Name:________________________________

Princeton University

ORF 523
Midterm Exam 1, Spring 2020

March 12, 2020, from 1:00 pm to 3:00 pm

(No more than 100 minutes can be spent on the exam.
The additional 20 minutes are for submission of the exam.)

Instructor:
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AIs:
Bachir El Khadir
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Emmanuel Ekwedike (grader)

Please read the exam rules below before you start.

1. The exam is to be submitted electronically on Blackboard before 3PM. We prefer a single PDF file, but pictures of solutions to individual problems are acceptable as well. In case of multiple submissions, only the latest version before 3PM will be graded.
2. Please remember to write your name on the first page of your solutions. Right next to it, please write out and sign the following pledge: *"I pledge my honor that I have not violated the honor code or the rules specified by the instructor during this examination."*
3. You cannot communicate with anyone during the exam.
4. You can only use the Internet for submission of the exam.
5. Each problem has 20 points. You can cite results shown in lecture or on problem sets without proof.

True or False? If "True," provide a proof. If "False," provide a counterexample and justify why your counterexample is valid.

1. Suppose $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$ are two quasiconvex functions. Then their pointwise maximum (i.e., the function $g(x) = \max\{f_1(x), f_2(x)\}$) is a quasiconvex function.

2. If $f : \mathbb{R}^n \to \mathbb{R}$ is a norm and $\Omega \subseteq \mathbb{R}^n$ is a compact convex set, the problem of minimizing f^2 over Ω has a unique optimal solution.[1]

3. A point $\bar{x} \in \mathbb{R}^n$ is a local minimum of a quadratic (i.e., degree-2) polynomial $p : \mathbb{R}^n \to \mathbb{R}$ if and only if there are no descent directions[2] at $\bar{x}$.

4. A point $\bar{x} \in \mathbb{R}^n$ is a local minimum of a cubic (i.e., degree-3) polynomial $p : \mathbb{R}^n \to \mathbb{R}$ if and only if there are no descent directions at $\bar{x}$.

5. Suppose $\Omega \subseteq \mathbb{R}^n$ is a closed convex set and c is a vector in $\mathbb{R}^n$. Consider the problem of minimizing $c^T x$ over Ω. If this problem has a finite optimal value, then it has an optimal solution.

[1] To avoid possible confusion, we note that f^2 denotes the square of the function f, not the composition of f with itself.

[2] We recall that a direction $d \in \mathbb{R}^n$ is a descent direction for the function p at the point $\bar{x}$ if there exists a scalar $\bar{\alpha} > 0$ such that $p(\bar{x} + \alpha d) < p(\bar{x})$ for all $\alpha \in (0, \bar{\alpha})$.