Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2020

October 16, 2020, 5PM EST - October 19, 2020, 9AM EST

(The exam is to be submitted on Gradecope within 2 hours of the download time. We strongly recommend that you leave about 20 minutes of these 2 hours for submission.)

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Please read the exam rules below before you start.

1. The exam should be submitted on Gradescope within two hours of the time you download it. You are free to write your solutions on paper or on a tablet, or to type them up (though we do not recommend that you type them because of limited time). Your submission can be a single PDF file (preferably with problems matched to pages), or multiple pictures of solutions to individual problems. Only the latest version submitted before your deadline will be graded.

2. Please remember to write your name on the first page of your solutions. Right next to it, please write out and sign the following pledge: *"I pledge my honor that I have not violated the honor code during this examination."*

3. You cannot communicate with anyone during the exam.

4. You can only use the Internet for submission of the exam.

5. You can cite results shown in lecture, lecture notes, precepts, or on problem sets without proof. You are not required to submit your "cheat sheet".

Problem 1 (20 pts): Consider the function $f(x_1, x_2) = g(x_1) + x_1x_2$, where $g : \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable. Show that regardless of the choice of the function g, the function f will not have a local minimum.

Problem 2 (30 pts): Consider the univariate function

$$f(x) = a_4x^4 + a_2x^2 + a_1x + a_0$$

where $a_0, a_1, a_2, a_4 \in \mathbb{R}$ are parameters.

(a) For what parameter values a_0, a_1, a_2, a_4 is f convex?

(b) For what parameter values a_0, a_1, a_2, a_4 is f strictly convex?

(c) Consider the case where f is convex and $a_0 = a_1 = a_2 = 1$. Suppose that to minimize f, we run Newton's method starting at $x_0 = 0$. After one iteration, what is the smallest possible value of f (given that you have the freedom to choose a_4)?

Problem 3 (25 pts): Consider the optimization problem

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & x \in \Omega, \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $\Omega \subseteq \mathbb{R}^n$. Classify the following statement as "true" or "false" by giving a proof or a counterexample:
If f is quasiconvex and Ω is a convex set, then every local minimum of (1) is a global minimum of (1).

Problem 4 (25 pts): Given a vector $y \in \mathbb{R}^n$, a vector $x_c \in \mathbb{R}^n$, a (symmetric) postive definite matrix $P \in \mathbb{R}^{n \times n}$, and a positive scalar $r \in \mathbb{R}$, consider the optimization problem

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & ||y - x||_2 \\ \text{s.t.} \quad & (x - x_c)^T P (x - x_c) \leq r, \end{aligned} \tag{2}$$

which finds the point in a given ellipsoid that is closest to y.

(a) Show that (2) is a convex optimization problem.

(b) Show that (2) has a unique optimal solution. (You can assume that an optimal solution exists, which is the case here.)