Name:________________________________

PRINCETON UNIVERSITY

ORF 363/COS 323
Midterm Exam, Fall 2024

OCTOBER 10, 2024, FROM 1:30 PM TO 2:50 PM

Instructor: A.A. Ahmadi

AIs: Aydin, Budway, Hua, Tziampazis

PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed.
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture, lecture notes, or problem sets without proof.

You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = \frac{1}{3}x_1^3 - x_1^2 + x_1x_2 + \frac{1}{2}x_2^2 + x_1 - x_2 - \frac{363}{323}$$

Problem 2: Consider an affine function $f(x) = a^T x + b$ for a nonzero vector $a \in \mathbb{R}^n$ and a scalar $b \in \mathbb{R}$. Are the following statements about the square of this function true or false? (In each case, provide a proof or a counterexample.)

(a) $f^2(x)$ is quasiconvex.

(b) $f^2(x)$ is convex.

(c) $f^2(x)$ is strictly convex.

Problem 3: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x) = x^T Q x + b^T x,$$

where the matrix Q and the vector b are given as:

$$Q = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}, \quad b = \begin{bmatrix} \alpha \\ 1 \end{bmatrix}.$$

Determine the range of α for which $f(x)$ is quasiconvex.

Problem 4: Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable convex function that satisfies $f(x) \leq 0$ for all $x \in \mathbb{R}^n$ and denote its gradient vector by $\nabla f(x)$. Show that $\nabla f(x) = 0$ for all $x \in \mathbb{R}^n$. (This implies that $f(x)$ must be a constant but you do not need to prove this.)