Name:______________________________

PRINCETON UNIVERSITY

ORF 363/COS 323
Midterm Exam, Fall 2019

OCTOBER 22, 2019, FROM 1:30 PM TO 2:50 PM

Instructor:
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AIs:
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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed (e.g., calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

Each question has 25 points. You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = \frac{1}{2}x_1^2 + x_1 x_2 - \frac{3}{2}x_2^2 + 2x_1 + 5x_2 + \frac{1}{3}x_2^3.$$

Problem 2: An $n \times n$ real symmetric matrix Q is said to be *copositive* if $x^T Q x \geq 0$ for all $x \in \mathbb{R}^n$ such that $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$.

(a) Show that the set of $n \times n$ symmetric copositive matrices is convex.
(b) Show that the set of 2×2 symmetric matrices that are not copositive is not convex.
(c) Show that the matrix

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 3 \\ 0 & 3 & 2 \end{pmatrix}$$

is copositive. (*Hint*: Write this matrix as a sum of two matrices that are more evidently copositive.)

Problem 3: Fill out the following table with "yes" or "no". Fully justify your answers.

	strictly convex	convex	quasiconvex
$f(x_1, x_2) = e^{x_1} + e^{x_2}$			
$f(x_1, x_2) = e^{x_1 + x_2}$			
$f(x_1, x_2) = e^{x_1 x_2}$			

Problem 4: Least squares with a Tikhonov regularizer
Consider the optimization problem

$$\min_{x \in \mathbb{R}^n} \; ||Ax - b||_2^2 + \lambda ||x||_2^2,$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $\lambda \geq 0$ (a scalar) are given.

(a) Is this a convex optimization problem? Why or why not?

(b) Give two different conditions, one on A and one on λ, under which the solution is unique.

(c) Assuming that the solution to the problem is unique, find an expression for the solution in terms of A, b, and λ. (This is similar to the expression we derived in class for the solution of the least squares problem.)