4.736	4.806	4.878	4.952	5.028	5.107
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.188	5.271	5.358	5.447	5.540	5.636	5.736	5.839	5.947	6.060
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
6.177	6.300	6.429	6.564	6.706	6.855	7.014	7.181	7.359	7.548
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.750	7.967	8.200	8.452	8.725	9.022	9.349	9.710	10.111	10.561
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
11.072	11.657	12.339	13.147	14.127	15.350	16.939	19.127	22.420	28.254

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$.

Table A4 (continued): VtF Critical Values: 1 percent level.

$\hat{F} = 7.38$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.869	1.916	2.021	2.108	2.184	2.252	2.315	2.374	2.430	2.484
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.536	2.586	2.635	2.683	2.730	2.776	2.822	2.867	2.912	2.956
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
3.001	3.045	3.089	3.133	3.177	3.221	3.265	3.310	3.354	3.399
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.445	3.490	3.536	3.582	3.629	3.676	3.724	3.773	3.822	3.871
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.922	3.973	4.025	4.077	4.131	4.186	4.241	4.298	4.356	4.415
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.475	4.537	4.600	4.664	4.730	4.798	4.867	4.938	5.012	5.087
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.164	5.244	5.327	5.412	5.499	5.590	5.684	5.782	5.883	5.989
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
6.098	6.212	6.332	6.456	6.587	6.724	6.868	7.020	7.180	7.350
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.530	7.722	7.926	8.145	8.381	8.635	8.910	9.210	9.539	9.902
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
10.305	10.756	11.267	11.851	12.529	13.330	14.297	15.496	17.041	19.140

$\hat{F} = 8.216$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
1.916	1.959	2.063	2.151	2.227	2.295	2.359	2.418	2.474	2.528
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.579	2.629	2.678	2.726	2.772	2.818	2.863	2.908	2.952	2.996
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
3.040	3.084	3.127	3.170	3.213	3.257	3.300	3.343	3.387	3.431
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.475	3.519	3.563	3.608	3.653	3.699	3.745	3.792	3.839	3.886
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.934	3.983	4.033	4.083	4.133	4.185	4.238	4.291	4.345	4.400
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.457	4.514	4.572	4.632	4.693	4.755	4.819	4.884	4.950	5.019
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
5.089	5.160	5.234	5.310	5.388	5.468	5.551	5.636	5.724	5.815
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
5.909	6.006	6.107	6.212	6.321	6.435	6.553	6.676	6.805	6.940
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
7.081	7.230	7.387	7.552	7.727	7.912	8.109	8.319	8.543	8.784
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
9.044	9.325	9.631	9.965	10.333	10.740	11.195	11.707	12.291	12.964

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$.

Table A4 (continued): VtF Critical Values: 1 percent level.

$\hat{F} = 10.062$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
2.000	2.037	2.134	2.222	2.298	2.366	2.428	2.486	2.541	2.594
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.644	2.693	2.740	2.786	2.831	2.875	2.918	2.961	3.003	3.045
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
3.086	3.127	3.168	3.209	3.249	3.290	3.330	3.370	3.410	3.451
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.491	3.531	3.572	3.613	3.653	3.695	3.736	3.778	3.819	3.862
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.904	3.947	3.990	4.034	4.078	4.123	4.168	4.214	4.260	4.307
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.354	4.402	4.450	4.500	4.550	4.601	4.652	4.705	4.758	4.813
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
4.868	4.924	4.981	5.040	5.099	5.160	5.222	5.286	5.350	5.417
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
5.484	5.554	5.625	5.698	5.773	5.849	5.928	6.009	6.093	6.179
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
6.267	6.358	6.453	6.550	6.651	6.755	6.863	6.975	7.091	7.212
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
7.338	7.470	7.607	7.751	7.901	8.059	8.225	8.399	8.584	8.779

$\hat{F} = 14.61$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
2.136	2.167	2.238	2.322	2.393	2.457	2.514	2.568	2.618	2.665
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.711	2.754	2.796	2.836	2.875	2.913	2.951	2.987	3.023	3.058
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
3.093	3.127	3.161	3.194	3.227	3.260	3.292	3.325	3.356	3.388
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.420	3.451	3.483	3.514	3.545	3.576	3.607	3.638	3.669	3.700
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.731	3.762	3.793	3.824	3.855	3.886	3.917	3.948	3.980	4.011
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
4.043	4.074	4.106	4.138	4.170	4.202	4.235	4.268	4.300	4.333
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
4.367	4.400	4.434	4.468	4.502	4.536	4.571	4.606	4.641	4.677
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
4.712	4.749	4.785	4.822	4.859	4.897	4.935	4.973	5.012	5.051
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
5.090	5.130	5.171	5.212	5.253	5.295	5.337	5.380	5.424	5.468
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
5.513	5.558	5.604	5.650	5.698	5.746	5.794	5.844	5.894	5.945

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0), \hat{F}; .01)}$.

Table A4 (continued): VtF Critical Values: 1 percent level.

$\hat{F} = 30.383$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
2.334	2.352	2.382	2.423	2.476	2.524	2.567	2.606	2.642	2.675
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.707	2.737	2.765	2.792	2.817	2.842	2.866	2.889	2.912	2.933
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.954	2.975	2.995	3.014	3.033	3.052	3.071	3.088	3.106	3.124
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
3.141	3.157	3.174	3.190	3.206	3.222	3.238	3.253	3.268	3.283
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
3.298	3.313	3.328	3.342	3.356	3.371	3.385	3.399	3.412	3.426
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
3.440	3.453	3.466	3.480	3.493	3.506	3.519	3.532	3.545	3.557
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
3.570	3.583	3.595	3.608	3.620	3.632	3.645	3.657	3.669	3.681
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
3.693	3.705	3.717	3.729	3.741	3.752	3.764	3.776	3.788	3.799
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
3.811	3.822	3.834	3.845	3.857	3.868	3.880	3.891	3.902	3.914
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
3.925	3.936	3.947	3.959	3.970	3.981	3.992	4.003	4.014	4.025

$\hat{F} = 252.342$

0.000	0.151	0.239	0.301	0.349	0.389	0.423	0.452	0.477	0.500
2.543	2.545	2.549	2.554	2.558	2.561	2.565	2.568	2.571	2.574
0.521	0.540	0.557	0.573	0.588	0.602	0.615	0.628	0.639	0.651
2.577	2.580	2.583	2.586	2.588	2.591	2.593	2.595	2.597	2.599
0.661	0.671	0.681	0.690	0.699	0.707	0.716	0.724	0.731	0.739
2.601	2.602	2.604	2.606	2.608	2.609	2.611	2.613	2.615	2.616
0.746	0.753	0.759	0.766	0.772	0.778	0.784	0.790	0.796	0.801
2.618	2.620	2.622	2.624	2.625	2.627	2.629	2.631	2.633	2.635
0.806	0.812	0.817	0.822	0.827	0.831	0.836	0.841	0.845	0.849
2.636	2.638	2.640	2.642	2.644	2.645	2.647	2.649	2.651	2.652
0.854	0.858	0.862	0.866	0.870	0.874	0.878	0.882	0.885	0.889
2.654	2.656	2.658	2.659	2.661	2.663	2.664	2.666	2.668	2.669
0.893	0.896	0.900	0.903	0.906	0.910	0.913	0.916	0.919	0.923
2.671	2.673	2.674	2.676	2.677	2.679	2.681	2.682	2.684	2.685
0.926	0.929	0.932	0.935	0.938	0.940	0.943	0.946	0.949	0.952
2.687	2.688	2.690	2.691	2.693	2.694	2.695	2.697	2.698	2.700
0.954	0.957	0.960	0.962	0.965	0.967	0.970	0.972	0.975	0.977
2.701	2.702	2.704	2.705	2.707	2.708	2.709	2.711	2.712	2.713
0.980	0.982	0.984	0.987	0.989	0.991	0.993	0.996	0.998	1.000
2.714	2.716	2.717	2.718	2.719	2.721	2.722	2.723	2.724	NA

Notes: Every table is associated with an F-statistic displayed immediately above it. Table entries are in pairs, where the top number is $\hat{\rho}(\beta_0)$ and the number below is $\sqrt{c(\hat{\rho}(\beta_0),\hat{F};.01)}$. For hypothesis testing, reject if and only if $|\hat{t}| > \sqrt{c(\hat{\rho}(\beta_0),\hat{F};.01)}$.

B.2 VtF Confidence Interval Adjustment factors

Table A5 displays 95% Confidence Interval Factors. To construct a confidence interval, calculate the first-stage $\hat{F}$, $\hat{r}$, 2SLS estimator $\hat{\beta}$, and its standard error $\widehat{se}(\hat{\beta})$. Then, if $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k^{-}(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k^{+}(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the confidence interval is: $[\hat{\beta} - k^{+}(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k^{-}(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$. The table displays the k^{+} and k^{-} for given $\hat{F}$ and $\hat{r}$. Each subtable is associated with an $\hat{F}$ value. For $|\hat{r}| = 0.xy$, x is the row digit and y is the column digit of the subtable. The corresponding cell then displays the (k^{-}, k^{+}) pair. When both k^{+} and k^{-} are below 1.96, the table entry is shaded gray. For these entries, the practitioner has the option to use the conventional t-ratio confidence interval and maintain valid inference, though the VtF interval will be smaller and less conservative.

Table A5: VtF 95% Confidence Interval Factors.

$\hat{F} = 5.618$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.285	2.263	2.241	2.220	2.198	2.177	2.156	2.136	2.115	2.095
	2.285	2.307	2.330	2.352	2.375	2.398	2.422	2.445	2.469	2.493
0.1	2.075	2.055	2.036	2.016	1.997	1.978	1.959	1.941	1.922	1.904
	2.518	2.542	2.567	2.592	2.617	2.643	2.669	2.695	2.721	2.747
0.2	1.886	1.868	1.851	1.834	1.816	1.799	1.783	1.766	1.750	1.734
	2.774	2.801	2.829	2.856	2.884	2.912	2.940	2.969	2.998	3.027
0.3	1.718	1.702	1.686	1.671	1.655	1.640	1.625	1.611	1.596	1.582
	3.056	3.086	3.115	3.145	3.176	3.206	3.237	3.268	3.299	3.331
0.4	1.568	1.554	1.540	1.526	1.513	1.507	1.507	1.508	1.512	1.516
	3.363	3.395	3.427	3.459	3.492	3.525	3.558	3.591	3.625	3.659
0.5	1.521	1.526	1.524	1.513	1.509	1.514	1.516	1.510	1.513	1.513
	3.693	3.727	3.762	3.796	3.831	3.866	3.902	3.937	3.973	4.009
0.6	1.510	1.510	1.510	1.511	1.510	1.511	1.511	1.511	1.512	1.513
	4.045	4.081	4.118	4.154	4.191	4.228	4.266	4.303	4.341	4.378
0.7	1.510	1.517	1.509	1.517	1.524	1.520	1.508	1.507	1.516	1.527
	4.416	4.454	4.493	4.531	4.570	4.608	4.647	4.686	4.725	4.765
0.8	1.538	1.551	1.563	1.577	1.591	1.606	1.621	1.638	1.656	1.675
	4.804	4.844	4.884	4.923	4.964	5.004	5.044	5.084	5.125	5.166
0.9	1.696	1.719	1.743	1.771	1.801	1.836	1.877	4.541	5.021	5.349
	5.206	5.247	5.288	5.329	5.371	5.412	5.453	5.495	5.537	5.578

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|, \hat{F})$ and the number below is $k_1(|\hat{r}|, \hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$.

Table A5 (continued): VtF 95% Confidence Interval Factors.

$\hat{F} = 8.081$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.017	2.002	1.988	1.974	1.960	1.946	1.933	1.919	1.906	1.892
	2.017	2.031	2.046	2.060	2.075	2.090	2.104	2.119	2.134	2.150
0.1	1.879	1.865	1.852	1.839	1.826	1.814	1.801	1.788	1.776	1.763
	2.165	2.180	2.196	2.211	2.227	2.243	2.259	2.275	2.291	2.307
0.2	1.751	1.739	1.726	1.714	1.702	1.691	1.679	1.667	1.656	1.644
	2.323	2.339	2.356	2.373	2.389	2.406	2.423	2.440	2.457	2.474
0.3	1.633	1.621	1.611	1.609	1.609	1.610	1.613	1.617	1.621	1.626
	2.491	2.509	2.526	2.544	2.561	2.579	2.597	2.615	2.633	2.651
0.4	1.632	1.632	1.627	1.617	1.611	1.615	1.620	1.619	1.613	1.618
	2.669	2.687	2.706	2.724	2.743	2.762	2.780	2.799	2.818	2.837
0.5	1.613	1.614	1.613	1.615	1.614	1.614	1.614	1.614	1.614	1.614
	2.856	2.876	2.895	2.914	2.934	2.953	2.973	2.993	3.013	3.033
0.6	1.615	1.614	1.614	1.613	1.617	1.615	1.622	1.612	1.616	1.625
	3.053	3.073	3.093	3.113	3.134	3.154	3.175	3.195	3.216	3.237
0.7	1.631	1.628	1.614	1.609	1.611	1.620	1.632	1.644	1.656	1.669
	3.258	3.279	3.300	3.321	3.342	3.363	3.384	3.406	3.427	3.449
0.8	1.683	1.697	1.712	1.728	1.744	1.762	1.781	1.801	1.822	1.845
	3.471	3.492	3.514	3.536	3.558	3.580	3.602	3.624	3.646	3.669
0.9	1.871	1.898	1.928	1.962	2.001	2.045	2.097	2.162	2.246	2.375
	3.691	3.714	3.736	3.759	3.781	3.804	3.827	3.850	3.872	3.895

$\hat{F} = 11.623$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.878	1.868	1.857	1.847	1.837	1.827	1.817	1.807	1.797	1.787
	1.878	1.888	1.899	1.909	1.920	1.930	1.941	1.952	1.962	1.973
0.1	1.777	1.768	1.758	1.748	1.739	1.729	1.720	1.710	1.701	1.695
	1.984	1.995	2.006	2.017	2.028	2.039	2.051	2.062	2.073	2.085
0.2	1.694	1.693	1.695	1.697	1.700	1.703	1.708	1.712	1.718	1.721
	2.096	2.108	2.119	2.131	2.142	2.154	2.166	2.178	2.190	2.202
0.3	1.720	1.715	1.707	1.697	1.697	1.701	1.706	1.708	1.701	1.699
	2.214	2.226	2.238	2.250	2.262	2.274	2.287	2.299	2.312	2.324
0.4	1.704	1.698	1.702	1.699	1.699	1.700	1.700	1.699	1.699	1.699
	2.337	2.349	2.362	2.375	2.387	2.400	2.413	2.426	2.439	2.452
0.5	1.699	1.699	1.699	1.699	1.699	1.701	1.700	1.703	1.701	1.702
	2.465	2.478	2.491	2.504	2.518	2.531	2.544	2.558	2.571	2.585
0.6	1.699	1.707	1.704	1.697	1.701	1.710	1.718	1.721	1.711	1.701
	2.598	2.612	2.626	2.639	2.653	2.667	2.681	2.695	2.709	2.723
0.7	1.695	1.694	1.698	1.709	1.720	1.732	1.745	1.758	1.772	1.786
	2.737	2.751	2.765	2.779	2.793	2.808	2.822	2.836	2.851	2.865
0.8	1.801	1.816	1.832	1.850	1.868	1.887	1.908	1.930	1.953	1.979
	2.880	2.894	2.909	2.924	2.938	2.953	2.968	2.983	2.997	3.012
0.9	2.007	2.037	2.071	2.108	2.151	2.200	2.259	2.332	2.429	2.579
	3.027	3.042	3.057	3.072	3.087	3.103	3.118	3.133	3.148	3.164

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A5 (continued): VtF 95% Confidence Interval Factors.

$\hat{F} = 16.719$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.797	1.789	1.781	1.773	1.765	1.763	1.761	1.761	1.761	1.763
	1.797	1.805	1.812	1.820	1.828	1.837	1.845	1.853	1.861	1.869
0.1	1.765	1.768	1.771	1.775	1.779	1.784	1.789	1.792	1.792	1.789
	1.877	1.886	1.894	1.902	1.911	1.919	1.927	1.936	1.944	1.953
0.2	1.783	1.776	1.767	1.764	1.766	1.770	1.776	1.778	1.773	1.766
	1.961	1.970	1.979	1.987	1.996	2.005	2.014	2.022	2.031	2.040
0.3	1.768	1.773	1.768	1.768	1.770	1.769	1.767	1.768	1.769	1.768
	2.049	2.058	2.067	2.076	2.085	2.094	2.103	2.112	2.121	2.131
0.4	1.767	1.768	1.767	1.767	1.767	1.768	1.767	1.768	1.767	1.768
	2.140	2.149	2.159	2.168	2.177	2.187	2.196	2.206	2.215	2.225
0.5	1.767	1.768	1.771	1.766	1.773	1.767	1.768	1.776	1.777	1.767
	2.234	2.244	2.253	2.263	2.273	2.283	2.292	2.302	2.312	2.322
0.6	1.765	1.771	1.780	1.787	1.792	1.791	1.779	1.770	1.764	1.761
	2.332	2.342	2.352	2.362	2.372	2.382	2.392	2.402	2.412	2.422
0.7	1.762	1.767	1.778	1.789	1.801	1.813	1.825	1.838	1.852	1.866
	2.432	2.443	2.453	2.463	2.473	2.484	2.494	2.505	2.515	2.525
0.8	1.880	1.896	1.912	1.929	1.947	1.966	1.986	2.007	2.030	2.055
	2.536	2.546	2.557	2.568	2.578	2.589	2.600	2.610	2.621	2.632
0.9	2.081	2.110	2.142	2.177	2.216	2.261	2.313	2.375	2.454	2.563
	2.643	2.653	2.664	2.675	2.686	2.697	2.708	2.719	2.730	2.741

$\hat{F} = 24.049$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.830	1.834	1.838	1.843	1.847	1.847	1.846	1.843	1.839	1.833
	1.830	1.826	1.823	1.820	1.817	1.815	1.814	1.813	1.813	1.813
0.1	1.826	1.818	1.817	1.818	1.821	1.825	1.831	1.831	1.828	1.820
	1.815	1.817	1.822	1.829	1.835	1.842	1.848	1.855	1.861	1.868
0.2	1.818	1.822	1.827	1.823	1.819	1.824	1.820	1.822	1.819	1.821
	1.875	1.881	1.888	1.895	1.902	1.908	1.915	1.922	1.929	1.936
0.3	1.820	1.820	1.820	1.820	1.820	1.820	1.820	1.820	1.820	1.820
	1.942	1.949	1.956	1.963	1.970	1.977	1.984	1.991	1.998	2.005
0.4	1.820	1.820	1.820	1.820	1.821	1.822	1.822	1.819	1.824	1.819
	2.012	2.019	2.026	2.034	2.041	2.048	2.055	2.062	2.070	2.077
0.5	1.824	1.825	1.818	1.822	1.829	1.832	1.824	1.818	1.817	1.824
	2.084	2.092	2.099	2.106	2.114	2.121	2.128	2.136	2.143	2.151
0.6	1.832	1.839	1.844	1.847	1.844	1.834	1.826	1.819	1.815	1.813
	2.158	2.166	2.173	2.181	2.189	2.196	2.204	2.211	2.219	2.227
0.7	1.813	1.817	1.825	1.836	1.846	1.857	1.869	1.881	1.893	1.906
	2.235	2.242	2.250	2.258	2.266	2.273	2.281	2.289	2.297	2.305
0.8	1.919	1.933	1.947	1.962	1.978	1.994	2.011	2.029	2.049	2.069
	2.313	2.321	2.329	2.337	2.345	2.353	2.361	2.369	2.377	2.385
0.9	2.091	2.114	2.139	2.166	2.196	2.228	2.264	2.305	2.352	2.407
	2.393	2.401	2.409	2.417	2.426	2.434	2.442	2.450	2.458	2.467

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|, \hat{F})$ and the number below is $k_1(|\hat{r}|, \hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$.

Table A5 (continued): VtF 95% Confidence Interval Factors.

$\hat{F} = 34.593$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.857	1.856	1.857	1.859	1.863	1.867	1.872	1.871	1.868	1.863
	1.857	1.862	1.868	1.874	1.878	1.883	1.886	1.888	1.889	1.888
0.1	1.858	1.859	1.862	1.867	1.864	1.858	1.860	1.864	1.859	1.861
	1.885	1.881	1.877	1.873	1.869	1.866	1.863	1.861	1.858	1.856
0.2	1.861	1.861	1.859	1.859	1.859	1.860	1.860	1.859	1.860	1.859
	1.854	1.853	1.852	1.852	1.852	1.852	1.854	1.856	1.859	1.864
0.3	1.860	1.859	1.859	1.860	1.859	1.859	1.860	1.860	1.860	1.860
	1.869	1.875	1.880	1.886	1.891	1.897	1.902	1.908	1.913	1.919
0.4	1.861	1.861	1.859	1.863	1.859	1.864	1.859	1.862	1.866	1.862
	1.924	1.930	1.936	1.941	1.947	1.952	1.958	1.964	1.969	1.975
0.5	1.857	1.861	1.868	1.872	1.869	1.862	1.857	1.856	1.860	1.868
	1.981	1.987	1.992	1.998	2.004	2.010	2.015	2.021	2.027	2.033
0.6	1.875	1.881	1.886	1.888	1.888	1.880	1.872	1.865	1.860	1.856
	2.039	2.044	2.050	2.056	2.062	2.068	2.074	2.080	2.086	2.092
0.7	1.853	1.852	1.852	1.855	1.862	1.872	1.881	1.891	1.901	1.912
	2.098	2.104	2.110	2.116	2.122	2.128	2.134	2.140	2.146	2.152
0.8	1.923	1.934	1.946	1.958	1.970	1.983	1.996	2.010	2.025	2.040
	2.158	2.164	2.171	2.177	2.183	2.189	2.195	2.202	2.208	2.214
0.9	2.056	2.073	2.091	2.110	2.130	2.151	2.173	2.198	2.224	2.252
	2.220	2.226	2.233	2.239	2.245	2.252	2.258	2.264	2.271	2.277

$\hat{F} = 49.759$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.887	1.886	1.888	1.892	1.896	1.894	1.890	1.887	1.890	1.893
	1.887	1.893	1.897	1.900	1.902	1.900	1.895	1.892	1.889	1.886
0.1	1.889	1.888	1.892	1.888	1.891	1.888	1.890	1.890	1.890	1.889
	1.885	1.885	1.886	1.891	1.896	1.901	1.906	1.909	1.913	1.915
0.2	1.889	1.889	1.889	1.889	1.888	1.889	1.888	1.888	1.888	1.888
	1.917	1.919	1.919	1.918	1.916	1.912	1.908	1.905	1.902	1.898
0.3	1.888	1.889	1.889	1.888	1.888	1.889	1.890	1.890	1.888	1.889
	1.896	1.893	1.891	1.888	1.886	1.885	1.883	1.882	1.881	1.881
0.4	1.890	1.889	1.889	1.893	1.889	1.888	1.893	1.896	1.891	1.887
	1.880	1.881	1.881	1.882	1.883	1.885	1.888	1.893	1.897	1.902
0.5	1.888	1.894	1.899	1.901	1.899	1.892	1.888	1.885	1.885	1.890
	1.906	1.911	1.915	1.920	1.925	1.929	1.934	1.938	1.943	1.948
0.6	1.897	1.903	1.909	1.914	1.917	1.919	1.918	1.912	1.905	1.899
	1.952	1.957	1.962	1.966	1.971	1.976	1.980	1.985	1.990	1.994
0.7	1.894	1.889	1.886	1.883	1.881	1.880	1.881	1.883	1.888	1.896
	1.999	2.004	2.009	2.013	2.018	2.023	2.028	2.032	2.037	2.042
0.8	1.905	1.914	1.922	1.932	1.941	1.951	1.961	1.971	1.982	1.993
	2.047	2.052	2.056	2.061	2.066	2.071	2.076	2.081	2.086	2.091
0.9	2.004	2.016	2.028	2.040	2.054	2.067	2.081	2.096	2.112	2.128
	2.095	2.100	2.105	2.110	2.115	2.120	2.125	2.130	2.135	2.140

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A5 (continued): VtF 95% Confidence Interval Factors.

$\hat{F} = 71.575$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.912	1.914	1.912	1.908	1.911	1.912	1.909	1.912	1.909	1.911
	1.912	1.909	1.908	1.913	1.916	1.917	1.914	1.911	1.908	1.907
0.1	1.909	1.910	1.911	1.910	1.909	1.910	1.910	1.910	1.910	1.909
	1.909	1.913	1.917	1.921	1.922	1.923	1.921	1.917	1.914	1.911
0.2	1.910	1.909	1.909	1.909	1.909	1.909	1.909	1.909	1.910	1.910
	1.909	1.907	1.906	1.906	1.907	1.909	1.914	1.918	1.922	1.926
0.3	1.910	1.910	1.910	1.910	1.911	1.911	1.909	1.912	1.909	1.913
	1.929	1.932	1.935	1.937	1.939	1.940	1.941	1.941	1.940	1.938
0.4	1.909	1.910	1.914	1.912	1.908	1.910	1.915	1.917	1.913	1.909
	1.935	1.932	1.928	1.926	1.923	1.920	1.918	1.915	1.913	1.911
0.5	1.907	1.910	1.916	1.920	1.923	1.923	1.917	1.912	1.909	1.906
	1.909	1.908	1.906	1.905	1.904	1.903	1.902	1.901	1.901	1.901
0.6	1.906	1.907	1.913	1.919	1.925	1.930	1.934	1.937	1.940	1.941
	1.901	1.902	1.903	1.904	1.905	1.907	1.910	1.914	1.918	1.922
0.7	1.940	1.935	1.930	1.924	1.919	1.915	1.911	1.908	1.905	1.903
	1.926	1.929	1.933	1.937	1.941	1.945	1.948	1.952	1.956	1.960
0.8	1.902	1.901	1.902	1.903	1.906	1.911	1.919	1.926	1.934	1.941
	1.964	1.968	1.971	1.975	1.979	1.983	1.987	1.991	1.995	1.999
0.9	1.949	1.958	1.966	1.975	1.983	1.992	2.002	2.011	2.021	2.031
	2.002	2.006	2.010	2.014	2.018	2.022	2.026	2.030	2.034	2.038

$\hat{F} = 104.67$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	1.928	1.925	1.926	1.926	1.926	1.925	1.926	1.926	1.926	1.925
	1.928	1.925	1.926	1.929	1.926	1.924	1.926	1.929	1.930	1.927
0.1	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925
	1.924	1.924	1.927	1.930	1.932	1.933	1.930	1.927	1.924	1.923
0.2	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925	1.925
	1.923	1.925	1.929	1.932	1.935	1.937	1.939	1.939	1.938	1.935
0.3	1.926	1.926	1.925	1.925	1.927	1.924	1.927	1.925	1.926	1.929
	1.932	1.929	1.926	1.924	1.923	1.922	1.921	1.921	1.922	1.924
0.4	1.925	1.924	1.928	1.930	1.926	1.924	1.925	1.929	1.932	1.932
	1.928	1.931	1.935	1.938	1.941	1.944	1.947	1.949	1.951	1.953
0.5	1.928	1.924	1.923	1.924	1.928	1.933	1.936	1.938	1.939	1.936
	1.955	1.956	1.957	1.957	1.957	1.957	1.955	1.953	1.950	1.947
0.6	1.931	1.927	1.924	1.922	1.921	1.922	1.924	1.929	1.935	1.939
	1.944	1.942	1.940	1.937	1.935	1.933	1.931	1.929	1.927	1.926
0.7	1.944	1.948	1.951	1.954	1.956	1.957	1.957	1.955	1.950	1.946
	1.924	1.923	1.922	1.920	1.919	1.919	1.918	1.917	1.917	1.917
0.8	1.941	1.937	1.933	1.930	1.926	1.924	1.921	1.919	1.918	1.917
	1.916	1.917	1.917	1.917	1.918	1.919	1.920	1.921	1.922	1.926
0.9	1.916	1.917	1.918	1.919	1.921	1.927	1.934	1.940	1.947	1.953
	1.929	1.932	1.935	1.938	1.941	1.944	1.947	1.951	1.954	1.957

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A6: VtF 99% Confidence Interval Factors.

$\hat{F} = 8.081$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	4.051	3.994	3.937	3.881	3.826	3.771	3.718	3.665	3.614	3.563
	4.051	4.110	4.170	4.230	4.292	4.354	4.417	4.482	4.547	4.613
0.1	3.513	3.464	3.415	3.367	3.321	3.275	3.229	3.185	3.141	3.098
	4.680	4.748	4.817	4.887	4.958	5.030	5.102	5.176	5.251	5.326
0.2	3.056	3.015	2.974	2.934	2.894	2.856	2.818	2.780	2.744	2.708
	5.403	5.480	5.558	5.638	5.718	5.799	5.881	5.964	6.047	6.132
0.3	2.673	2.638	2.604	2.570	2.538	2.505	2.474	2.443	2.412	2.382
	6.217	6.304	6.391	6.479	6.567	6.657	6.747	6.838	6.930	7.022
0.4	2.353	2.324	2.296	2.268	2.241	2.214	2.188	2.162	2.136	2.111
	7.115	7.209	7.304	7.399	7.495	7.592	7.689	7.787	7.886	7.985
0.5	2.087	2.063	2.039	2.016	1.994	1.972	1.954	1.942	1.935	1.929
	8.084	8.185	8.285	8.387	8.489	8.591	8.694	8.797	8.901	9.006
0.6	1.924	1.920	1.916	1.914	1.912	1.910	1.909	1.909	1.909	1.910
	9.111	9.216	9.321	9.428	9.534	9.641	9.748	9.856	9.964	10.073
0.7	1.911	1.913	1.916	1.919	1.923	1.929	1.934	1.941	1.951	1.962
	10.182	10.291	10.400	10.510	10.620	10.731	10.842	10.953	11.064	11.176
0.8	1.975	1.989	2.004	2.020	2.036	2.053	3.564	9.031	9.563	10.018
	11.288	11.400	11.512	11.625	11.738	11.851	11.964	12.078	12.192	12.306
0.9	10.428	10.806	11.162	11.500	11.824	12.136	12.439	12.732	13.019	13.299
	12.420	12.535	12.650	12.765	12.880	12.995	13.110	13.226	13.342	13.458

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A6 (continued): VtF 99% Confidence Interval Factors.

$\hat{F} = 11.623$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	3.182	3.153	3.124	3.094	3.066	3.037	3.009	2.981	2.954	2.926
	3.182	3.212	3.242	3.273	3.303	3.334	3.366	3.397	3.429	3.461
0.1	2.899	2.872	2.846	2.820	2.794	2.768	2.742	2.717	2.692	2.668
	3.493	3.526	3.559	3.592	3.625	3.659	3.693	3.727	3.762	3.796
0.2	2.643	2.619	2.595	2.572	2.548	2.525	2.502	2.480	2.458	2.435
	3.831	3.867	3.902	3.938	3.974	4.011	4.047	4.084	4.121	4.158
0.3	2.414	2.392	2.371	2.350	2.329	2.308	2.288	2.268	2.248	2.228
	4.196	4.234	4.272	4.310	4.349	4.388	4.427	4.466	4.506	4.546
0.4	2.209	2.190	2.171	2.152	2.134	2.118	2.105	2.095	2.089	2.084
	4.586	4.626	4.666	4.707	4.748	4.789	4.830	4.872	4.914	4.956
0.5	2.079	2.074	2.070	2.067	2.064	2.061	2.059	2.057	2.056	2.056
	4.998	5.041	5.083	5.126	5.169	5.212	5.256	5.299	5.343	5.387
0.6	2.055	2.055	2.056	2.057	2.058	2.060	2.063	2.066	2.069	2.074
	5.432	5.476	5.521	5.565	5.610	5.656	5.701	5.746	5.792	5.838
0.7	2.079	2.084	2.090	2.098	2.108	2.120	2.133	2.148	2.163	2.179
	5.884	5.930	5.977	6.023	6.070	6.117	6.164	6.211	6.258	6.305
0.8	2.196	2.213	2.232	2.251	2.271	2.293	2.315	2.339	2.365	2.393
	6.353	6.401	6.449	6.497	6.545	6.593	6.642	6.690	6.739	6.788
0.9	2.423	2.455	2.491	2.531	2.575	2.626	2.685	2.758	6.353	6.918
	6.837	6.886	6.935	6.985	7.034	7.084	7.134	7.183	7.233	7.283

$\hat{F} = 16.719$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.829	2.809	2.790	2.771	2.752	2.733	2.714	2.695	2.677	2.658
	2.829	2.848	2.868	2.888	2.908	2.928	2.949	2.969	2.990	3.010
0.1	2.640	2.622	2.604	2.586	2.568	2.550	2.533	2.516	2.498	2.481
	3.031	3.052	3.073	3.095	3.116	3.138	3.159	3.181	3.203	3.225
0.2	2.464	2.448	2.431	2.414	2.398	2.382	2.365	2.349	2.333	2.318
	3.247	3.269	3.292	3.315	3.337	3.360	3.383	3.406	3.429	3.453
0.3	2.302	2.286	2.272	2.258	2.246	2.236	2.228	2.222	2.217	2.212
	3.476	3.500	3.524	3.547	3.571	3.596	3.620	3.644	3.669	3.693
0.4	2.208	2.203	2.199	2.196	2.193	2.190	2.188	2.186	2.184	2.182
	3.718	3.743	3.768	3.793	3.818	3.843	3.869	3.894	3.920	3.946
0.5	2.181	2.180	2.180	2.179	2.180	2.180	2.181	2.182	2.183	2.185
	3.972	3.998	4.024	4.050	4.077	4.103	4.130	4.156	4.183	4.210
0.6	2.187	2.190	2.193	2.196	2.200	2.205	2.210	2.216	2.222	2.229
	4.237	4.264	4.291	4.319	4.346	4.374	4.401	4.429	4.457	4.485
0.7	2.239	2.250	2.263	2.277	2.292	2.308	2.324	2.341	2.359	2.378
	4.513	4.541	4.569	4.598	4.626	4.655	4.683	4.712	4.741	4.770
0.8	2.397	2.418	2.439	2.462	2.485	2.511	2.538	2.566	2.597	2.630
	4.799	4.828	4.857	4.886	4.916	4.945	4.975	5.004	5.034	5.064
0.9	2.666	2.706	2.749	2.798	2.853	2.916	2.992	3.085	3.207	3.393
	5.094	5.124	5.154	5.184	5.214	5.244	5.275	5.305	5.336	5.366

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A6 (continued): VtF 99% Confidence Interval Factors.

$\hat{F} = 24.049$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.643	2.628	2.614	2.600	2.586	2.572	2.559	2.545	2.531	2.518
	2.643	2.657	2.671	2.686	2.700	2.715	2.729	2.744	2.759	2.774
0.1	2.504	2.491	2.477	2.464	2.451	2.438	2.425	2.412	2.399	2.386
	2.789	2.804	2.819	2.834	2.849	2.865	2.880	2.896	2.911	2.927
0.2	2.375	2.364	2.355	2.346	2.339	2.333	2.327	2.323	2.319	2.315
	2.943	2.959	2.974	2.990	3.006	3.022	3.039	3.055	3.071	3.088
0.3	2.311	2.307	2.304	2.300	2.298	2.295	2.292	2.290	2.288	2.287
	3.104	3.121	3.137	3.154	3.171	3.188	3.205	3.222	3.239	3.256
0.4	2.285	2.284	2.283	2.282	2.281	2.281	2.280	2.280	2.281	2.281
	3.273	3.290	3.308	3.325	3.342	3.360	3.378	3.395	3.413	3.431
0.5	2.282	2.283	2.284	2.286	2.287	2.289	2.292	2.295	2.298	2.301
	3.449	3.467	3.485	3.503	3.521	3.539	3.558	3.576	3.595	3.613
0.6	2.305	2.310	2.315	2.320	2.325	2.331	2.339	2.348	2.358	2.371
	3.632	3.650	3.669	3.688	3.707	3.726	3.745	3.764	3.783	3.802
0.7	2.384	2.398	2.414	2.429	2.446	2.463	2.481	2.499	2.518	2.538
	3.821	3.840	3.860	3.879	3.899	3.918	3.938	3.958	3.977	3.997
0.8	2.560	2.582	2.605	2.629	2.655	2.683	2.712	2.743	2.777	2.813
	4.017	4.037	4.057	4.077	4.097	4.117	4.137	4.158	4.178	4.198
0.9	2.852	2.895	2.943	2.996	3.056	3.126	3.209	3.312	3.449	3.660
	4.219	4.239	4.260	4.280	4.301	4.322	4.342	4.363	4.384	4.405

$\hat{F} = 34.593$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.533	2.522	2.511	2.500	2.490	2.479	2.470	2.460	2.451	2.443
	2.533	2.544	2.555	2.566	2.577	2.588	2.599	2.610	2.622	2.633
0.1	2.436	2.429	2.423	2.418	2.413	2.409	2.405	2.402	2.399	2.395
	2.644	2.656	2.667	2.679	2.690	2.702	2.713	2.725	2.737	2.749
0.2	2.392	2.388	2.385	2.382	2.380	2.377	2.375	2.373	2.371	2.369
	2.760	2.772	2.784	2.796	2.808	2.820	2.832	2.844	2.857	2.869
0.3	2.368	2.366	2.365	2.363	2.362	2.362	2.361	2.360	2.360	2.360
	2.881	2.894	2.906	2.918	2.931	2.943	2.956	2.968	2.981	2.994
0.4	2.360	2.360	2.360	2.360	2.361	2.362	2.362	2.364	2.365	2.367
	3.007	3.019	3.032	3.045	3.058	3.071	3.084	3.097	3.110	3.123
0.5	2.368	2.370	2.372	2.375	2.378	2.381	2.384	2.388	2.392	2.397
	3.136	3.150	3.163	3.176	3.190	3.203	3.216	3.230	3.243	3.257
0.6	2.401	2.406	2.411	2.417	2.424	2.433	2.442	2.453	2.466	2.479
	3.271	3.284	3.298	3.312	3.325	3.339	3.353	3.367	3.381	3.395
0.7	2.493	2.507	2.523	2.538	2.555	2.572	2.589	2.608	2.627	2.646
	3.409	3.423	3.437	3.451	3.466	3.480	3.494	3.508	3.523	3.537
0.8	2.667	2.689	2.711	2.735	2.760	2.787	2.815	2.845	2.877	2.911
	3.552	3.566	3.581	3.595	3.610	3.624	3.639	3.654	3.669	3.683
0.9	2.948	2.989	3.033	3.081	3.136	3.198	3.270	3.355	3.463	3.608
	3.698	3.713	3.728	3.743	3.758	3.773	3.788	3.803	3.818	3.833

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A6 (continued): VtF 99% Confidence Interval Factors.

$\hat{F} = 49.759$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.485	2.481	2.477	2.473	2.470	2.467	2.464	2.462	2.459	2.456
	2.485	2.490	2.495	2.501	2.507	2.513	2.520	2.528	2.536	2.544
0.1	2.453	2.450	2.447	2.444	2.442	2.440	2.438	2.436	2.434	2.433
	2.552	2.561	2.570	2.579	2.588	2.597	2.606	2.615	2.624	2.633
0.2	2.431	2.429	2.428	2.427	2.425	2.424	2.423	2.423	2.422	2.421
	2.642	2.652	2.661	2.670	2.679	2.689	2.698	2.708	2.717	2.726
0.3	2.421	2.420	2.420	2.420	2.420	2.420	2.420	2.420	2.420	2.421
	2.736	2.745	2.755	2.765	2.774	2.784	2.794	2.803	2.813	2.823
0.4	2.422	2.422	2.423	2.424	2.426	2.427	2.428	2.430	2.432	2.434
	2.832	2.842	2.852	2.862	2.872	2.882	2.892	2.902	2.912	2.922
0.5	2.436	2.439	2.441	2.444	2.447	2.451	2.455	2.459	2.463	2.467
	2.932	2.942	2.952	2.962	2.973	2.983	2.993	3.003	3.014	3.024
0.6	2.471	2.475	2.481	2.487	2.494	2.503	2.512	2.522	2.533	2.545
	3.034	3.045	3.055	3.066	3.076	3.086	3.097	3.108	3.118	3.129
0.7	2.558	2.572	2.586	2.600	2.615	2.631	2.646	2.663	2.680	2.698
	3.139	3.150	3.161	3.171	3.182	3.193	3.204	3.215	3.226	3.236
0.8	2.716	2.735	2.755	2.776	2.798	2.821	2.845	2.870	2.896	2.925
	3.247	3.258	3.269	3.280	3.291	3.302	3.313	3.324	3.336	3.347
0.9	2.955	2.987	3.021	3.058	3.099	3.143	3.192	3.247	3.309	3.383
	3.358	3.369	3.380	3.392	3.403	3.414	3.425	3.437	3.448	3.460

$\hat{F} = 71.575$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.497	2.495	2.492	2.490	2.488	2.487	2.485	2.483	2.481	2.480
	2.497	2.500	2.502	2.505	2.507	2.510	2.512	2.515	2.517	2.520
0.1	2.478	2.477	2.476	2.474	2.473	2.472	2.471	2.470	2.469	2.468
	2.523	2.526	2.529	2.533	2.537	2.541	2.546	2.550	2.556	2.561
0.2	2.467	2.467	2.466	2.466	2.465	2.465	2.465	2.464	2.464	2.464
	2.567	2.573	2.579	2.586	2.593	2.599	2.607	2.614	2.621	2.629
0.3	2.464	2.464	2.465	2.465	2.465	2.466	2.466	2.467	2.468	2.468
	2.636	2.644	2.651	2.659	2.666	2.674	2.682	2.689	2.697	2.705
0.4	2.469	2.470	2.472	2.473	2.474	2.476	2.478	2.479	2.481	2.483
	2.712	2.720	2.728	2.735	2.743	2.751	2.759	2.767	2.774	2.782
0.5	2.486	2.488	2.490	2.493	2.496	2.500	2.503	2.507	2.510	2.514
	2.790	2.798	2.806	2.814	2.822	2.830	2.838	2.846	2.854	2.862
0.6	2.517	2.521	2.525	2.530	2.536	2.543	2.550	2.558	2.567	2.576
	2.870	2.878	2.886	2.895	2.903	2.911	2.919	2.927	2.936	2.944
0.7	2.587	2.598	2.609	2.621	2.634	2.647	2.660	2.674	2.688	2.703
	2.952	2.960	2.969	2.977	2.985	2.994	3.002	3.010	3.019	3.027
0.8	2.718	2.733	2.750	2.766	2.783	2.801	2.820	2.839	2.859	2.880
	3.036	3.044	3.053	3.061	3.070	3.078	3.087	3.095	3.104	3.113
0.9	2.902	2.925	2.949	2.975	3.002	3.031	3.061	3.094	3.129	3.167
	3.121	3.130	3.139	3.147	3.156	3.165	3.173	3.182	3.191	3.200

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|,\hat{F})$ and the number below is $k_1(|\hat{r}|,\hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|,\hat{F})\widehat{se}(\hat{\beta})]$.

Table A6 (continued): VtF 99% Confidence Interval Factors.

$\hat{F} = 104.67$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.514	2.513	2.511	2.510	2.509	2.508	2.507	2.506	2.505	2.504
	2.514	2.515	2.517	2.518	2.520	2.521	2.522	2.524	2.526	2.527
0.1	2.504	2.503	2.502	2.501	2.501	2.500	2.500	2.499	2.499	2.499
	2.529	2.531	2.533	2.536	2.538	2.540	2.542	2.544	2.546	2.548
0.2	2.498	2.498	2.498	2.498	2.498	2.498	2.498	2.498	2.498	2.498
	2.550	2.553	2.555	2.557	2.559	2.562	2.565	2.568	2.571	2.574
0.3	2.499	2.499	2.500	2.500	2.501	2.501	2.502	2.503	2.504	2.505
	2.578	2.582	2.586	2.590	2.594	2.599	2.604	2.608	2.614	2.619
0.4	2.506	2.507	2.508	2.509	2.510	2.512	2.513	2.515	2.517	2.519
	2.624	2.630	2.636	2.641	2.647	2.653	2.659	2.665	2.672	2.678
0.5	2.520	2.522	2.524	2.527	2.529	2.532	2.535	2.538	2.541	2.544
	2.684	2.690	2.697	2.703	2.709	2.715	2.722	2.728	2.734	2.741
0.6	2.547	2.550	2.553	2.556	2.560	2.564	2.569	2.574	2.579	2.585
	2.747	2.753	2.760	2.766	2.772	2.779	2.785	2.792	2.798	2.805
0.7	2.592	2.600	2.608	2.616	2.625	2.635	2.645	2.655	2.666	2.678
	2.811	2.818	2.824	2.831	2.837	2.844	2.850	2.857	2.863	2.870
0.8	2.689	2.701	2.713	2.726	2.738	2.752	2.765	2.779	2.793	2.808
	2.877	2.883	2.890	2.896	2.903	2.910	2.916	2.923	2.930	2.936
0.9	2.823	2.839	2.855	2.872	2.890	2.908	2.927	2.947	2.967	2.989
	2.943	2.950	2.957	2.963	2.970	2.977	2.984	2.991	2.997	3.004

$\hat{F} = 252.342$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.546	2.545	2.545	2.545	2.544	2.544	2.544	2.544	2.543	2.543
	2.546	2.546	2.546	2.547	2.547	2.548	2.548	2.549	2.549	2.550
0.1	2.543	2.543	2.543	2.543	2.543	2.543	2.543	2.543	2.543	2.543
	2.550	2.551	2.552	2.552	2.553	2.554	2.554	2.555	2.556	2.556
0.2	2.543	2.543	2.543	2.543	2.543	2.544	2.544	2.544	2.544	2.545
	2.557	2.558	2.559	2.560	2.561	2.561	2.562	2.563	2.564	2.565
0.3	2.545	2.546	2.546	2.546	2.547	2.547	2.548	2.548	2.549	2.550
	2.566	2.567	2.568	2.569	2.570	2.571	2.572	2.574	2.575	2.576
0.4	2.550	2.551	2.552	2.553	2.553	2.554	2.555	2.556	2.557	2.558
	2.578	2.579	2.580	2.582	2.583	2.585	2.586	2.587	2.589	2.590
0.5	2.559	2.560	2.561	2.562	2.564	2.565	2.566	2.567	2.569	2.570
	2.592	2.593	2.594	2.596	2.597	2.599	2.600	2.601	2.603	2.604
0.6	2.571	2.573	2.575	2.576	2.578	2.580	2.582	2.584	2.586	2.588
	2.606	2.607	2.609	2.611	2.613	2.615	2.616	2.619	2.621	2.623
0.7	2.591	2.593	2.595	2.597	2.599	2.601	2.603	2.606	2.608	2.611
	2.625	2.628	2.630	2.633	2.635	2.638	2.641	2.644	2.646	2.649
0.8	2.614	2.617	2.620	2.624	2.628	2.632	2.636	2.641	2.646	2.651
	2.653	2.656	2.659	2.662	2.666	2.669	2.672	2.676	2.680	2.683
0.9	2.657	2.662	2.668	2.675	2.681	2.688	2.695	2.702	2.710	2.718
	2.687	2.691	2.694	2.698	2.702	2.706	2.710	2.714	2.718	2.722

Notes: Every table is associated with an $\hat{F}$ value displayed above it. Each table entry corresponds to an $|\hat{r}|$ value, where the row label gives the first digit following the decimal point and the column label gives the second digit. Table entries are in pairs, where the top number is $k_2(|\hat{r}|, \hat{F})$ and the number below is $k_1(|\hat{r}|, \hat{F})$. If $\hat{r} > 0$, the confidence interval is: $[\hat{\beta} - k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$. If $\hat{r} < 0$, the CI is: $[\hat{\beta} - k_2(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta}), \hat{\beta} + k_1(|\hat{r}|, \hat{F})\widehat{se}(\hat{\beta})]$.

C Additional Numerical Results

C.1 Confidence Interval Properties and Performance Across different Data Generating Processes

Because our interest is in how the relative performance of VtF and AR intervals change under different data generating processes, we focus on the 25th, 50th, and 75th percentiles of the distribution of $\text{length}_{VtF}, \text{length}_{tF}, \text{length}_{AR}$, conditional on $F > q$, under 16 different designs corresponding to $\rho = 0, 0.5, 0.8, 0.9$ and $f_0 = 1, 3, 6, 9$. For each of these data generating processes, and any null hypothesis β_0, one can draw 1 million pairs of $t_{AR}(\beta_0), f$ and for each of those pairs, determine whether β_0 is accepted or rejected under VtF, tF, and AR. By iterating this process over a grid of β_0 values using the same 1 million observations, one can obtain 1 million confidence sets, and from these confidence sets, once can compute the relevant percentiles of the length measure of interest.

The first row of each cell in Table A7 lists the 25th, 50th, and 75th percentiles of the interval length for VtF, tF, AR, as measured in terms of the natural log of length. We find the following:

1. The percentiles of lengths for VtF are shorter than those of tF by a considerable amount, particularly for the relatively "weak instrument" designs of $f_0 = 1, 3$, with an advantage that ranges from 23 log points at the 25th percentile for $f_0 = 3, \rho = .9$ to 110 log points at the 75th percentile at $f_0 = 1, \rho = 0$.
2. VtF's advantage over AR is roughly half the advantage that it has over tF, which advantage in lengths ranging from about 10 log points for the 25th percentile for $f_0 = 3, \rho = .9$ to 76 log points at the 75th percentile for the $f_0 = 1, \rho = 0$ design.
3. As the instrument gets stronger, for the designs with $f_0 = 6$, the ordering that VtFs length percentiles are shorter than those of ARs which are in turn, shorter than those of tF, is continues to hold, but overall the magnitudes of the gains are more modest. For example, the medians for VtF, tF, and AR for the $f_0 = 6, \rho = .9$ design range between -.28 to -.344.
4. For even stronger instrument designs ($f_0 = 9$), the largest difference in the percentiles is about 7.4 log points for the 75th percentiles of the VtF and tF procedures, for the $f_0 = 9, \rho = 0$ design. All other differences in the percentiles are smaller.

The limitation of these comparisons of percentiles is that they provide no information on the likelihood that a dataset will produce a VtF confidence interval that is shorter than that of AR. For example, without further information, it could be that despite the marginal distribution of VtF lengths being shifted to the left, compared to that of AR, the likelihood that VtF will produce a shorter interval is not particularly high.

We investigate this by reporting the percentiles of the difference in natural logarithm of lengths

$$\ln(\text{length}_{tF}) - \ln(\text{length}_{VtF}), \ln(\text{length}_{AR}) - \ln(\text{length}_{VtF})$$

again conditional on $F > 3.84$. Table A7 shows that in every one of the 16 designs, as would be expected, the percentiles of the $tF - VtF$ difference are all positive. More notably, in every single one of the 16 designs, the 25th, 50th, and 75th percentiles of the $AR - VtF$ differences are also all positive, which indicates it is significantly more frequent that VtF will produce small intervals than AR.

To measure this frequency, we also compute, and find that across all designs, we find that $\Pr\left[\text{length}_{AR} > \text{length}_{VtF} | 1.96^2 < F \leq 104.67\right]$ is more than .97, and in many designs the monte carlo integration implies a probability of 1. We impose the condition on $F \leq 104.67$ to concentrate on scenarios where blindly recommending the use of the t-test without additional assumptions would not be advisable.

The magnitudes of the difference are not trivial for designs with weak instruments ($f_0 = 1, 3$), with a median $AR - VtF$ difference ranging from 16.5 log points for the $f_0 = 3, \rho = .9$ design to 46 log points for the $f_0 = 1, \rho = 0$ design. With instrument strength at $f_0 = 6$, the largest magnitude of the $AR - VtF$ difference is about 13 log points for the 75th percentile of the difference, for $f_0 = 6, \rho = 0$, and for $f_0 = 9$, while all of the reported percentiles of the $AR - VtF$ differences are positive, none are larger than 6 log points.

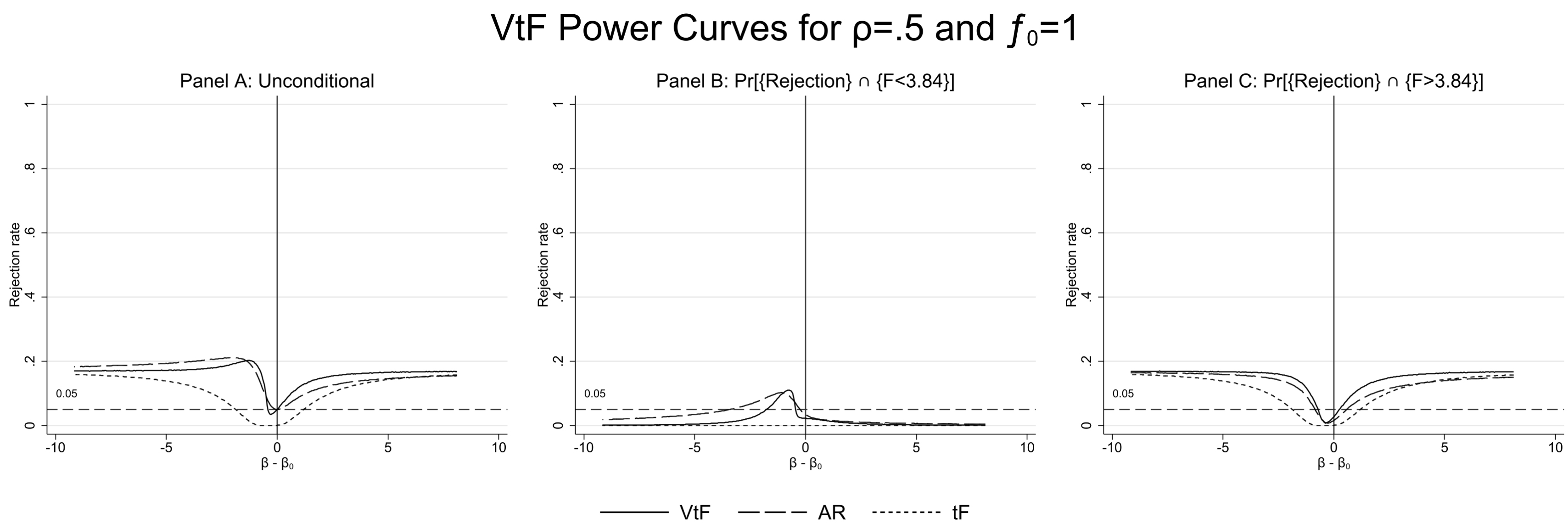
Table A7: Confidence Interval Performance: VtF, tF, AR, and t, Simulations

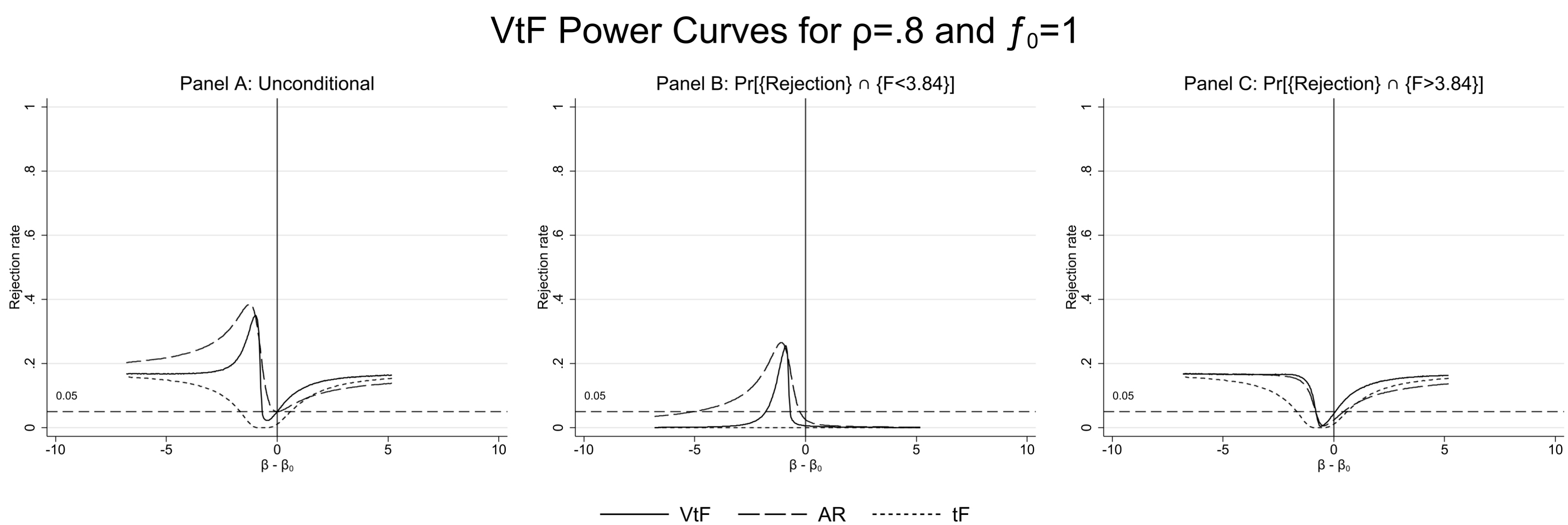
	f_0=1, ρ =				f_0=3, ρ =				f_0=6, ρ =				f_0=9, ρ =			
	0.000	0.500	0.800	0.900	0.000	0.500	0.800	0.900	0.000	0.500	0.800	0.900	0.000	0.500	0.800	0.900
tF (25th pctile)	1.177	1.193	1.253	1.347	0.455	0.530	0.787	1.064	–0.414	–0.306	0.014	0.317	–0.857	–0.731	–0.385	–0.072
tF (50th pctile)	1.623	1.642	1.724	1.841	0.800	0.905	1.227	1.536	–0.267	–0.133	0.230	0.549	–0.779	–0.636	–0.264	0.058
tF (75th pctile)	2.164	2.189	2.287	2.428	1.263	1.407	1.799	2.137	–0.094	0.075	0.483	0.820	–0.687	–0.521	–0.121	0.211
AR (25th pctile)	0.818	0.841	0.931	1.081	0.225	0.316	0.629	0.941	–0.476	–0.361	–0.023	0.289	–0.851	–0.721	–0.365	–0.049
AR (50th pctile)	1.232	1.269	1.427	1.648	0.505	0.649	1.067	1.418	–0.354	–0.208	0.177	0.507	–0.784	–0.634	–0.253	0.073
AR (75th pctile)	1.823	1.888	2.146	2.451	0.904	1.145	1.693	2.081	–0.213	–0.023	0.414	0.761	–0.705	–0.530	–0.119	0.217
t (25th pctile)	0.410	0.422	0.470	0.554	0.060	0.130	0.386	0.666	–0.522	–0.416	–0.095	0.208	–0.873	–0.748	–0.401	–0.089
t (50th pctile)	0.561	0.575	0.638	0.748	0.247	0.345	0.670	0.982	–0.412	–0.280	0.084	0.404	–0.809	–0.666	–0.295	0.027
t (75th pctile)	0.675	0.692	0.799	0.958	0.451	0.591	0.989	1.328	–0.289	–0.119	0.290	0.626	–0.735	–0.567	–0.168	0.164
VtF (25th pctile)	0.493	0.513	0.597	0.739	0.016	0.128	0.482	0.837	–0.560	–0.454	–0.091	0.256	–0.894	–0.761	–0.420	–0.101
VtF (50th pctile)	0.768	0.796	0.923	1.125	0.254	0.414	0.865	1.254	–0.459	–0.302	0.120	0.486	–0.832	–0.687	–0.311	0.028
VtF (75th pctile)	1.067	1.113	1.329	1.621	0.562	0.798	1.355	1.762	–0.347	–0.121	0.365	0.750	–0.761	–0.596	–0.177	0.181
$tF-VtF$ (25th pctile)	0.661	0.646	0.589	0.519	0.430	0.372	0.276	0.203	0.146	0.136	0.094	0.058	0.036	0.031	0.035	0.027
$tF-VtF$ (50th pctile)	0.823	0.803	0.731	0.643	0.543	0.474	0.348	0.265	0.192	0.166	0.108	0.064	0.053	0.050	0.046	0.029
$tF-VtF$ (75th pctile)	1.051	1.023	0.919	0.795	0.697	0.609	0.441	0.347	0.250	0.201	0.124	0.073	0.074	0.074	0.054	0.031
$AR-VtF$ (25th pctile)	0.324	0.325	0.326	0.326	0.207	0.177	0.140	0.102	0.084	0.083	0.049	0.013	0.042	0.042	0.050	0.036
$AR-VtF$ (50th pctile)	0.460	0.465	0.486	0.505	0.255	0.238	0.201	0.165	0.105	0.092	0.059	0.022	0.048	0.052	0.056	0.045
$AR-VtF$ (75th pctile)	0.738	0.751	0.792	0.813	0.344	0.351	0.345	0.311	0.131	0.103	0.066	0.034	0.056	0.064	0.058	0.053
$t-VtF$ (25th pctile)	–0.386	–0.408	–0.512	–0.642	–0.112	–0.202	–0.356	–0.429	0.036	–0.000	–0.075	–0.124	0.020	0.014	0.009	–0.018
$t-VtF$ (50th pctile)	–0.206	–0.220	–0.277	–0.357	–0.010	–0.072	–0.193	–0.270	0.044	0.021	–0.036	–0.082	0.023	0.019	0.015	–0.000
$t-VtF$ (75th pctile)	–0.083	–0.092	–0.127	–0.182	0.043	–0.002	–0.096	–0.171	0.054	0.033	–0.005	–0.048	0.026	0.025	0.018	0.013
$Pr[Len_{VtF} > Len_{AR}]$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.022	0.000	0.000	0.000	0.003
$Pr[VtF \subseteq t]$	0.025	0.022	0.011	0.004	0.237	0.088	0.003	0.000	0.953	0.486	0.043	0.004	1.000	0.974	0.606	0.316
$Pr[VtF \supseteq t]$	0.264	0.230	0.139	0.095	0.094	0.031	0.050	0.326	0.000	0.000	0.299	0.880	0.000	0.000	0.039	0.327
$Pr[Len_{VtF} < Len_t, VtF \nsubseteq t]$	0.057	0.052	0.035	0.017	0.223	0.156	0.017	0.001	0.039	0.263	0.159	0.022	0.000	0.023	0.235	0.179
$Pr[Len_{VtF} > Len_t, VtF \nsupseteq t]$	0.654	0.696	0.814	0.884	0.446	0.725	0.931	0.674	0.008	0.250	0.500	0.094	0.000	0.004	0.120	0.179
$Pr[VtF disjoint]$	0.005	0.008	0.023	0.051	0.001	0.008	0.039	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

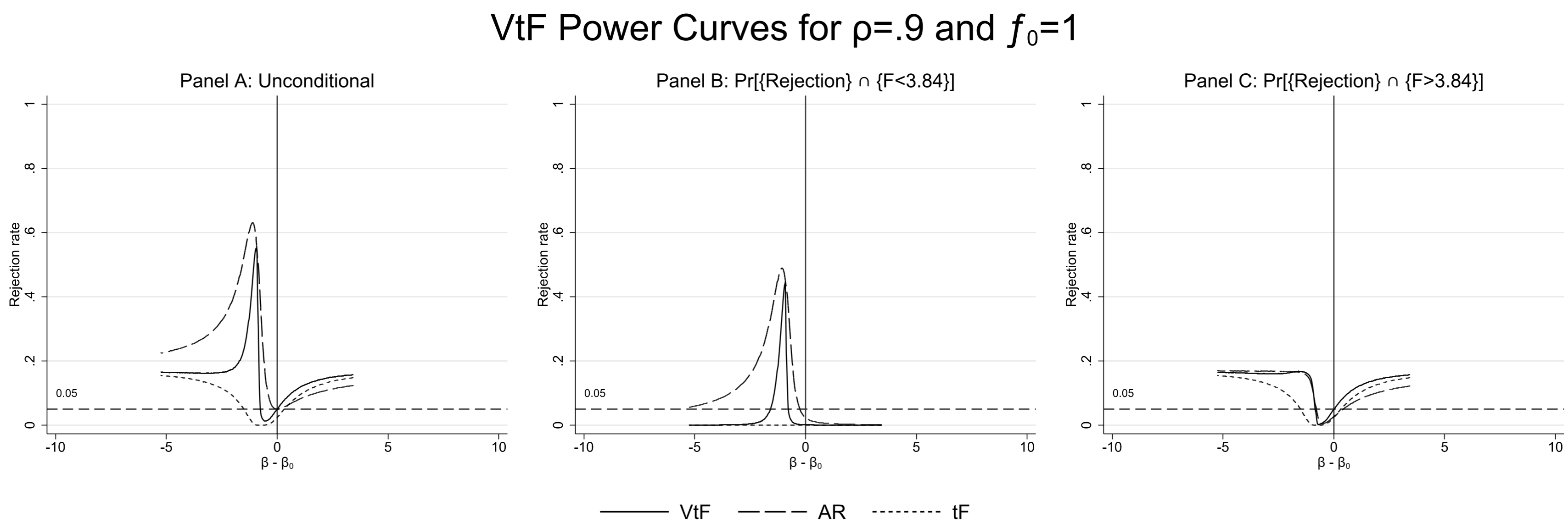
Note: For each design, 1 million random draws were taken, representing 1 million datasets. All quantities in the table are conditional on the F-statistic being between 1.96^2 and 104.67.

C.2 Power Curves with Decompositions Across different Data Generating Processes

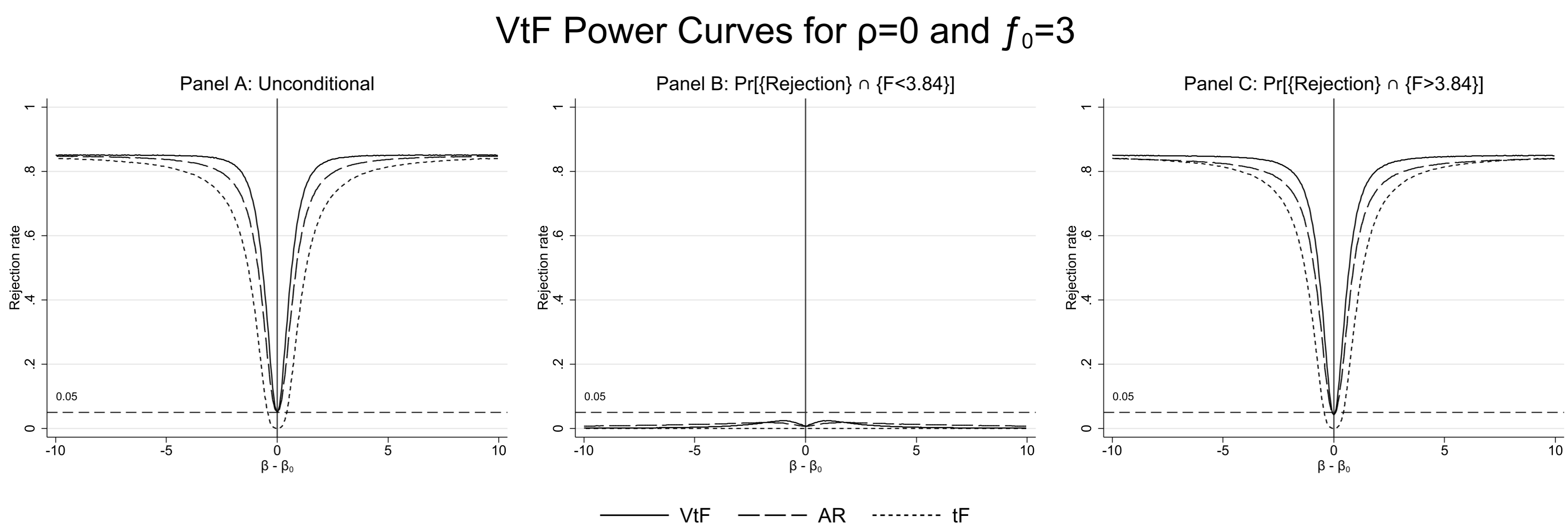
Figure A2: VtF Power Curves for Values of ρ and f_0





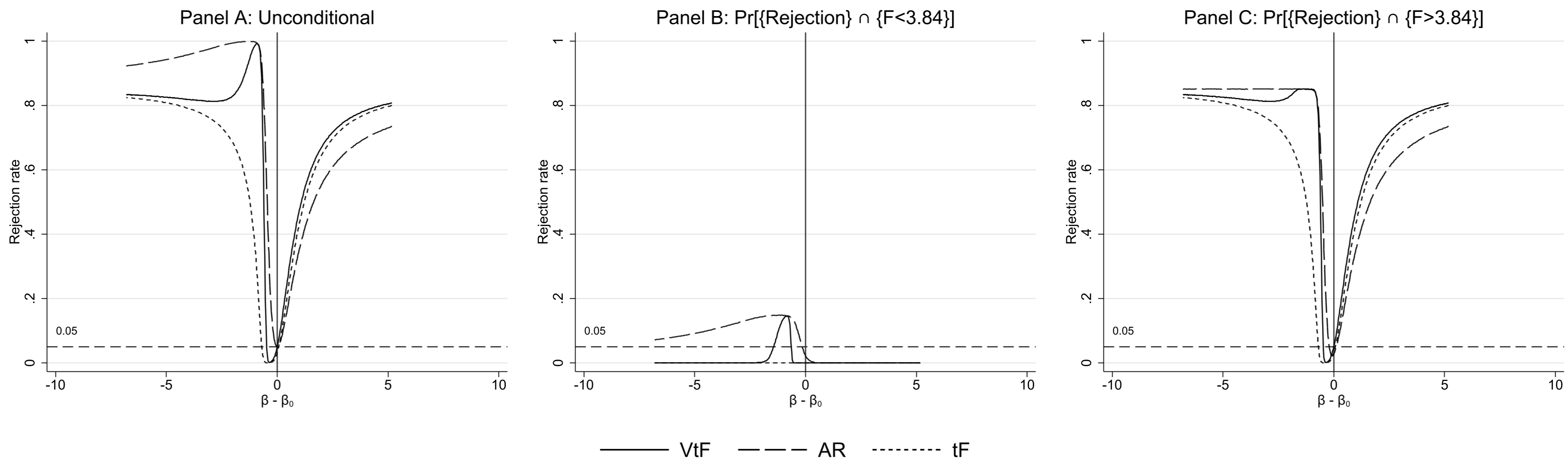


VtF Power Curves for ρ=.9 and ƒ0=1
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF

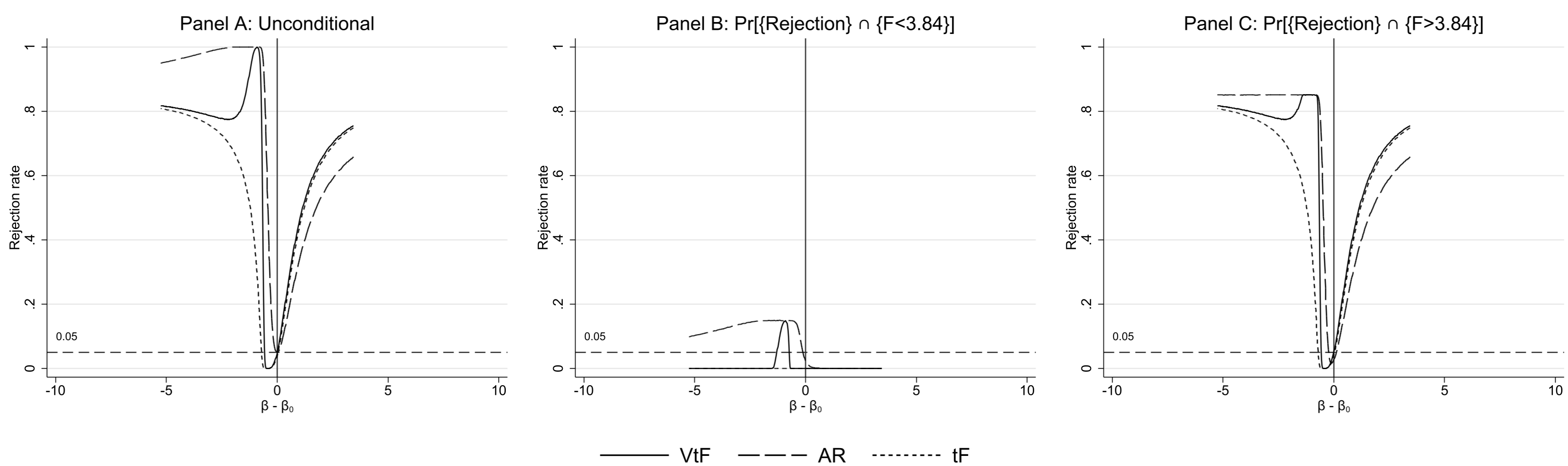


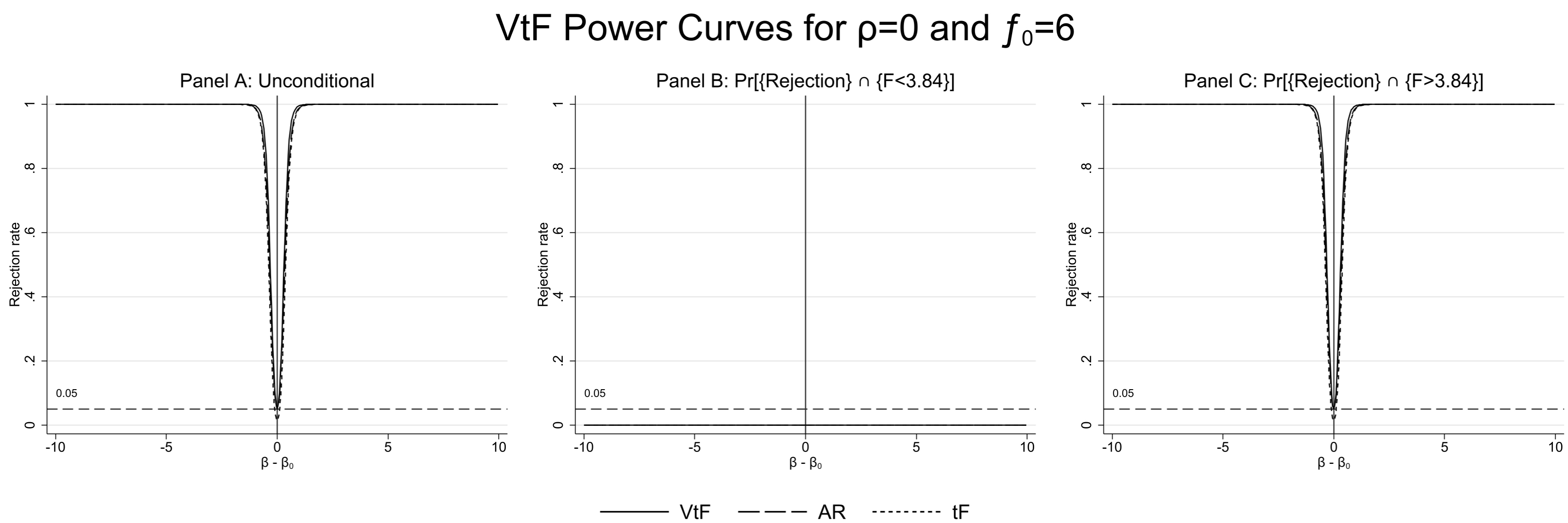
VtF Power Curves for ρ=0 and ƒ0=3
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF

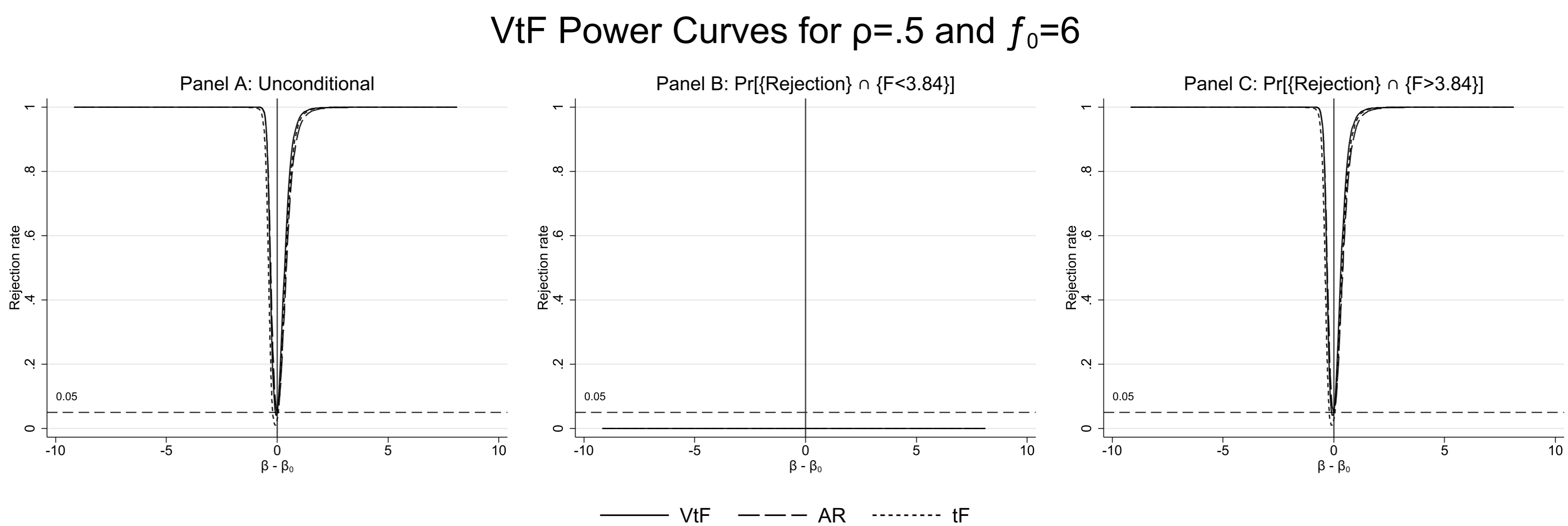
VtF Power Curves for ρ=.8 and f_0=3

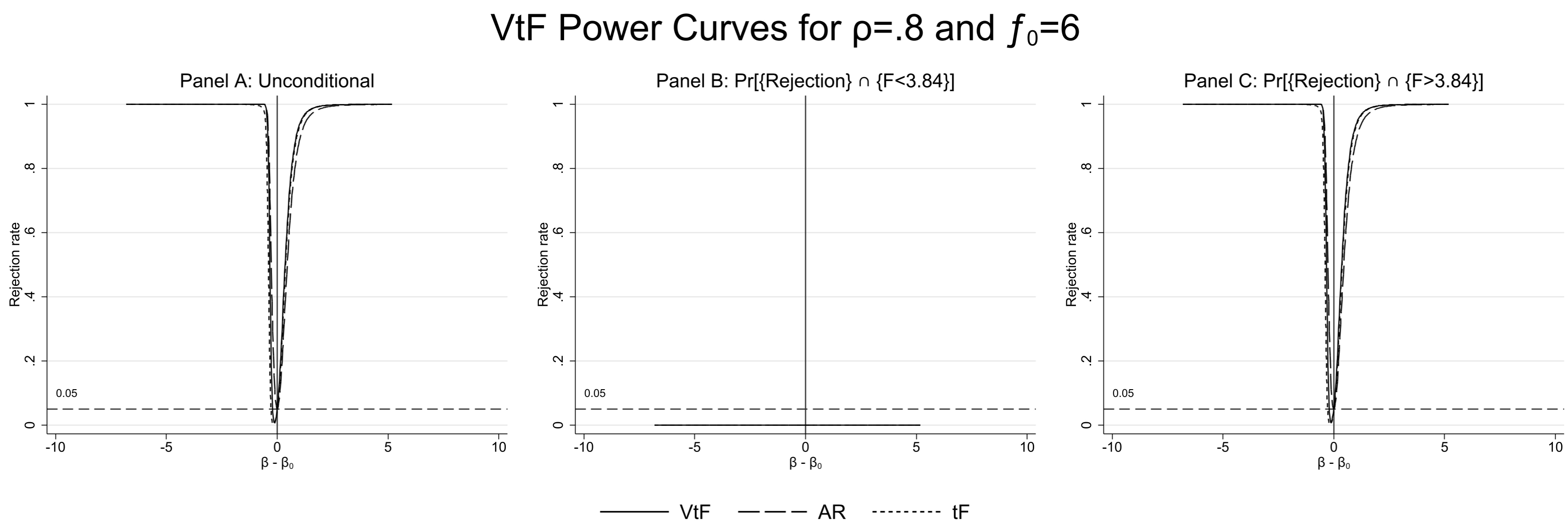


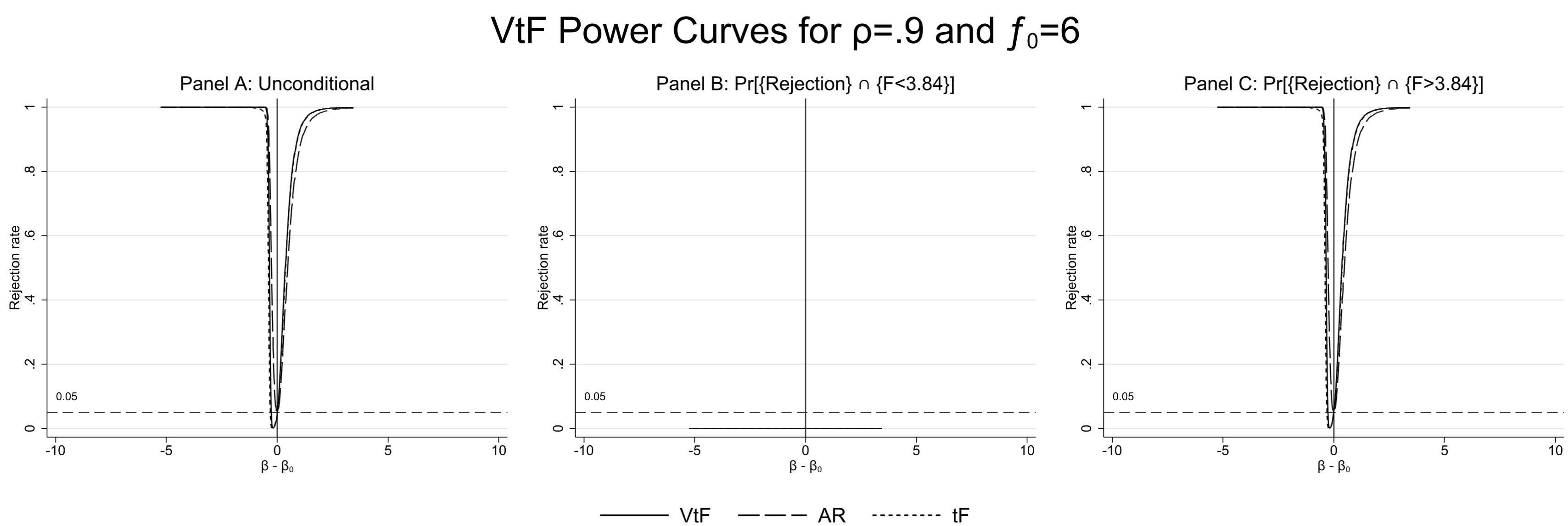
VtF Power Curves for ρ=.9 and f_0=3

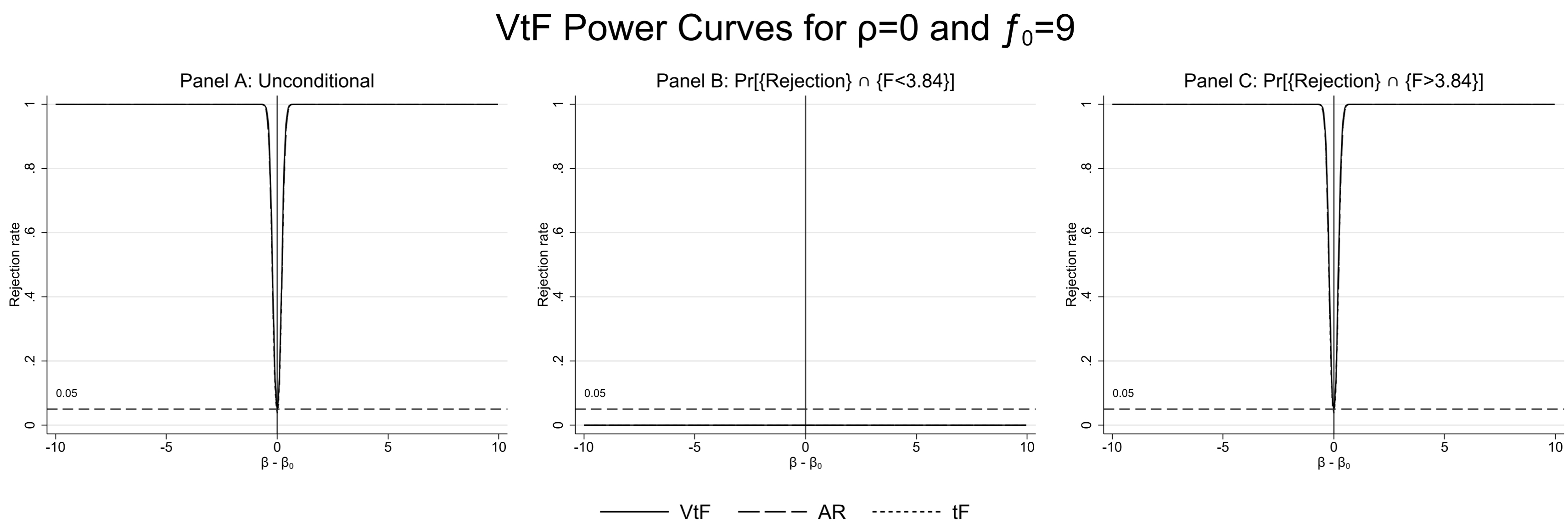


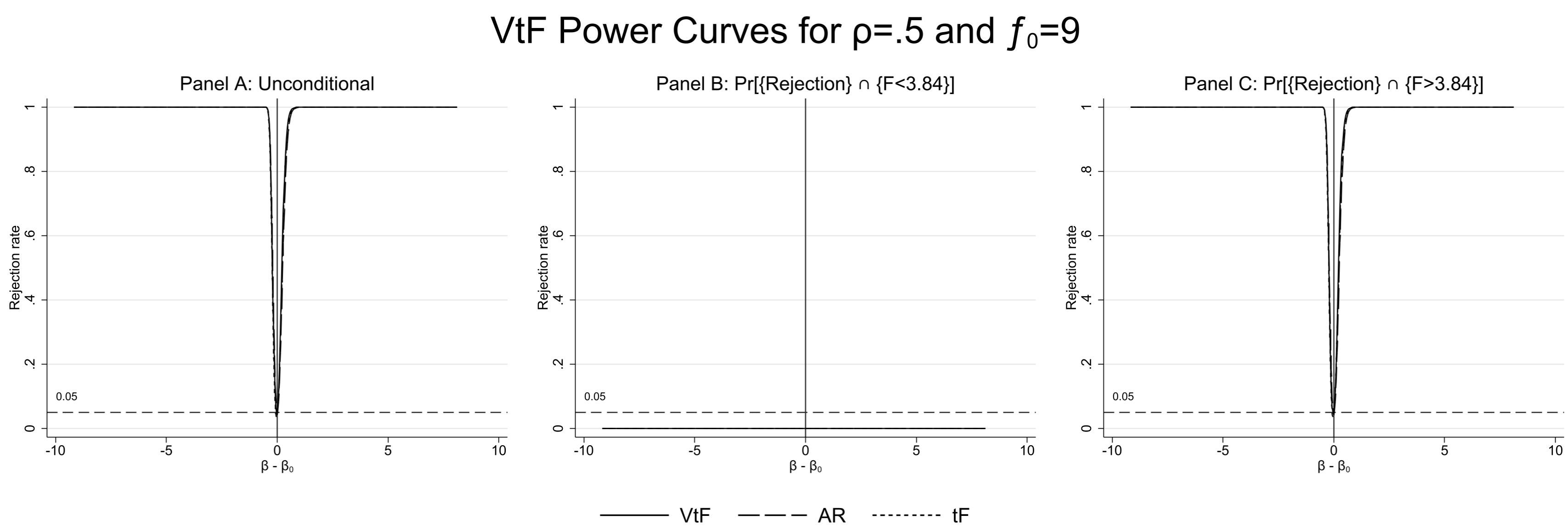

VtF Power Curves for ρ=0 and ƒ0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
0
.2
.4
.6
.8
1
0.05
-10
-5
0
5
10
β - β0
VtF
AR
tF


VtF Power Curves for ρ=.5 and ƒ0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
0
.2
.4
.6
.8
1
0.05
-10
-5
0
5
10
β - β0
VtF
AR
tF

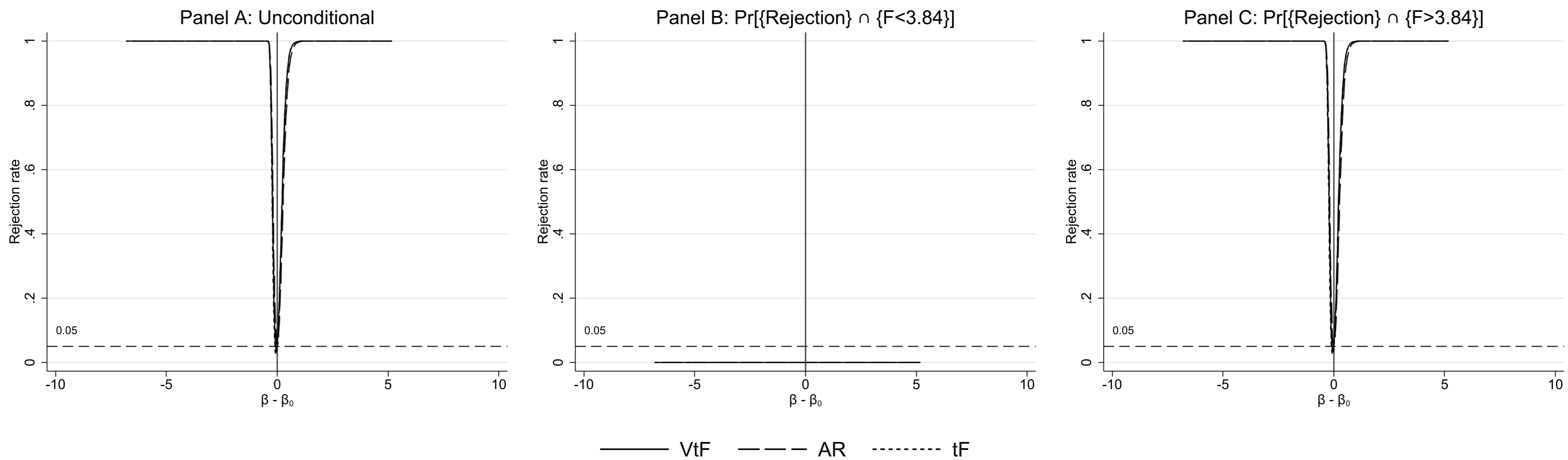

VtF Power Curves for ρ=.8 and ƒ0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF


VtF Power Curves for ρ=.9 and ƒ0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF

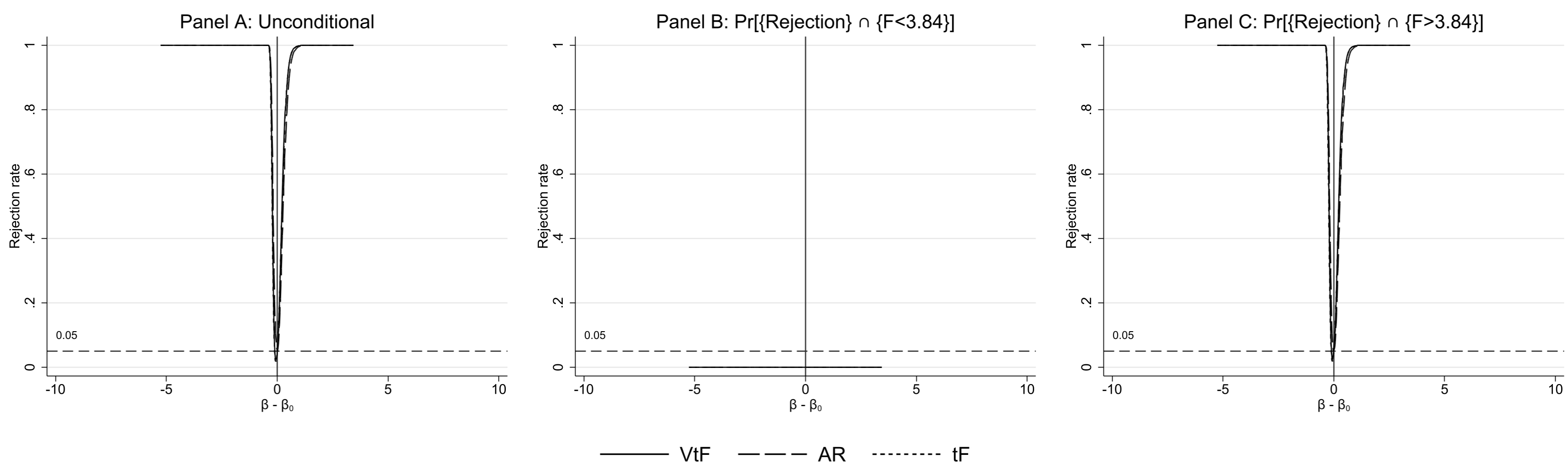

VtF Power Curves for ρ=0 and ƒ0=9
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF


VtF Power Curves for ρ=.5 and ƒ0=9
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
β - β0
0.05
VtF
AR
tF

VtF Power Curves for ρ=.8 and f_0=9



VtF Power Curves for ρ=.9 and f_0=9



C.3 Performance of Conditional Wald

While this paper has mainly focused on a comparison between VtF and the procedure generally recommended for just identified IV in the econometrics literature, AR, we also explored the performance of one other procedure, Conditional Wald (CW). This procedure was introduced in Moreira (2003) but Andrews, Moreira and Stock (2007) recommended against its use in testing for two reasons – power sometimes falls below size (biasedness) and generally its poor power, as viewed from using the first approach (fixed reduced form covariance) to visualizing power.[37] We are aware of only one empirical study that has implemented it (Cruz and Moreira (2005)). More recently, Van de Sijpe and Windmeijer (2023) consider the performance of CW in homoskedastic, overidentified IV models using the "second approach" to visualizing power and find that from that perspective CW has favorable behavior, and in some cases superior to that of the conditional likelihood ratio procedure of Moreira (2003). Van de Sijpe and Windmeijer (2023) also provide simulation evidence for various data generating processes that demonstrate on the basis of median confidence set lengths (including the unbounded sets) that CW appears to outperform the conditional likelihood ratio intervals.

Like VtF, Conditional Wald uses the t-ratio, and rejects the null that $\beta = \beta_0$ if the magnitude of the t-ratio is larger than a data-dependent critical value function. Motivated by the approach of SY (2005) common in empirical work, the critical value function for VtF is constructed to depend on the first-stage F-statistic. The critical value function for CW depends on a different statistic, denoted Q and defined in Appendix D, that arises from the conditioning argument of Moreira (2002, 2009). More formally, CW rejects if and only if

$$\hat{t}^2 > c_{CW}\left(\hat{\rho}(\beta_0), \hat{Q}\right)$$

where $\hat{Q} = \hat{f} - \hat{\rho}\,(\beta_0)\,\hat{t}_{AR}\,(\beta_0)$, and where the function c_{CW} satisfies

$$\Pr_{\Delta(\beta_0)=0,\rho(\beta_0),f_0}\left[t^2 > c_{CW}\left(\rho\left(\beta_0\right), Q\right) | Q = Q_0\right] = \alpha$$

for all values of Q_0 on the real line.

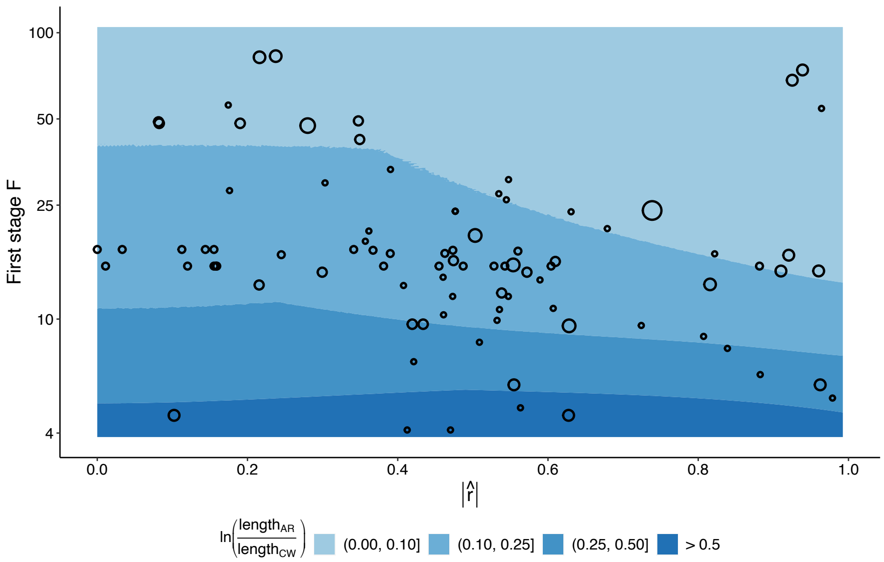
At both a theoretical and practical level, it is important to note the distinction between this procedure and VtF. Although both are based on, and require the computation of $\hat{t}$ and the function $\hat{\rho}(\cdot)$, CW requires the additional computation of two statistics: $\hat{f}$ and $\hat{t}_{AR}(\beta_0)$, whereas VtF requires only $\hat{F} = \hat{f}^2$. $\hat{Q}$ is needed to implement CW, while the quantity Q is only used to derive the VtF critical value function and is actually not computed for implementation of VtF since the critical value function takes $\hat{F}$ as an argument. This distinction is the reason why one cannot use

[37] see Section IV.B for a discussion of three approaches to presenting power curves.

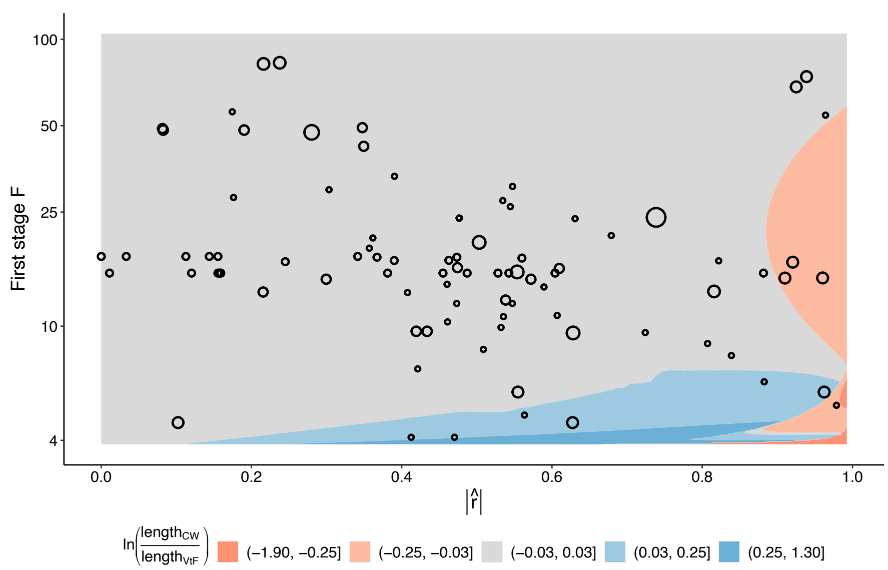
the conditional quantile method of Moreira (2003) to derive VtF, and a completely different way of deriving the critical value function is required, as discussed in more detail in Appendix D.

Figure A9: Confidence Interval Lengths

(a) CW versus AR



(b) CW versus VtF



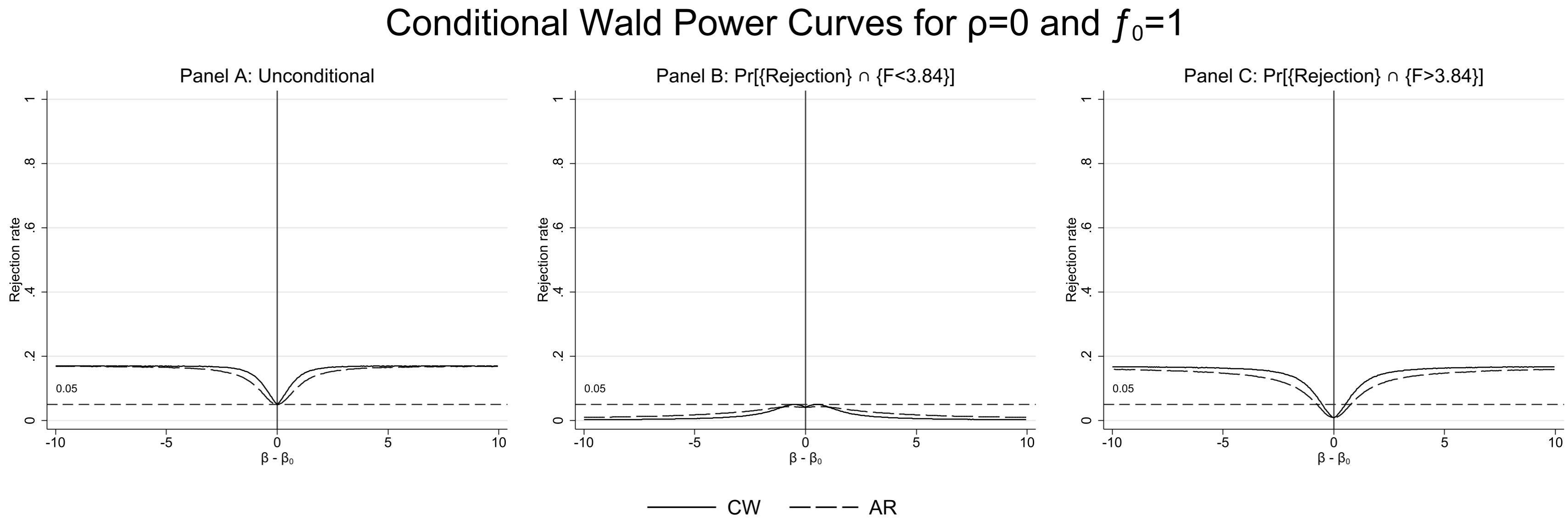
Note: Circles represent 89 specifications for which first-stage F-statistics are between 1.96^2 and 104.67, with the size of circle proportional to the inverse of the number of specifications in each study. The degree of shading indicates the difference in the log(CI lengths) (AR minus CW for figure (a), CW minus VtF for figure (b)). The vertical axis is a log-scale. The range of $|\hat{r}|$ is $[0, 0.995]$.

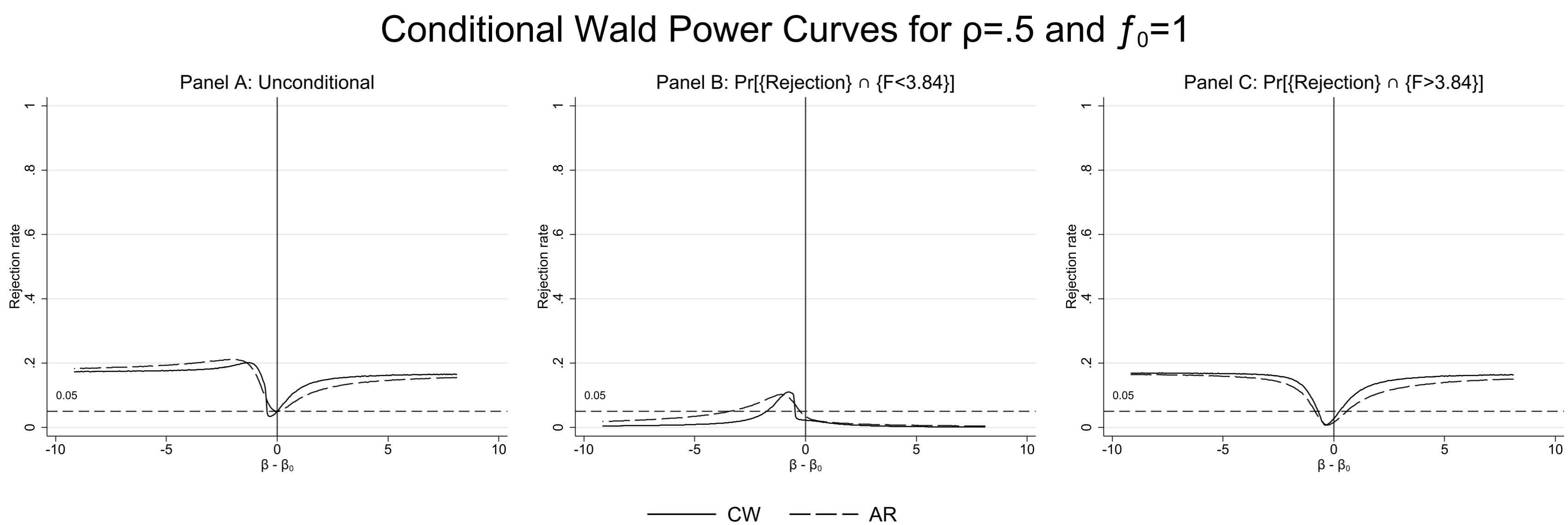
In our analysis, we find that CW *also* appears to outperform AR (see Figure A9a), and in many respects behaves similarly to VtF (see Figure A9b). Like VtF, CW is outside the class of unbiased and invariant tests. And, like all the tests VtF, AR, and tF, CW generates bounded confidence sets when $\hat{F} > q_{1-\alpha}$ and unbounded confidence sets when $\hat{F} < q_{1-\alpha}$, which allows for a decomposition of CW power on equal footing with our findings in section IV.C. As with VtF, the CW generally has a visible power advantage over AR when focusing on the bounded confidence region $F > q_{1-\alpha}$ (see Figure A10).

We also note one theoretical difference between VtF and Conditional Wald. CW's expected confidence set length conditional on being bounded is always greater than that of VtF. Indeed, like AR, we discovered that Conditional Wald's conditional expected length is *infinite*, and like AR (see LMMP (2022)), this is driven by the rate at which the bounded confidence length can explode as $\hat{F}$ approaches $q_{1-\alpha}$; this leads to longer confidence sets for values of $\hat{F}$ close to 1.96^2.

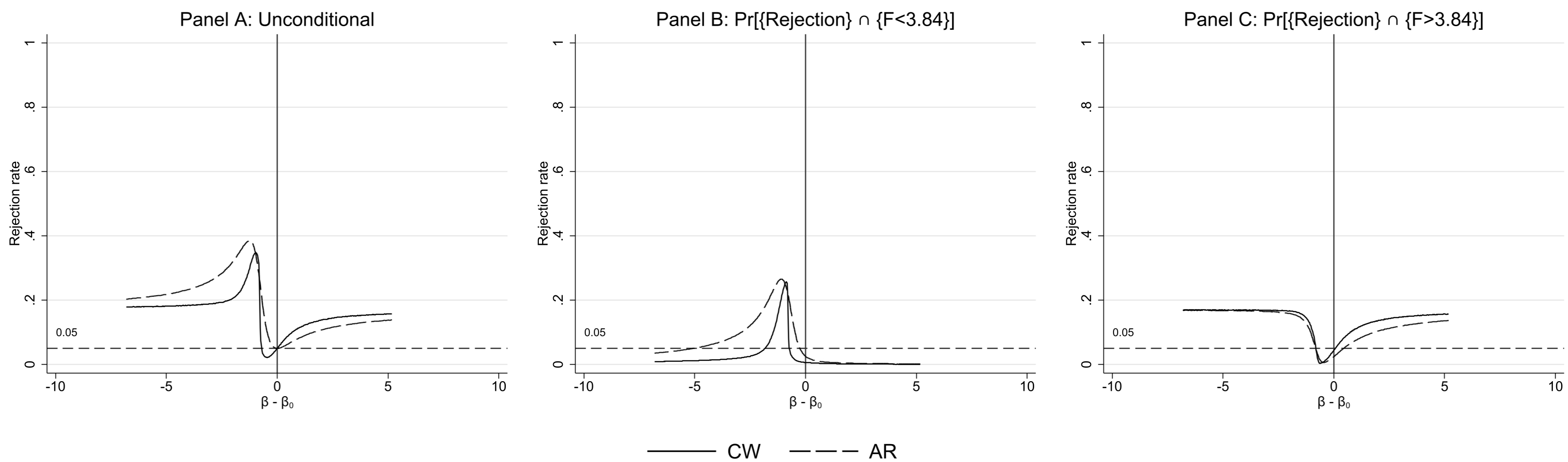
Finally, in comparing how CW confidence sets compare to the usual ± 1.96 intervals, we found that the set of datasets (values of $(\hat{F}, \hat{r})$) such that the CW confidence sets are entirely contained within the usual ± 1.96 intervals is smaller and contained within the corresponding set for VtF intervals. Therefore, a practitioner will discover that adjustments beyond the ± 1.96 intervals will be more frequent if the adjustment is based on CW rather than VtF.

Figure A10: Conditional Wald Power Curves for Values of ρ and f_0

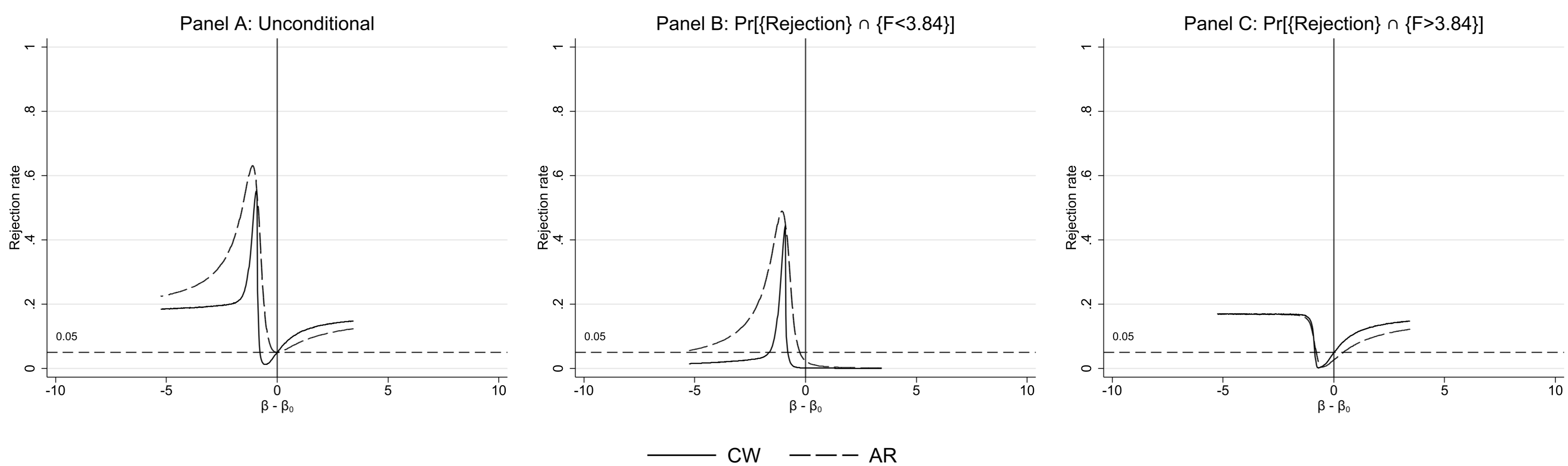




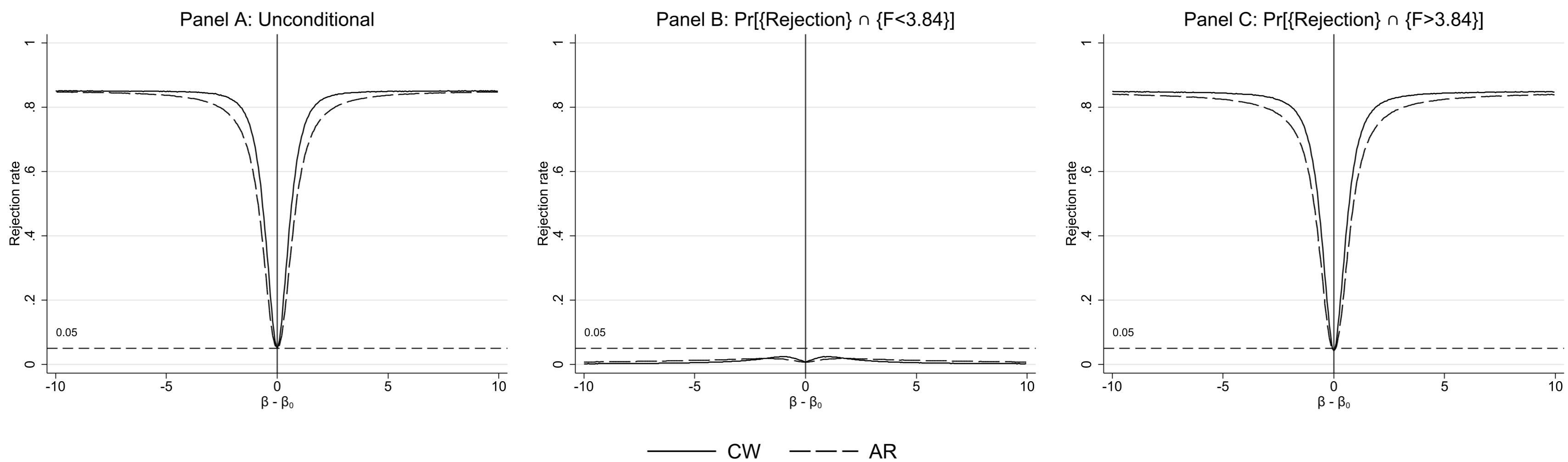
Conditional Wald Power Curves for ρ=.8 and f_0=1



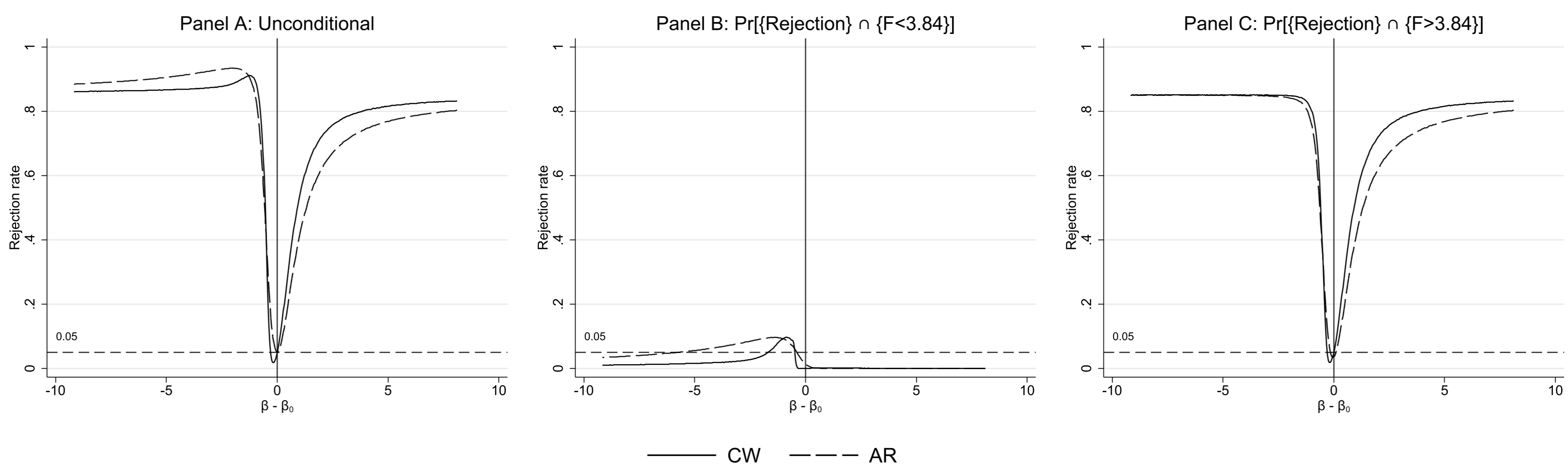
Conditional Wald Power Curves for ρ=.9 and f_0=1



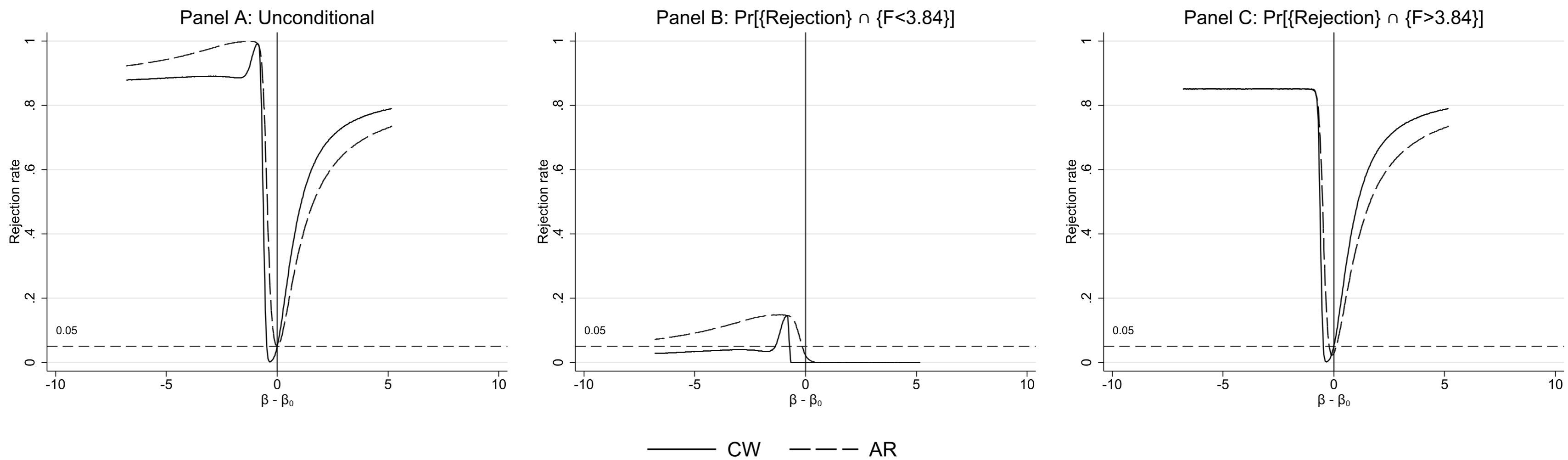
Conditional Wald Power Curves for ρ=0 and f_0=3



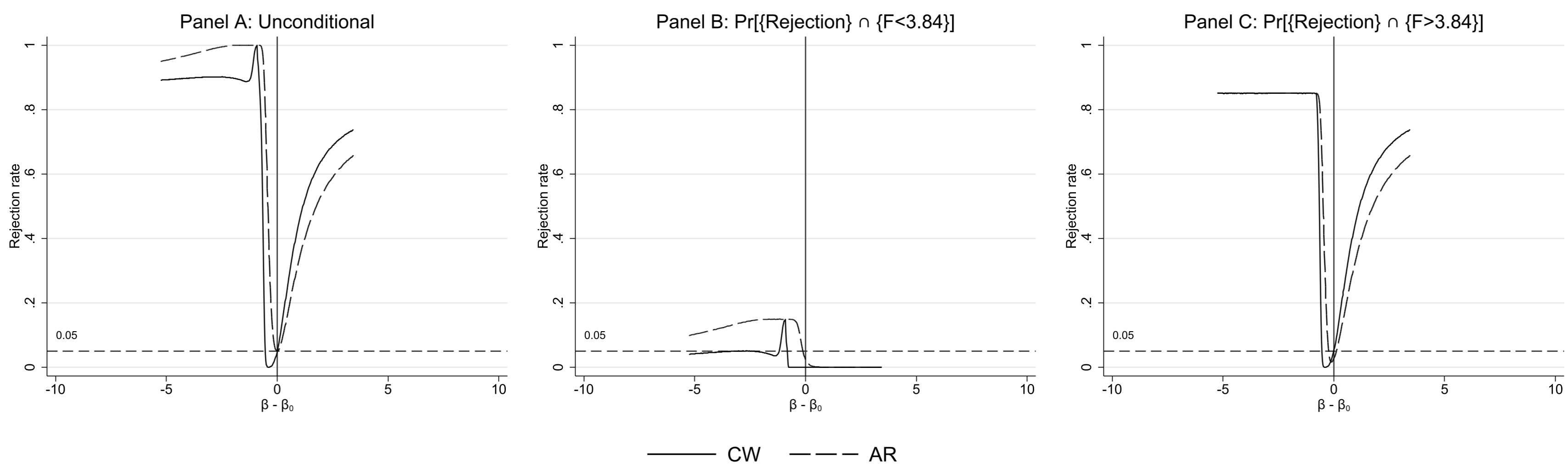
Conditional Wald Power Curves for ρ=.5 and f_0=3



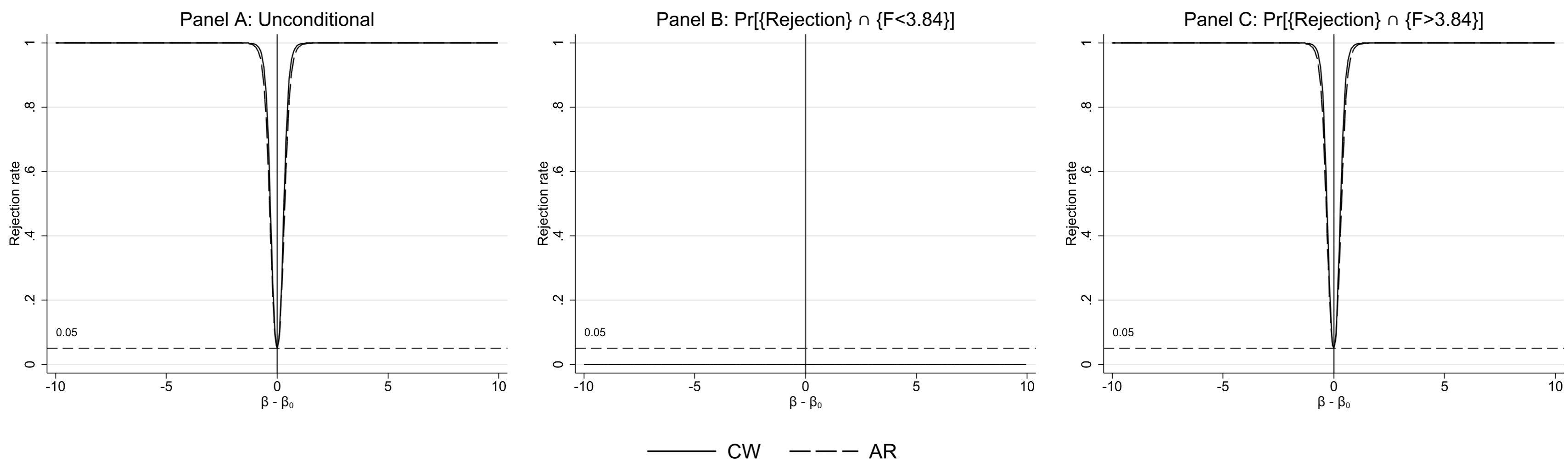
Conditional Wald Power Curves for ρ=.8 and f_0=3



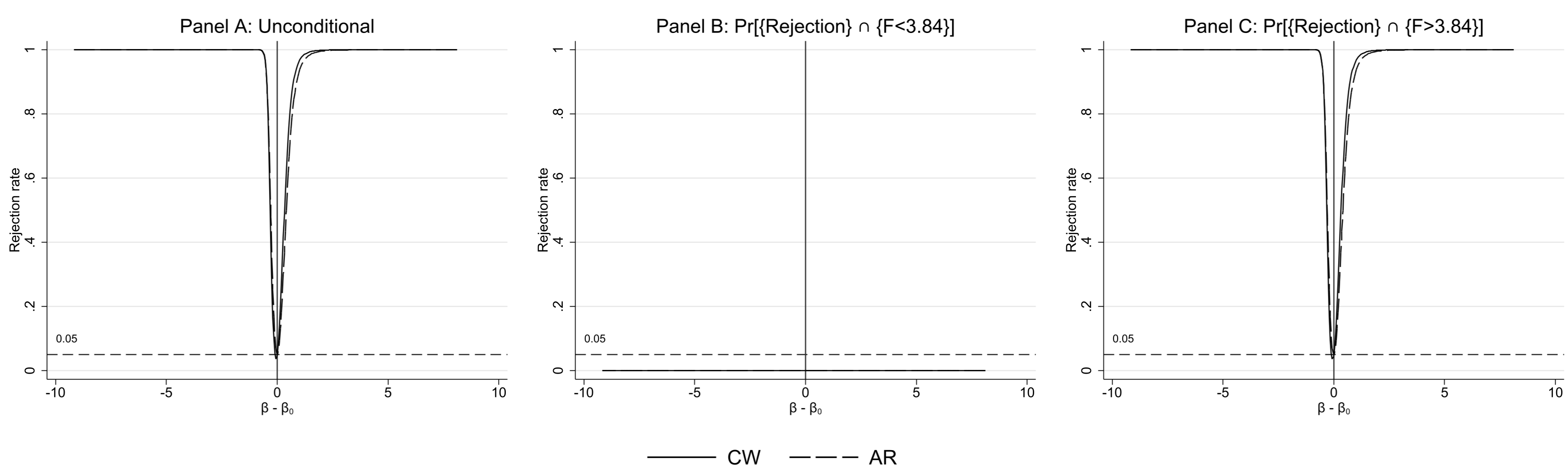
Conditional Wald Power Curves for ρ=.9 and f_0=3

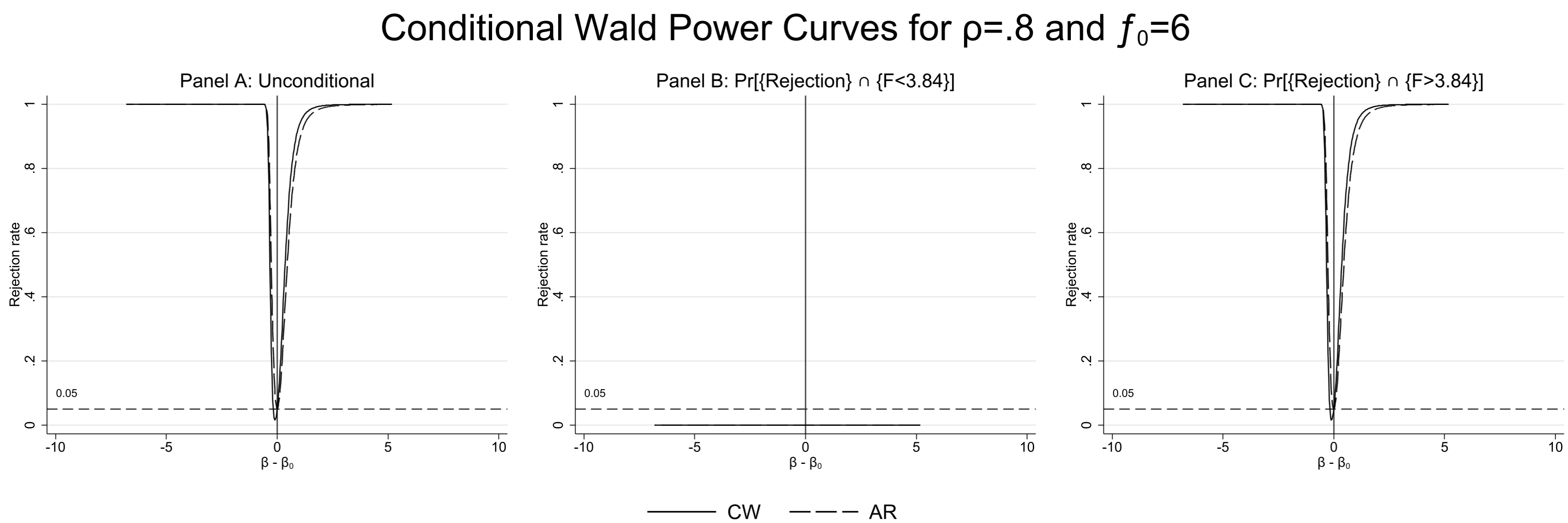


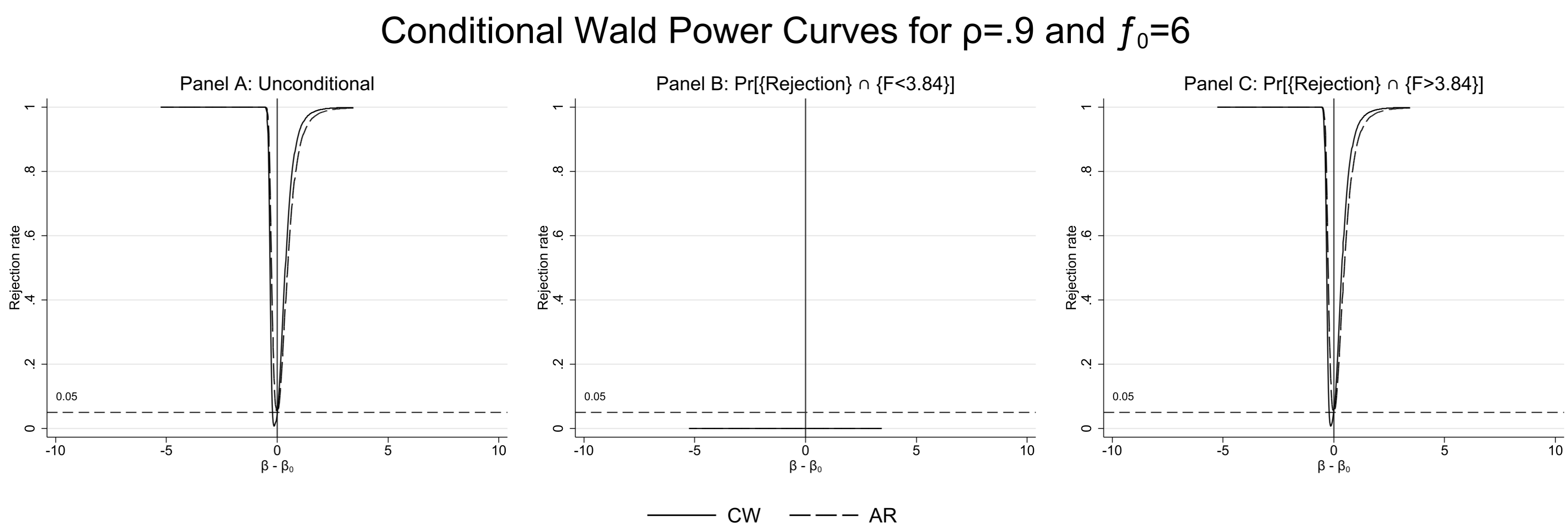
Conditional Wald Power Curves for ρ=0 and f_0=6



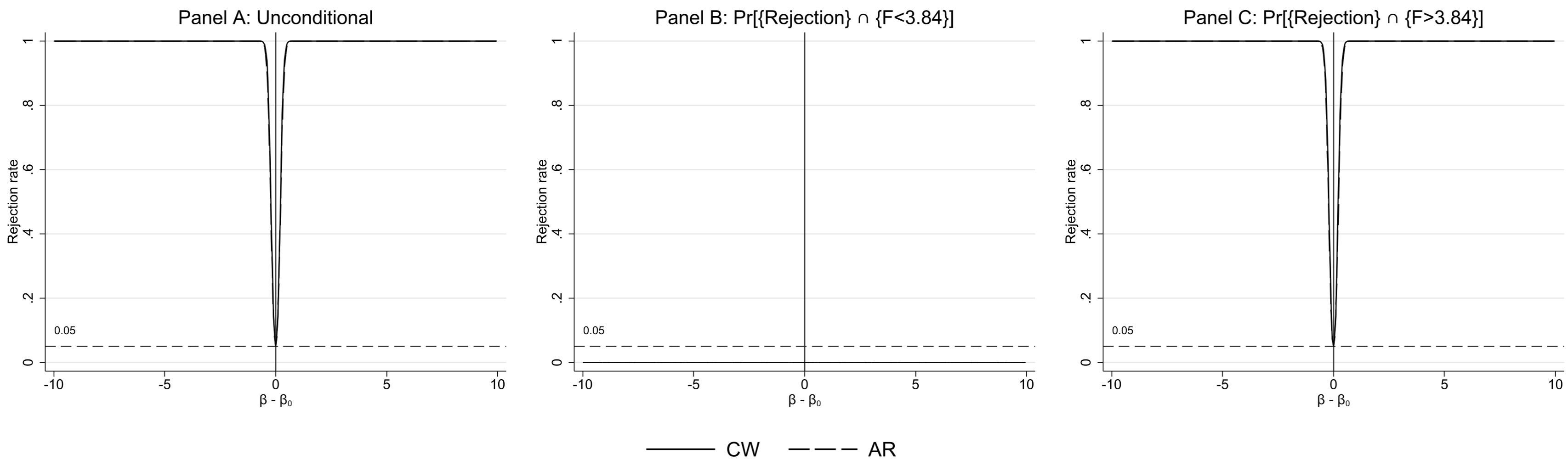
Conditional Wald Power Curves for ρ=.5 and f_0=6



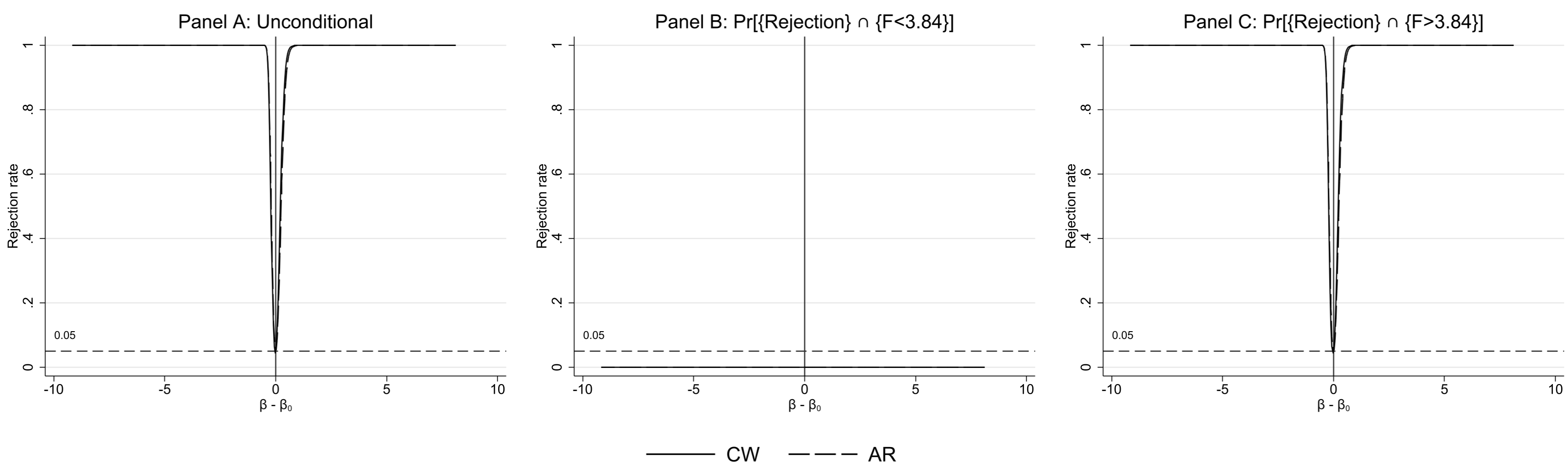

Conditional Wald Power Curves for ρ=.8 and f_0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
0
.2
.4
.6
.8
1
0.05
-10
-5
0
5
10
β - $β_0$
CW
AR


Conditional Wald Power Curves for ρ=.9 and f_0=6
Panel A: Unconditional
Panel B: Pr[{Rejection} ∩ {F<3.84}]
Panel C: Pr[{Rejection} ∩ {F>3.84}]
Rejection rate
0
.2
.4
.6
.8
1
0.05
-10
-5
0
5
10
β - $β_0$
CW
AR

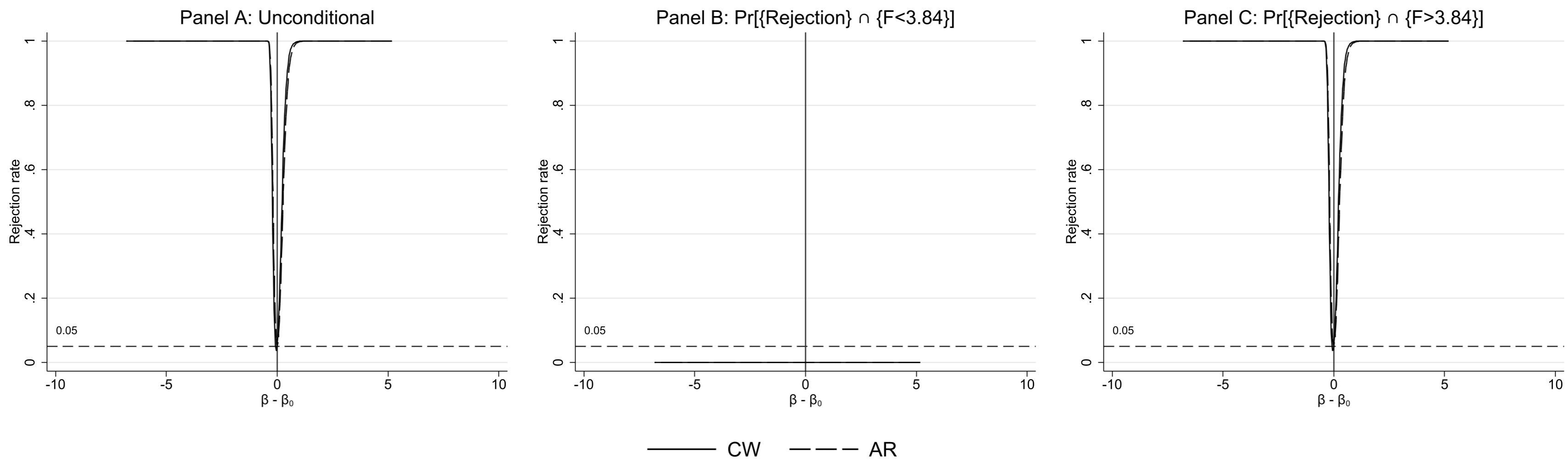
Conditional Wald Power Curves for ρ=0 and f_0=9



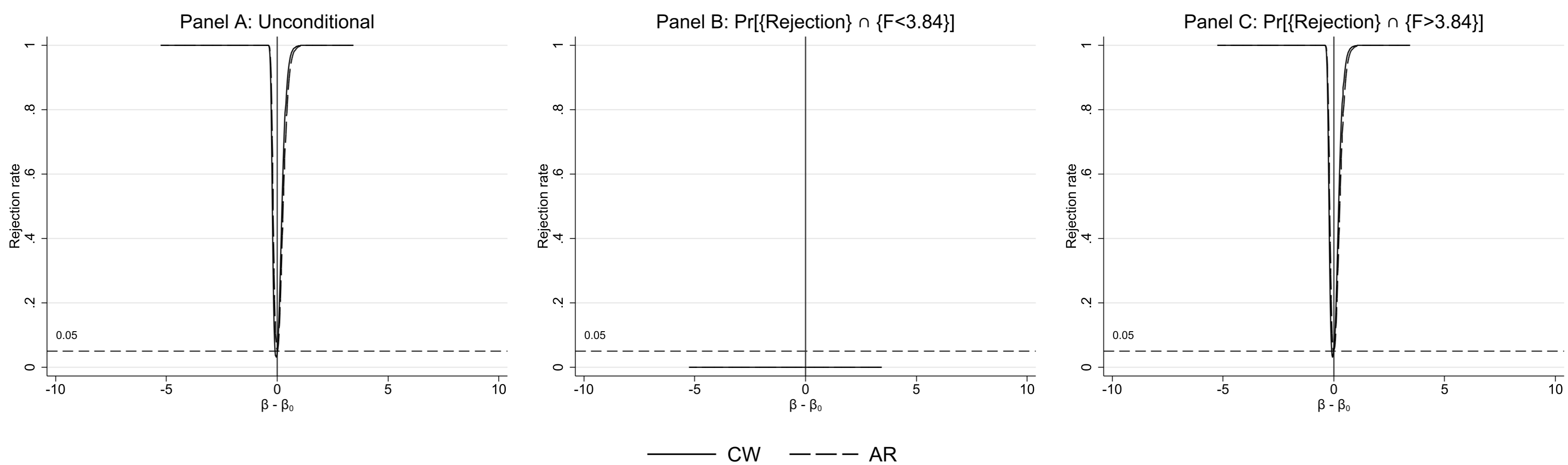
Conditional Wald Power Curves for ρ=.5 and f_0=9



Conditional Wald Power Curves for ρ=.8 and f_0=9



Conditional Wald Power Curves for ρ=.9 and f_0=9



D Derivation and Details for VtF Inference Procedures

D.1 VtF Critical Value Function

For $0 < |\rho(\beta_0)| < 1$, establishing existence, uniqueness and the precise form of the VtF critical value function c proceeds in three separate stages: 1) use the theoretical result of Moreira (2009) to establish that the existence and uniqueness of c can equivalently be shown for a conditional version of statement (7); 2) use the strategy from LMMP (2022) and the theorem of Fefferman (2021) to show that there exists a function defined on a small domain that satisfies the conditional similarity statement, with a known starting point and derivative; and 3) use the starting point and derivative, and the conditional similarity condition to uniquely compute the entire function.

Below it is straightforward to see that c is symmetric in $\rho(\beta_0)$ about zero, so it suffices to focus on the derivation for $\rho(\beta_0) > 0$ and the case of $\rho(\beta_0) < 0$ will follow directly. The case of $\rho(\beta_0) = 0$ can be solved directly and is provided below. The case of $|\rho(\beta_0)| = 1$ is covered in LMMP (2022).

Equivalence of unconditional and conditional similarity. First, we use Theorem 1 from Moreira (2009) to establish that any test that satisfies the unconditional similarity (7), must also satisfy conditional similarity

$$\Pr_{\Delta(\beta_0)=0,\rho(\beta_0),f_0}\left[t^2 > c\left(\rho\left(\beta_0\right), F; \alpha\right) | Q = Q_0\right] = \alpha \tag{11}$$

for all values of Q_0, where Q is the statistic

$$Q = f - \rho(\beta_0)\, t_{AR}(\beta_0).$$

This means that instead of there being a single restriction that needs to be satisfied, there is instead a family of restrictions that the critical value function satisfies. This makes it more plausible that such a function could be pinned down and uniquely defined within a class.[38]

This is an important result, because even if we could establish existence and a form of uniqueness for the conditional statement, without the result from Moreira (2009), it would leave open the possibility that a completely different function could also satisfy (7), which would then lead to the questions of whether in fact such an alternative existed, and if so, how to compute it, and ultimately, which version a practitioner should use. But the result of Moreira (2009) tells us that any notion of uniqueness of a critical value function that satisfies (11) will carry over to the same notion of uniqueness of a critical value function that satisfies (7).

Existence and Uniqueness of an Invariant Curve. Next we seek the function c such that

[38] In our notation, $\Delta(\beta_0) = 0$ just indicates that the probability in Equation (11) is evaluated under the null.

the acceptance region $t^2 \leq c\left(\rho(\beta_0), f^2\right)$ has 95 percent probability under the null, conditional on $Q = Q_0$ for all values of Q_0. This stage of the development follows a similar strategy to that of the development of the tF critical value function in LMMP (2022) (with some important modifications and technical differences). That is, we show that there is a correspondence between the critical value function and an invariant curve solution to a mapping (defined by (7)) that can be viewed as a discrete dynamical system with a fixed point. This reduces the problem to establishing the existence and uniqueness of the invariant curve, and we use the results in Fefferman (2021) to accomplish this.

For a fixed value of $\rho(\beta_0) \neq 0$, we can substitute out $t_{AR}(\beta_0) = \frac{f-Q}{\rho(\beta_0)}$ in the expression for t^2 in (10) to yield an acceptance region in terms of f and Q given by

$$t^2\left(\frac{f-Q}{\rho(\beta_0)}, f, \rho(\beta_0)\right) \leq c\left(\rho(\beta_0), f^2\right).$$

At $Q = 0$, we choose an initial value for the function c that forms the most powerful test (conditional on $Q = 0$), which corresponds to an acceptance region in the form of a bounded interval in the f-space, $\{f \,|\, f^2 \leq \rho(\beta_0)^2 q_{1-\alpha})\}$,[39] where $q_{1-\alpha}$ is the $(1-\alpha)$th quantile of a chi-squared with one degree of freedom. Then we seek the critical value function that extends from this initial value for small values of Q and maintains a bounded interval acceptance region in the f-space.

The discrete dynamical system is specified by a mapping from a point on the curve c to another point on the same curve. Given $\rho(\beta_0) > 0$, take Q small and the acceptance region in f to be a bounded interval. For some $\bar{f} > 0$, let $c = c\left(\rho(\beta_0), \bar{f}^2\right)$, so $(\bar{f}^2, c)$ is a point on the critical value function, which corresponds to an upper endpoint of an acceptance region. The corresponding value of Q for this acceptance region can be obtain by finding a solution Q_0 at the boundary of the acceptance region via:

$$t^2\left(\frac{\bar{f}-Q_0}{\rho(\beta_0)}, \bar{f}, \rho(\beta_0)\right) = c.$$

The upper endpoint $\bar{f}$ and Q_0 determine the upper endpoint of the acceptance region in the $t_{AR}(\beta_0)$ space: $\frac{\bar{f}-Q_0}{\rho(\beta_0)}$. Also, the statistic Q is by construction independent of $t_{AR}(\beta_0)$ which has a standard normal distribution under the null. So the lower endpoint of the acceptance region can be found by solving for $\underline{f}$ in

$$\Phi\left(\frac{\bar{f}-Q_0}{\rho(\beta_0)}\right) - \Phi\left(\frac{\underline{f}-Q_0}{\rho(\beta_0)}\right) = 1-\alpha.$$

For sufficiently small Q_0, $\underline{f}$ will be negative.

[39] Equivalently, we initialize c so that $c(\rho(\beta_0), \rho(\beta_0)^2 q_{1-\alpha}) = \frac{\rho(\beta_0)^2}{1-\rho(\beta_0)^2} q_{1-\alpha}$.

Now the critical value function evaluated at $\underline{f}^2$ is given by

$$c' = t^2\left(\frac{\underline{f}-Q_0}{\rho\left(\beta_0\right)}, \underline{f}, \rho\left(\beta_0\right)\right).$$

These steps describe a mapping from $(\bar{f}^2, c)$ to $(\underline{f}^2, c')$.

This mapping can even be applied starting at a point (F, c) not on the critical value function curve, and so the mapping defines a discrete dynamical system for each $\rho\left(\beta_0\right)$ with a fixed point $(F, c) = (\rho\left(\beta_0\right)^2 q_{1-\alpha}, \frac{\rho(\beta_0)^2}{1-\rho(\beta_0)^2} q_{1-\alpha})$, which is the initial value described above. It turns out that the linearization of this mapping around the fixed point is non-contracting and non-expanding. So, the characterization of the solution to this system involves higher-order expansions and results from Fefferman (2021). These results show existence and uniqueness for an initial segment of the critical value function starting at the known fixed point. Theorem **??** in Appendix D.3 provides a formal statement of these findings along with a local approximation to the critical value function at the fixed point given by

$$c(F;\rho(\beta_0)) = -\frac{\left(\rho^2\left(\beta_0\right) - q_{1-\alpha}\left(1-\rho^2\left(\beta_0\right)\right)\right)}{q_{1-\alpha}\left(1-\rho^2\left(\beta_0\right)\right)^2}(F-\rho^2(\beta_0)q_{1-\alpha}) + \frac{\rho^2(\beta_0)q_{1-\alpha}}{1-\rho^2(\beta_0)} + O((F-\rho^2(\beta_0)q_{1-\alpha})^{3/2})$$

as $F \downarrow \rho^2(\beta_0)q_{1-\alpha}$.

Extension of the curve globally. With the coordinates of the fixed point and the derivative of the invariant curve (i.e., the critical value function) evaluated at $Q_0 = 0$ in hand, it is then straightforward to repeatedly apply the inverse mapping, starting with the initial segment in the neighborhood of the fixed point, to determine the value of c for any value of f^2. As the critical value function is extended away from the fixed point for larger values of Q_0, eventually the function $t^2\left(\frac{f-Q_0}{\rho(\beta_0)}, f, \rho\left(\beta_0\right)\right)$ in the f-space crosses the derived critical value function. In particular, the t^2 function takes the form of a 'W' and the middle 'hump' of this function increases with larger values of Q_0 until it intersects the critical value function c. When this happens, the acceptance region becomes a union of bounded disjoint intervals. This feature is easily accounted for computationally and does not change the essential operation of this algorithm for uniquely extending out the critical value function. At the point where the additional crossing occurs, the critical value function is still continuous but can exhibit a kink or point of non-differentiability.

Specification of the curve for small $|\mathbf{F}|$. The steps above describe the derivation of the critical value function c for $|f^2| \geq \rho^2(\beta_0)q_{1-\alpha}$. It remains to describe c for $|f^2| < \rho^2(\beta_0)q_{1-\alpha}$. From above, the VtF test accepts for $|f^2| < \rho^2(\beta_0)q_{1-\alpha}$. That is, c must satisfy $t^2\left(t_{AR}\left(\beta_0\right), f, \rho\left(\beta_0\right)\right) <$

$c\left(\rho(\beta_0), f^2\right)$. Using (10), it is straightforward to show that

$$\max_{t_{AR}(\beta_0)} t^2\left(t_{AR}\left(\beta_0\right), f, \rho\left(\beta_0\right)\right) = \frac{f^2}{1-\rho^2\left(\beta_0\right)}.$$

It follows that any choice of c satisfying

$$c\left(\rho(\beta_0), f^2\right) > \frac{f^2}{1-\rho^2\left(\beta_0\right)}$$

for $|f^2| < \rho^2(\beta_0)q_{1-\alpha}$ will lead to acceptance over this region. So, there are many equivalent choices for c in this region, e.g. $c\left(\rho(\beta_0), f^2\right) = \frac{\rho^2(\beta_0)q_{1-\alpha}}{1-\rho^2(\beta_0)}$ or ∞ for $|f^2| < \rho^2(\beta_0)q_{1-\alpha}$. In Figure 4, the VtF critical value function is not drawn over this region of the f-space since an infinity of equivalent choices for c are possible.

VtF **critical value function when** $\rho(\beta_0) = 0$**.** The specific case of $\rho\left(\beta_0\right) = 0$ is derived in a different way since the substitution $t_{AR}\left(\beta_0\right) = \frac{f-Q}{\rho(\beta_0)}$ is not possible in that case. Instead, we note that when $\rho\left(\beta_0\right) = 0$,

$$t^2 = \frac{t_{AR}\left(\beta_0\right)^2 f^2}{f^2 + t_{AR}\left(\beta_0\right)^2}$$

and $Q = f$ is independent of $t_{AR}\left(\beta_0\right)$. Rejection is given by

$$\frac{t_{AR}\left(\beta_0\right)^2 f^2}{f^2 + t_{AR}\left(\beta_0\right)^2} > c\left(0, f^2\right),$$

which is identical to

$$t_{AR}\left(\beta_0\right)^2 > \frac{c\left(0, f^2\right) f^2}{f^2 - c\left(0, f^2\right)}. \tag{12}$$

For any value of $Q = f$, this must occur with probability α. So for any value of f

$$\frac{c\left(0, f^2\right) f^2}{f^2 - c\left(0, f^2\right)} = q_{1-\alpha} \quad \Longrightarrow \quad c\left(0, F\right) = \frac{q_{1-\alpha}}{1+\frac{q_{1-\alpha}}{F}}.$$

Remark. From (12), it follows that AR and VtF are identical tests for values β_0 corresponding to $\rho\left(\beta_0\right) = 0$. For any β_0 such that $\rho\left(\beta_0\right) \neq 0$, the tests are different.

D.2 Confidence Interval Construction

Once the VtF critical value function c_α has been obtained, confidence intervals can be constructed from inversion of the test. In this section, we show how – up to location ($\hat{\beta}$) and scale ($\hat{se}(\hat{\beta})$) quantities – the resulting confidence intervals can be expressed as a function of two statistics, $\hat{F}$, and $\hat{r}$. This data dimension reduction is precisely what allows us to produce the two-dimensional "heatmaps" in Figures 6a, 6b, and 6d, which contain a data-realization by data-realization comparison of confidence interval lengths. This data dimension reduction also underlies the ability to produce the inflation factors in Figure A9.

By definition, the VtF confidence sets are the values of β_0 that satisfy

$$\hat{t}^2 = \frac{\hat{t}_{AR}^2(\beta_0)\hat{f}^2}{\hat{f}^2 - 2\hat{\rho}(\beta_0)\hat{t}_{AR}(\beta_0)\hat{f} + \hat{t}_{AR}^2(\beta_0)} \leq c_\alpha\left(\hat{\rho}(\beta_0), \hat{f}^2\right).$$

To arrive at the reduction of dimensionality in the confidence interval expression, it is useful to start by working with an equivalent linear transformation of the parameter β. Let

$$\beta^* = \beta \cdot \frac{\sqrt{\frac{\hat{\sigma}_{22}}{\hat{\sigma}_{11}}}}{\sqrt{1-\hat{\rho}_{RF}^2}} - \frac{\hat{\rho}_{RF}}{\sqrt{1-\hat{\rho}_{RF}^2}}$$

and analogously define β_0^* and $\hat{\beta}^*$ (by replacing β above with β_0 and $\hat{\beta}$).

With a slight abuse of notation, the VtF confidence sets for the parameter β^* are the values of β_0^* that satisfy

$$\hat{t}^2 = \frac{\hat{t}_{AR}\left(\beta_0^*\right)^2\hat{f}^2}{\hat{f}^2 - 2\hat{\rho}\left(\beta_0^*\right)\hat{t}_{AR}\left(\beta_0^*\right)\hat{f} + \hat{t}_{AR}\left(\beta_0^*\right)^2} \leq c_\alpha\left(-\frac{\beta_0^*}{\sqrt{1+(\beta_0^*)^2}}, \hat{F}\right).$$

where we have used the fact that this particular linear transformation implies $\hat{\rho}(\beta_0^*) = -\frac{\beta_0^*}{\sqrt{1+(\beta_0^*)^2}}$. The linear transformation also leads to a simplified expression for $\hat{t}_{AR}\left(\beta_0^*\right)$ that depends only on $\hat{f}, \hat{\beta}^*$, and β_0^*,

$$\hat{t}_{AR}(\beta_0^*) = \frac{\hat{f}\left(\hat{\beta}^* - \beta_0^*\right)}{\sqrt{1+\beta_0^{*2}}}.$$

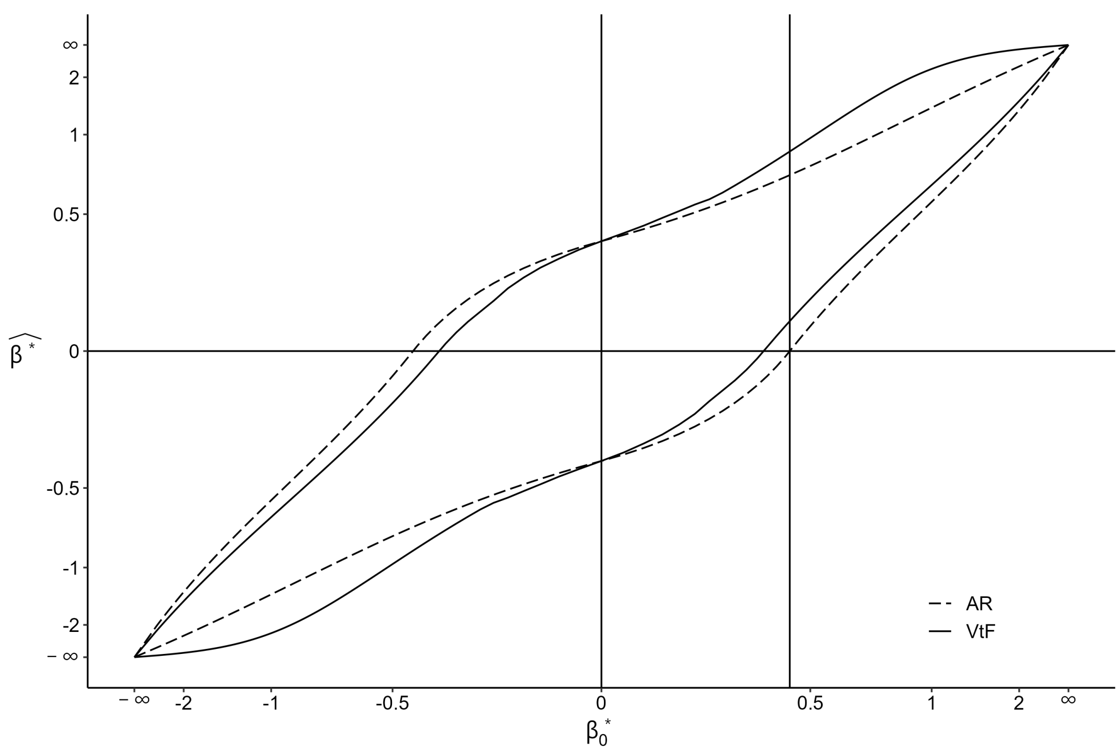
Therefore, both sides of the inequality above are defined only by three quantities, $\hat{f}, \hat{\beta}^*$, and β_0^*, and hence the points that define the boundaries of the confidence set are a function of $\hat{f}, \hat{\beta}^*$. And since $\hat{t}^2$ is symmetric in $\hat{f}$ around $\hat{f} = 0$ and $\hat{r} = -\hat{\beta}^*/\sqrt{1+\hat{\beta}^{*2}}$ (so that there is a one-to-one mapping between $\hat{r}$ and $\hat{\beta}^*$), the boundaries of the confidence set are determined by the realization of the statistics $\hat{F}, \hat{r}$. A similar data dimension reduction follows for AR and tF allowing for the comparison of confidence interval lengths for all data realizations through only the values of $\hat{F}, |\hat{r}|$ (or only $\hat{F}$ in the case of tF), see Figures 6a, 6b, and 6d.

If β_U denotes the upper endpoint of the VtF confidence interval, then the corresponding "inflation factor" is $k^+ = (\beta_U - \hat{\beta})/\hat{se}(\hat{\beta})$. Since the transformation to the β^* space is linear, it is straightforward to show that the upper inflation factor can be equivalently expressed as $k^+ = (\beta_U^* - \hat{\beta}^*)/\hat{se}(\hat{\beta}^*)$. Since $\hat{se}(\hat{\beta}^*) = \frac{1}{\sqrt{\hat{F}}}\sqrt{1+\hat{\beta}^{*2}}$ and the upper endpoint β_U^* has already been shown to be a function of $\hat{F}, \hat{r}$, it follows that the upper inflation factor is uniquely determined by $\hat{F}, \hat{r}$: $k^+ = k^+(\hat{F}, \hat{r})$. A similar argument follows for the lower endpoint, so that the VtF confidence interval is given by

$$\left[\hat{\beta} - k^-\left(\hat{F}, \hat{r}\right) \cdot \hat{se}\left(\hat{\beta}\right), \hat{\beta} + k^+\left(\hat{F}, \hat{r}\right) \cdot \hat{se}(\hat{\beta})\right].$$

The inversion of the test statistic provides one perspective on what drives the interval length advantage of VtF over AR. For a fixed value of $\hat{F} = (3.5)^2$, Figure A18 plots the acceptance region, in terms of values of $\hat{\beta}^*$ for AR and VtF, for all values of β_0^*. The transformation $\frac{x}{\sqrt{1+x^2}}$ is used for both the horizontal and vertical axes, so that we can visualize values of β_0^* and $\hat{\beta}^*$ as large as ∞ in magnitude. The boundaries of the two procedures' acceptance regions are identical at $\beta_0^* = 0$, when VtF and AR are identical procedures. The set of points on any vertical line between the critical value boundaries represents the values of $\hat{\beta}^*$ that would lead to acceptance of that null hypothesis. At the same time, the set of points on any horizontal line between the boundaries represents the confidence interval.

Figure A18: Acceptance Region and Confidence Interval: VtF vs AR with $\hat{F} = (3.5)^2$

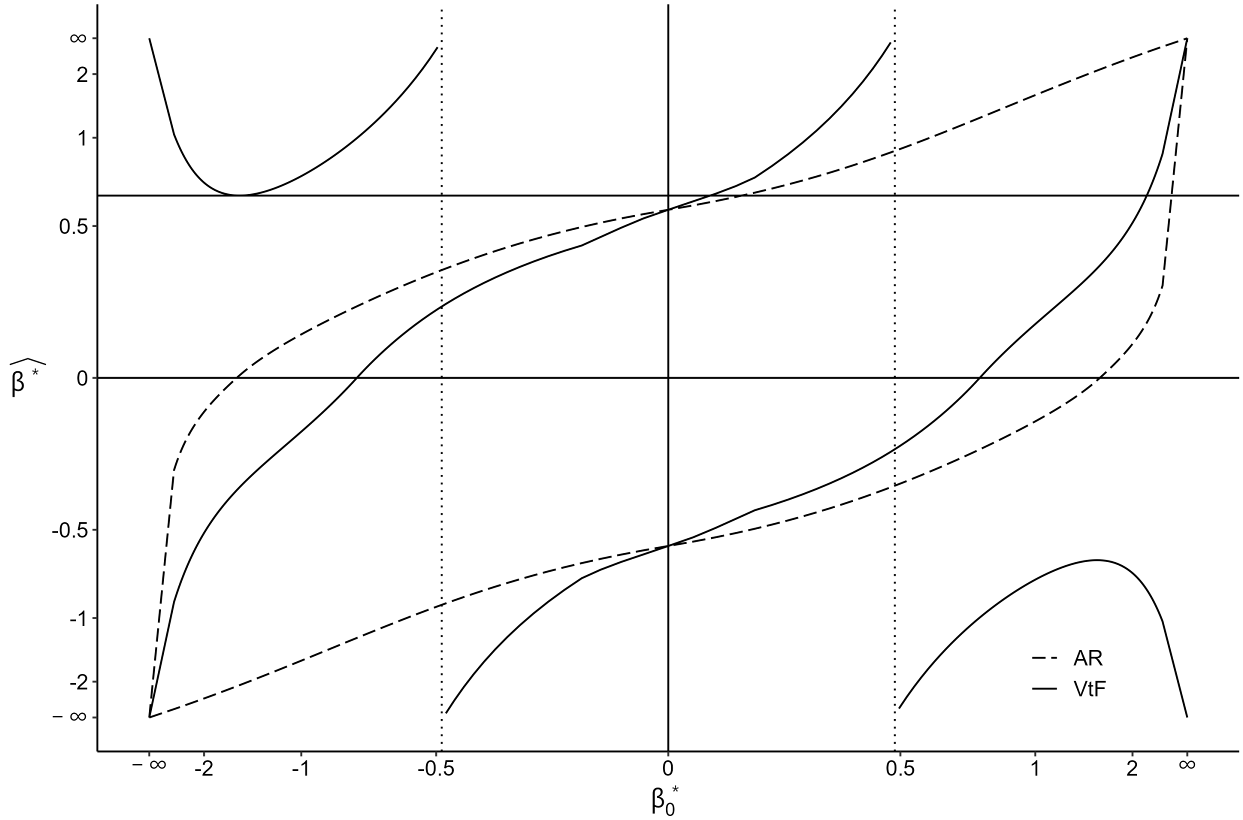


Note: Horizontal and vertical axis use a $\frac{x}{\sqrt{1+x^2}}$ scale, and are labeled using the normalization β^* described in the text.

Relative to *AR*, the boundaries for acceptance for VtF are rotated in the counter-clockwise direction around the points of intersection at $\beta_0^* = 0$. This means that for any hypothesis other than $\beta_0^* = 0$, the acceptance regions for $\hat{\beta}^*$ under the two procedures will not contain one another. On the other hand, this very same counter-clockwise rotation implies that there must exist a region of realizations of $\hat{\beta}^*$ such that the VtF *confidence interval* is entirely contained within the *AR* interval – and hence also is shorter in length. And even for magnitudes outside this range of $\hat{\beta}^*$, it is still possible for the VtF interval to be shorter, even if it is not contained within the *AR* interval. (Note that when VtF is not contained within the *AR* interval, it is more difficult to visualize relative length because the horizontal axis is not a linear scale.)

Remark. Even though VtF and *AR* have bounded confidence sets under the same condition $F > q_{1-\alpha}$, VtF can produce disjoint intervals as a confidence set. This is illustrated in Appendix A19, where the value of $\hat{F}$ is 2^2. For any value of β_0^*, *AR*'s acceptance region for $\hat{\beta}^*$ is a convex set. By contrast, for VtF, when the magnitude of β_0^* is sufficiently large, the acceptance region is a non-convex set. This occurs because for those sufficiently large values of β_0^*, $\hat{F} < c_\alpha\left(\beta_0^*, \hat{F}\right)$. As a consequence, for sufficiently large magnitudes for $\hat{\beta}^*$, the confidence set will consist of more than one bounded interval. Although these disjoint intervals are valid in the sense of containing the true parameter β^* at the intended confidence level, throughout the paper (e.g. Table 2, A7 A5, Figures 2, 6a, 6b, 6d), when we compare VtF's confidence set performance to other confidence interval methods, we have simply convexified the VtF interval; we use the smallest single interval that covers the entire VtF confidence set. Strictly speaking, these simplified intervals are conservative, but we surmise that practitioners will ultimately be more interested in reporting global rather than local "bounds" as confidence sets.

Figure A19: Acceptance Region and Confidence Interval: VtF vs AR
with $\hat{F} = (2)^2$



Note: Note: Horizontal and vertical axis use a $\frac{x}{\sqrt{1+x^2}}$ scale, and are labeled using the normalization β^* described in the text.

D.3 Further Details and Properties of the VtF Critical Value Function

(See authors for further details)