PRINCETON UNIVERSITY

Workshop Proposal - 55th Conference on Decision and Control

Workshop title. Solving large-scale semidefinite programs in controls, robotics, and machine learning.

Motivation and workshop goals. Exciting recent developments at the interface of optimization and control have shown that several fundamental problems in dynamics and control, such as stability analysis, collision avoidance, robust performance, and controller synthesis can be addressed by a synergy of classical tools from Lyapunov theory and modern algebraic techniques in optimization. The success of algebraic methods stems from the fact that at the heart of many control problems lies the task of optimizing a polynomial function over a *semialgebraic set*, i.e., a set described by polynomial equations or inequalities. Until not too long ago, such problems were believed to be intractable from a computational perspective. Around the year 2000, however, the concept of *"sum of squares (sos) optimization"* was developed [18], [12], [19], which showed that semialgebraic problems can often be successfully addressed by *semidefinite programming*—a subclass of convex optimization problems for which global solution methods are available. This discovery was general enough that it impacted not just numerous problems of controls [10], [4], [11], [1], but also combinatorial optimization [9], game theory [20], statistics and machine learning [15], software verification [21], filter design [22], quantum computation [6], robotics [23], and many others.

Despite the wonderful advances in algebraic techniques for polynomial optimization and their successful interplay with problems in control and other areas, a single challenge has limited the horizon of possibilities for the field and that is *scalability*. Indeed, it is well known that the size of the semidefinite programs (SDPs) resulting from sum of squares techniques (although polynomial in the data size) grows quickly and this severely limits the scale of the problems that can be efficiently and reliably solved with available SDP solvers. This drawback deprives large-scale systems of the application of algebraic techniques and perhaps equally importantly shuts the door on the enormous opportunities that lie ahead if we could use these tools for real-time optimization.

In this workshop, after a review of fundamentals by some of the leaders of the field, a select group of researchers will give tutorials on recent exciting algorithmic innovations which were developed to circumvent the scalability issues encountered in semidefinite and sum of squares programming. We will showcase new problems in controls that are now within reach and also layout challenges that remain. In addition, our plan is to demonstrate how the recent more-scalable algorithms for semidefinite programming are facilitating the transfer of ideas from controls into two nearby fields of interest, namely robotics and machine learning.

Workshop content and structure. The full-day workshop is divided into three separate modules:

- In Module 1 (consisting of one session), we are very excited to have two pioneers of the field, Professors Pablo Parrilo (MIT) and Jean Bernard Lasserre (LAAS, CNRS), review the fundamentals of semidefinite and sum of squares programming and describe recent trends.
- Module 2 (consisting of three sessions) is organized around three different axes for improving scalability of semidefinite programs:
  - The first axis is around introducing a lower-complexity SDP hierarchy for polynomial optimization [13], and around replacing SDPs all together by simpler optimization problems such as linear programs (LP) or second order cone programs (SOCP) which can be solved at larger scales. Here, we give tutorials on the so-called "dsos" and "sdsos" optimization techniques [3], [2], [16], which serve as LP and SOCP-based alternatives to sos optimization.
  - The second axis is on introduction of systematic techniques for exploiting sparsity and other structural properties of semidefinite programs in order to reduce their size [8], [5]. One of the works presented in this session [17] is the recipient of the 2015 O. Hugo Schuck Best Paper Award of the American Automatic Control Council.
  - The third axis of improvement concerns recent algorithmic developments for large-scale SDPs which are meant to replace standard interior point based methods. In this session, we discuss the new QSDPNAL algorithm [14] and an extension of the Frank-Wolfe method to low-rank matrix completion problems arising in machine learning [7].

- In Module 3 (consisting of one session), we present a wide variety of applications of large-scale semidefinite and sum of squares programs to machine learning, robotics, and controls. Our first speaker in the session is from the Google Brain Team and will give us interesting insights into the current challenges in machine learning faced by the industry. Our second speaker will present applications in robotics related to autonomous flying and collision avoidance. Our third speaker will conclude with insights on the interplay between big data and sparsity in systems theory.

Workshop organizers.

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Targeted audience and prerequisites. This workshop is aimed at researchers with an interest in convex optimization-based approaches to nonconvex, semialgebraic problems of controls, machine learning, and robotics.

We have purposefully chosen to schedule a review of fundamentals session at the beginning of the day so that the only prerequisite required for the workshop would be a basic knowledge of convex optimization. As such, graduate students and junior researchers are definitely part of our target audience, though the workshop may also be of interest to more senior researchers.

The tentative schedule of our workshop is given below.

Module 1: Fundamentals of semidefinite and SOS programming	
Pablo Parrilo	9:00 – 9:30am
Jean-Bernard Lasserre	9:30 – 10:10am
Break	10:10 – 10:30am
Module 2: SDP and scalability	
Approximating SDPs with simpler optimization problems	
Amir Ali Ahmadi	10:30 – 11:00am
Georgina Hall	11:00 – 11:30am
Lunch break	11:30am – 1:00pm
Exploiting structure in SDPs	
Pablo Parrilo	1:00 – 1:30pm
Antonis Papachristodoulou	1:30 – 2:00pm
Break	2:00 – 2:15pm
Better algorithms for SDPs	
Defeng Sun	2:15 – 2:45pm
Rob Freund	2:45 – 3:15pm
Break	3:15 – 3:30pm
Module 3: Applications to control, machine learning, and robotics	
Vikas Sindhwani	3:30 – 4:00pm
Aniruhda Majumdar	4:00 – 4:30pm
Mario Sznaier	4:30 – 5:00pm

The speakers' biographies and tentative abstracts are given below in the order of appearance.

Pablo A. Parrilo is a Professor of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology. He is currently Associate Director of the Laboratory for Information and Decision Systems (LIDS), and is also affiliated with the Operations Research Center (ORC). He received an Electronics Engineering undergraduate degree from the University of Buenos Aires (Argentina), and a PhD in Control and Dynamical Systems from the California Institute of Technology. His research interests include optimization methods for engineering applications, control and identification of uncertain complex systems, robustness analysis and synthesis, and the development and application of computational tools based on convex optimization and algorithmic algebra to practically relevant engineering problems. Prof. Parrilo has received several distinctions, including a Finmeccanica Career Development Chair, the Donald P. Eckman Award of the American Automatic Control Council, the SIAM Activity Group on Control and Systems Theory (SIAG/CST) Prize, the IEEE Antonio Ruberti Young Researcher Prize, and the Farkas Prize of the INFORMS Optimization Society. He is an IEEE Fellow.

Title (First talk): Sums of squares techniques and polynomial optimization

Abstract (First talk): Optimization and decision problems involving multivariate polynomials are ubiquitous in many areas of engineering and applied mathematics. In recent years there has been much interest in the use of convex optimization based symbolic-numeric techniques for this class of problems. We survey the basic features of these algebraic approaches, involving sum of squares (SOS) and semidefinite programming, emphasizing geometric aspects and a few selected applications in systems and control theory.

Title (Second talk): Exploiting structure in sum of squares programs

Abstract (Second talk): We discuss a number of techniques for exploiting structure in the formulation of SOS/SDP programs. Among others, these include structured sparsity (Newton polytopes), equality constraints (SOS on quotient rings), group theoretic structure (symmetry reduction), and facial reduction. These techniques can notably improve the size and numerical conditioning of the resulting SDPs, and are illustrated using several control-oriented applications.

Jean B. Lasserre is a "Directeur de Recherche" at LAAS, a CNRS laboratory in Toulouse (France). He is also a member of IMT, the Institute of Mathematics of Toulouse. He was twice a one-year visitor in the Electrical Engineering Dept. of the University of California at Berkeley. He wrote several articles, notably in Applied Math, Control and Optimization, and is the author or co-author of eight books, including *Moments, Positive Polynomials and Their Applications*, Imperial College Press, London, 2009 and *An Introduction to Polynomial and Semi-Algebraic Optimization*, Cambridge University Press, Cambridge, UK, 2015. He is a SIAM Fellow, and the recipient of the 2009 Lagrange prize in Continuous Optimization, the 2015 Kachiyan prize of the Optimization Society of INFORMS and the 2015 John Von Neumann Theory prize of the INFORMS Society. He is also a Laureate of the European Research Council (ERC) for his ERC-Advanced Grant project TAMING.

Title: BSOS: a bounded-degree SOS hierarchy for polynomial optimization.

Abstract: The powerful SOS-based hierarchy of semidefinite programs based on Putinar's positivity certificate is penalized by the fast growth of the size of the semidefinite matrices involved. The BSOS hierarchy uses a different positivity certificate (mixing Krivine-Handelman's and Putinar's), and involves semidefinite matirces of fixed size. In contrast to the Krivine-Handelman LP-hierarchy, finite convergence for SOS-convex programs is guaranteed.

Amir Ali Ahmadi is an Assistant Professor at the Department of Operations Research and Financial Engineering at Princeton University and an Associated Faculty member of the Department of Computer Science. Amir Ali received his PhD in EECS from MIT and was a Goldstine Fellow at the IBM Watson Research Center prior to joining Princeton. His research interests are in optimization, computational aspects of dynamics and control, and computational complexity theory. Amir Ali's recent awards include the INFORMS Computing Society Prize (for best series of papers at the interface of operations research and computer science), the AFOSR Young Investigator Program Award, the NSF CAREER Aaward, the Google Faculty Award, the Goldstine Fellowship of IBM Research, the teaching award of Princeton's Engineering Council, the Best Conference Paper Award of the IEEE International Conference on Robotics and Automation, and the prize for one of two most outstanding papers published in the SIAM Journal on Control and

Optimization in 2013-2015. Amir Ali is also a three-time recipient of the NSF Oberwolfach Fellowship and a best student paper award finalist at the 47th IEEE Conference on Decision and Control.

Title: DSOS and SDSOS optimization: more tractable alternatives to SOS optimization

Abstract: "DSOS and SDSOS optimization" techniques are more scalable alternatives to sum of squares (SOS) optimization which instead of semidefinite programming rely on linear and second order cone programming. They have been successfully used (for example) to find a stabilizing controller for a model of a humanoid robot with 30 state variables, and can handle quartic polynomial optimization problems with 50-70 variables in the order of a few minutes. In this talk, we review the theoretical and numerical aspects of these algorithms and present new directions for improving their approximation quality in an iterative fashion.

Georgina Hall is a fourth-year graduate student and a Gordon Wu fellow in the department of Operations Research at Princeton University, under the supervision of Professor Amir Ali Ahmadi. She obtained a B.S. and an M.S. from the Ecole Centrale, Paris, in 2011 and 2013 respectively. Her interests lie in convex relaxations of NP-hard problems, particularly those arising in polynomial optimization. She is a recipient of the Engineering Council Teaching Award, the Graduate School Excellence in Teaching Award, and the Médaille de l'Ecole Centrale from the French Académie des Sciences.

Title: Iterative LP and SOCP-based approximations to SDPs

Abstract: We develop techniques for approximating SDPs with LPs and SOCPs. Our algorithms iteratively grow an inner approximation to the PSD cone using a column generation scheme and/or a change of basis scheme involving Cholesky decompositions.

Antonis Papachristodoulou holds an M.A./M.Eng. degree in Electrical and Information Sciences from the University of Cambridge and a Ph.D. degree in Control and Dynamical Systems from the California Institute of Technology. Currently, he is an Associate Professor in Engineering Science at the University of Oxford and a Tutorial Fellow at Worcester College. He is an EPSRC Fellow for Growth in Synthetic Biology and Director of the

EPSRC & BBSRC Centre for Doctoral Training in Synthetic Biology. His research interests include large-scale nonlinear systems analysis, sum of squares programming, synthetic and systems biology, networked systems, and flow control. In 2015 he received the European Control Award for his contributions to robustness analysis and applications to networked control systems and systems biology and the O. Hugo Schuck Best Paper Award.

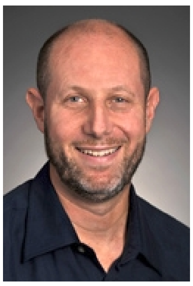
Title: Exploiting chordal sparsity for the analysis and design of large-scale networked systems

Abstract: Many systems analysis and some design questions can be formulated as Linear Matrix Inequalities and solved using semidefinite programming. For large system instances, it is essential to exploit or even impose sparsity and structure within the problem in order to solve the associated programs efficiently. In this talk we will present recent results on the analysis and design of networked systems, where chordal sparsity can be used to decompose the resulting SDPs, and solve an equivalent set of smaller semidefinite constraints.

Defeng Sun is Professor at the Department of Mathematics, National University of Singapore. His main research interest lies in large scale matrix optimization and statistical learning. Currently he serves as associate editor to Mathematical Programming, both Series A and Series B, SIAM Journal on Optimization and others.

Title: A two-phase proximal augmented Lagrangian method for linear and convex quadratic semidefinite programming problems.

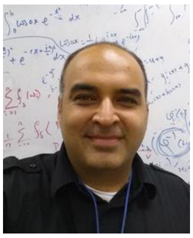
Abstract: In this talk we shall introduce a two-phase proximal augmented Lagrangian method, called QSDPNAL, for solving large scale linear and convex quadratic semidefinite programming problems of many equality and inequality constraints. QSDPNAL is built on several recently developed ingredients in conic optimization: the semi-smooth Newton-CG method, the symmetric Gauss-Seidel decomposition theorem and the robust calmness property in sensitivity analysis.



Robert Freund is the Theresa Seley Professor in Management Science at the Sloan School of Management at MIT. He received his B.A. in Mathematics from Princeton University and M.S. and Ph.D. degrees in Operations Research at Stanford University. His main research interests are in convex optimization, computational complexity and related computational science, convex geometry, large-scale nonlinear optimization, and related mathematical systems. He received the Longuet-Higgins Prize in computer vision (2007) as well as numerous teaching and education awards at MIT in conjunction with the course and textbook (co-authored with Dimitris Bertsimas) Data, Models, and Decisions: the Fundamentals of Management Science. He has served as Co-Editor of the journal Mathematical Programming and Associate Editor of several optimization and operations research journals. He is the former Co-Director of MIT Operations Research Center, the MIT Program in Computation for Design and Optimization, and the former Chair of the INFORMS Optimization Section. He also served a term as Deputy Dean of the Sloan School at MIT (2008-11).

Title: : An Extended Frank-Wolfe Method, and its Application to Low-Rank Matrix Completion

Abstract: Motivated by the problem of computing low-rank matrix completion solutions, we present an extension of the Frank-Wolfe method that is designed to induce near-optimal solutions on low-dimensional faces of the feasible region. We also present computational guarantees for the method that trade off efficiency in computing near-optimal solutions with upper bounds on the dimension of minimal faces of iterates. We then present computational results for large-scale matrix completion problems that demonstrate significant speed-ups in computing low-rank near-optimal solutions.



Vikas Sindhwani is Research Scientist in the Google Brain team in New York. His interests are broadly in core mathematical foundations of statistical learning, and in end-to-end design aspects of building large-scale, robust machine intelligence systems. He received the best paper award at Uncertainty in Artificial Intelligence (UAI) 2013, the IBM Pat Goldberg Memorial Award in 2014, and was co-winner of the Knowledge Discovery and Data Mining (KDD) Cup in 2009. He previously led the Machine Learning group at IBM Research, NY, and has a PhD in CS from the University of Chicago. His publications are available at: `http://vikas.sindhwani.org/`.

Title: : Geometric Reasoning in Complex 3D Environments using Sum-of-squares Programming

Abstract: Motivated by applications in Robotics and Computer Vision,

we study problems related to real-time spatial reasoning of a 3D environment using sublevel sets of polynomials. These include: tightly containing a cloud of points (e.g., representing an obstacle) with convex or nearly-convex basic semialgebraic sets, computation of Euclidean distances between two such sets, separation of two convex basic semalgebraic sets that overlap, and tight containment of the union of several basic semialgebraic sets with a single convex one. We use algebraic techniques from sum of squares (sos) optimization that reduce all these tasks to semidefinite programs of small size and present numerical experiments in realistic scenarios. Joint work with Amirali Ahmadi, Georgina Hall and Ameesh Makadia (`https://arxiv.org/abs/1611.07369`).

Anirudha Majumdar is a Ph.D. candidate in the Electrical Engineering and Computer Science department at MIT. He is a member of the Robot Locomotion Group at the Computer Science and Artificial Intelligence Lab and is advised by Prof. Russ Tedrake. Ani received his undergraduate degree in Mechanical Engineering and Mathematics from the University of Pennsylvania, where he was a member of the GRASP lab. His research is primarily in robotics: he works on algorithms for controlling highly dynamics robots such as unmanned aerial vehicles with formal guarantees on the safety of the system. Ani's research has been recognized by the Siebel Foundation Scholarship and the Best Conference Paper Award at the International Conference on Robotics and Automation (ICRA) 2013.

Title: : Optimization techniques for controlling agile robots with formal safety guarantees

Abstract: In this talk, I will describe algorithms for the synthesis of feedback controllers that come with associated formal guarantees on the stability of the robot and show how these controllers and certificates of stability can be used for robust planning in environments previously unseen by the system. In order to make these results possible, our approach leverages computational tools such as sums-of-squares (SOS) programming and semidefinite programming. I will describe this work in the context of the problem of high-speed unmanned aerial vehicle (UAV) flight through cluttered environments previously unseen by the robot. In this context, our approach allows us to guarantee that the robot will fly through its environment in a collision-free manner despite uncertainty in the dynamics (e.g., wind gusts or modeling errors). The resulting hardware demonstrations on a fixed-wing airplane

constitute one of the first examples of provably safe and robust control for robotic systems with complex nonlinear dynamics that need to plan in real-time in environments with complex geometric constraints.

Mario Sznaier is the Dennis Picard Chaired Professor at the Electrical and Computer Engineering Department, Northeastern University. Prior to joining Northeastern University, Dr. Sznaier was a Professor of Electrical Engineering at the Pennsylvania State University and also held visiting positions at the California Institute of Technology. His research interest include robust identification and control of hybrid systems, robust optimization, and dynamical vision. Dr. Sznaier is currently serving as an associate editor for the journal Automatica, chair of the IEEE Control Systems Society Technical Committee on Computational Aspects of Control Systems Design, General Chair of the 2016 IEEE Multi Systems Conference and Program Chair of the 2017 IEEE Conference on Decision and Control. In 2012 he received a distinguished member award from the IEEE Control Systems Society for his contributions to robust control, identification and dynamic vision.

Title: The interplay between big data and sparsity in system theory.

Abstract: Arguably, one of the hardest challenges faced now by the systems community stems from the exponential explosion in the availability of data, fueled by recent advances in sensing and actuation capabilities. Simply stated, classical techniques are ill equipped to handle very large volumes of (heterogeneous) data, due to poor scaling properties and to impose the structural constraints required to implement ubiquitous sensing and control.

The goal of this talk is to explore how this curse of dimensionality can be potentially overcome by exploiting the twin blessings of self-similarity (high degree of spatio-temporal correlation in the data) and inherent underlying sparsity. While these ideas have already been recently used in machine learning (for instance in the context of dimensionality reduction and variable selection), they have hitherto not been fully exploited in systems theory. By appealing to a deep connection to semi-algebraic optimization, rank minimization and matrix completion we will show that, in the context of systems theory, the limiting factor is given by the "memory" of the system rather than the size of the data itself, and discuss the implications of this fact. These concepts will be illustrated examining examples of "easy" and "hard" problems, including the synthesis of filters and controllers subject to information

flow constraints, and identification of classes of non-linear systems.

The talk will conclude with an application of these ideas to the non-trivial problem of extracting actionable information from very large data streams. In particular, we will show how exploiting sparsity leads to tractable, scalable solutions to the problems of anomaly detection and activity analysis from video streams

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